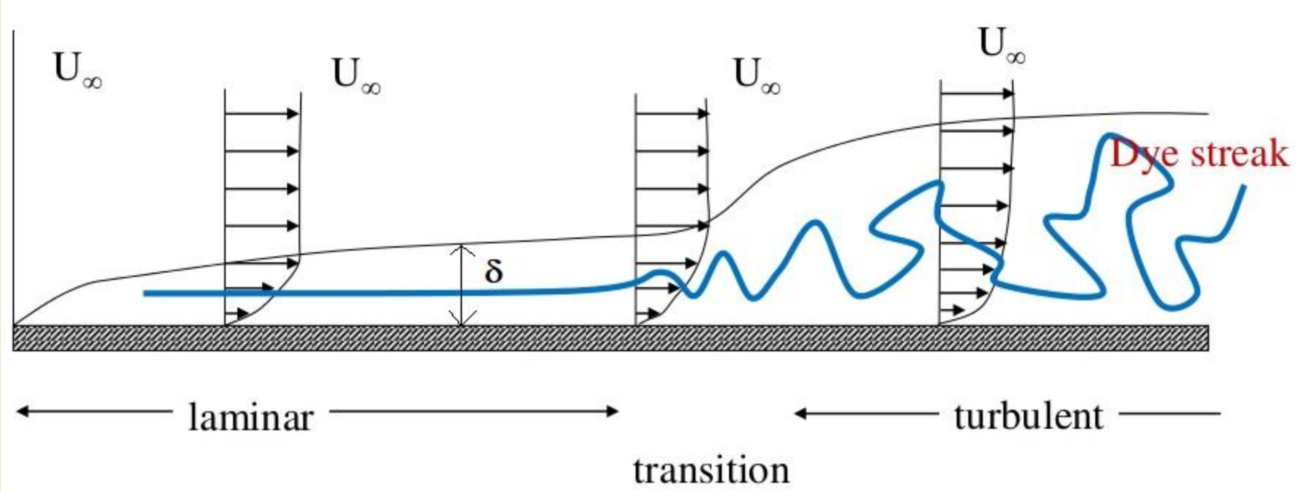
Forced Convection

Physical mechanism of forced convection

- The easiest way to visualize the convective heat transfer to and from a surface is to assume the existence of a stagnant fluid film at the wall, though no such film exists in reality
- In case of heat transfer from a surface, most of the resistance to heat transfer is offered by a narrow zone of fluid near the surface called the *boundary layer*

- Let us consider the flow of fluid over a wide flat plate at zero angle of incidence (plate is oriented along the direction of the flow of the bulk fluid)
- Fluid velocity is zero at the surface of the plate (no slip condition) and gradually increases with distance from the plate
- At sufficiently large distance from the plate, the fluid velocity becomes equal to the free stream velocity
- The region above the plate surface within which this change of velocity from zero to the free stream velocity occurs is called ***velocity or momentum or hydrodynamic boundary layer***
- The thickness of the boundary layer is the distance from the plate at which 99% of free stream velocity is attained and is known as **boundary layer thickness, δ**

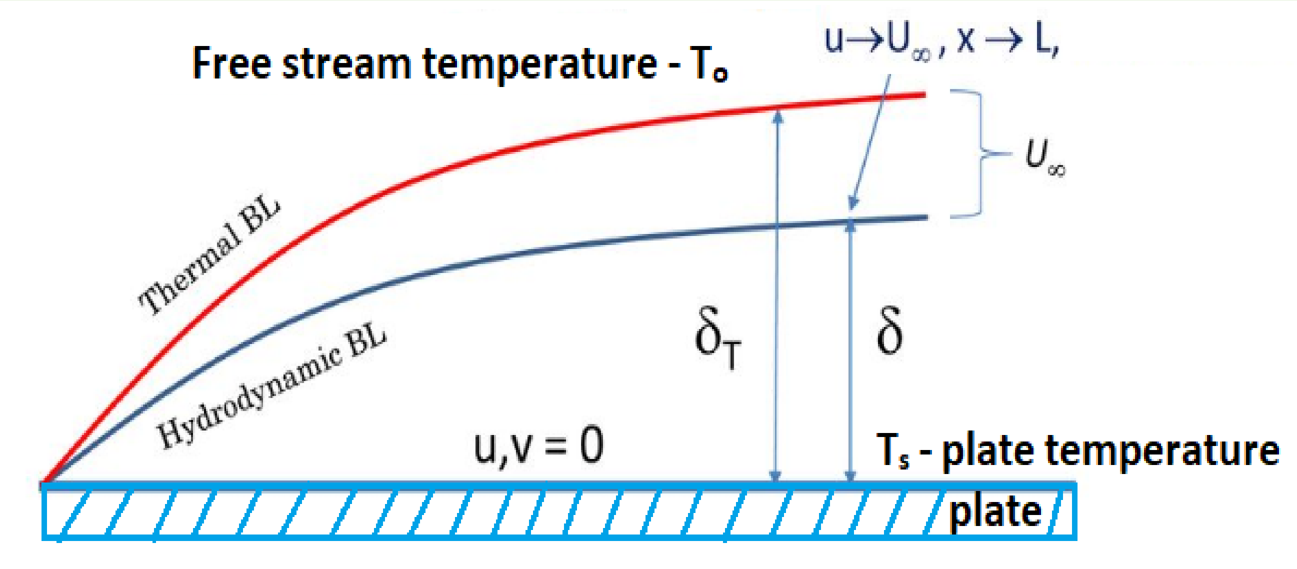


- The characteristics of the boundary layer flow depend upon the Reynolds Number $\left(Re_x = \frac{\rho v x}{\mu}\right)$ where x is the distance from the edge

  Flow is laminar $10^5 < Re_x < 5 \times 10^5$

  Flow is in the transition region $Re_x = 5 \times 10^5\ to\ 10^6$

  Flow is turbulent $Re_x \geq 10^6$
- Heat transfer from a hot plate to a flowing fluid occurs principally by convection
- A thermal boundary layer is formed in the liquid similar to the hydrodynamic boundary layer
- The temperature of the fluid will be same as the temperature of the plate, and at the edge of the thermal boundary layer, the temperature of the fluid is very close to the bulk temperature
- The *thickness of the thermal boundary layer is, however, different from that of the hydrodynamic boundary layer and depends on the thermal properties of the fluid*
- The convective transfer of heat takes place through the boundary layer
- The velocity distribution in a two-dimensional boundary layer governs the heat transfer coefficient in the boundary layer
- When the value of the dimensionless group, Prandtl Number, $\left(Pr = \frac{\mu C_p}{k} = \frac{\mu/\rho}{k/\rho C_p} = \frac{momentum\ diffusivity}{thermal\ diffusivity}\right)$, Pr < 1, the momentum boundary layer remains within the thermal boundary layer. If Pr>1, the reverse is true



Estimation of heat transfer coefficient

- A large number of correlations have been developed over the years to corelate the heat transfer coefficient with relevant properties, parameters and variables of systems
- These relevant properties, parameters and variables are clubbed or grouped into ***dimensionless groups***
- These dimensionless groups are identified by means of ***dimensional analysis*** which is used to corelate the variables that influence a particular physical process

Dimensional Analysis

- The first step to dimensional analysis is to identify the relevant variables influencing the process
- In case of convective heat transfer to a fluid flowing through a circular pipe, it has been found from experiments that heat transfer coefficient, $\boldsymbol{h}$ depends on the fluid velocity, $\boldsymbol{v}$; pipe diameter, $\boldsymbol{d}$; thermal conductivity of the fluid, $\boldsymbol{k}$; specific heat of fluid, $\boldsymbol{C_p}$; density of fluid, $\boldsymbol{\rho}$; and viscosity of fluid, $\boldsymbol{\mu}$
- So, $\boldsymbol{h}$, $\boldsymbol{v}$, $\boldsymbol{d}$, $\boldsymbol{k}$, $\boldsymbol{C_p}$, $\boldsymbol{\rho}$ and $\boldsymbol{\mu}$ are the variables and properties which are then used to form the ***dimensionless groups***
- One of the procedures used to form the dimensionless groups are is the ***Buckingham Pi Theorem***
- The ***Buckingham Pi Theorem*** states that the number of independent dimensionless groups (p) which will be formed by combining physical variables is equal to the difference between the number of physical variables (n) and the number of primary dimensions (m) used to express them

$$p = n - m$$

- The dimensionless groups (π_1, π_2, π_3) can be expressed as $\phi(\pi_1, \pi_2, \pi_3, \ldots\ldots) = 0$
- One group is often written as a function of the other two $\pi_1 = f(\pi_2, \pi_3, \ldots.)$
- Some of the limitations of the ***Buckingham Pi Theorem*** are:
  - The variables which influence the process needs to be known apriori
  - It does not give any information on the mechanism

The application of the ***Buckingham Pi Theorem method*** can be understood by the following example where the ***fluid flow equation is developed*** using this method

- An incompressible fluid is flowing inside a circular tube of inside diameter D, the significant variables are pressure drop, ΔP, velocity, v, diameter, D, tube length, L, viscosity, μ, density, ρ.
- How is ΔP related to the other parameters?

Variable	Unit	Dimension
ΔP	$N/m^2 \left(kg/ms^2\right)$	$ML^{-1}T^{-2}$
v	m/s	LT^{-1}
D	m	L
L	m	L
μ	kg/ms	$ML^{-1}T^{-1}$
ρ	kg/m^3	ML^{-3}

Total no of variables $= n = 6$
Total number of fundamental dimensions used $= m = 3$
Therefore, number of fundamental groups $= 6 - 3 = 3$

$$\pi_1 = f(\pi_2, \pi_3)$$

- A core group of variables (m no of variables) are selected which will appear in each dimensionless group (repeating variable)

  They must contain all the fundamental dimensions

- Usually the repeating variables should not should not form a non-dimensional parameter
- No repeating variable must have the same dimensions
- While choosing the repeating variables, the variable whose effect one desires to isolate is often excluded (eg. ΔP)

- For this case, L and D have the same dimension
- $v, D \; and \; \rho$ are used as repeating variables (as $m = 3$)

$$\pi_1 = D^a v^b \rho^c \Delta P^1$$
$$\pi_2 = D^d v^e \rho^f L$$
$$\pi_3 = D^g v^h \rho^i \mu$$

Considering the first group (π_1)

$$\pi_1 = D^a v^b \rho^c \Delta P^1$$

$$M^o L^o T^o = L^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT^2}\right)^1$$

Equating the exponents, we get,

$$For \quad L \Rightarrow \qquad a+b-3c-1=0$$
$$For \quad M \Rightarrow \qquad 0+0+c+1=0$$
$$For \quad T \Rightarrow \qquad 0-b+0-2=0$$

Solving the equations, we have, $c=-1, b=-2, a=0$

Substituting these values, the first dimensionless group becomes, $\pi_1 = D^0 v^{-2} \rho^{-1} \Delta P^1$

$$\boxed{\pi_1 = \frac{\Delta P}{\rho v^2}}$$

Considering the second group ($\boldsymbol{\pi_2}$) $\pi_2 = D^d v^e \rho^f L$

$$M^o L^o T^o = L^d \left(\frac{L}{T}\right)^e \left(\frac{M}{L^3}\right)^f (L)^1$$

Equating the exponents, we get, $For \quad L \Rightarrow \quad d-e-3f+1=0, \quad For \quad M \Rightarrow \quad f=0, \quad For \quad T \Rightarrow -e=0$

Solving the equations, we have, $\mathrm{e}=0, \ f=0, \ d=-1$

Substituting these values, the second dimensionless group becomes, $\pi_2 = D^{-1} v^0 \rho^0 L^1$

$$\boxed{\pi_2 = \frac{L}{D}}$$

Considering the third group (π_3) $$\pi_3 = D^g v^h \rho^i \mu$$

$$M^o L^o T^o = L^g \left(\frac{L}{T}\right)^h \left(\frac{M}{L^3}\right)^i \left(\frac{M}{LT}\right)^1$$

Equating the exponents, we get,

$$For \quad L \Rightarrow \qquad g + h - 3i - 1 = 0$$

$$For \quad M \Rightarrow \qquad 0 + 0 + i + 1 = 0$$

$$For \quad T \Rightarrow \qquad 0 - h + 0 - 1 = 0$$

Solving the equations, we have, $h = -1, \quad i = -1, \quad g = -1$

Substituting these values, the first dimensionless group becomes, $\pi_3 = D^{-1} v^{-1} \rho^{-1} \mu^1$

$$\pi_3 = \frac{\mu}{Dv\rho}$$

Putting the three groups together we have,

$$\pi_1 = f(\pi_2 \, , \pi_3)$$

$$\left(\frac{\Delta P}{\rho v^2}\right) = f\left[\left(\frac{L}{D}\right)\left(\frac{\mu}{Dv\rho}\right)\right]$$

Buckingham's π-Theorem Method can be applied for forced and free convection processes to determine the **heat transfer coefficient**

- ***Rate of heat transfer by convection*** to an incompressible fluid travelling in ***turbulent flow in a pipe*** has been found to be influenced by v, ρ, k, C_p, μ and d (inner diameter) of the pipe
- What is the relationship between heat transfer coefficient (h) and other variables?

Variable	Unit	Dimension
h	$W/m^2°C$	$M\theta^{-1}T^{-3}$
v	m/s	LT^{-1}
ρ	kg/m^3	ML^{-3}
d	m	L
k	$W/m°C$	$ML\theta^{-1}T^{-3}$
C_p	$J/kg°C$	$L^2T^{-2}\theta^{-1}$
μ	kg/ms	$ML^{-1}T^{-1}$

Total no of variables $= n = 7$
Total number of fundamental dimensions used $= m = 4$
Therefore, number of fundamental groups $= p = 7 - 4 = 3$

$$\pi_1 = f(\pi_2, \pi_3)$$

Repeating variables selected = d, v, k, μ

$$\pi_1 = d^a v^b k^c \mu^e h$$

$$\pi_2 = d^f v^g k^i \mu^j \rho$$

$$\pi_3 = d^l v^m k^n \mu^p C_p$$

Considering the first group (π_1)

$$\pi_1 = d^a v^b k^c \mu^e h$$

$$M^o L^o T^o \theta^o = L^a \left(\frac{L}{T}\right)^b \left(\frac{ML}{T^3\theta}\right)^c \left(\frac{M}{LT}\right)^e \left(\frac{M}{\theta T^3}\right)$$

Equating the exponents, we get,

$$For \quad L \Rightarrow \qquad a + b + c - e + 0 = 0$$
$$For \quad M \Rightarrow \qquad 0 + 0 + c + e + 1 = 0$$
$$For \quad T \Rightarrow \qquad 0 - b - 3c - e - 3 = 0$$
$$For \quad \theta \Rightarrow \qquad 0 + 0 - c + 0 - 1 = 0$$

Solving the equations, we have, $c = -1,\ e = 0,\ b = 0,\ a = 1$

Substituting these values, the first dimensionless group becomes, $\pi_1 = d^1 v^0 k^{-1} \mu^0 h$

$$\boldsymbol{\pi_1 = \frac{hd}{k} = Nusselt\ Number = Nu}$$

Considering the second group (π_2)

$$\pi_2 = d^f v^g k^i \mu^j \rho$$

$$M^o L^o T^o \theta^o = L^f \left(\frac{L}{T}\right)^g \left(\frac{ML}{T^3\theta}\right)^i \left(\frac{M}{LT}\right)^j \left(\frac{M}{L^3}\right)$$

Equating the exponents, we get,

$$For \quad L \Rightarrow \qquad f + g + i - j - 3 = 0$$

$$For \quad M \Rightarrow \qquad 0 + 0 + i + j + 1 = 0$$

$$For \quad T \Rightarrow \qquad 0 - g - 3i - j + 0 = 0$$

$$For \quad \theta \Rightarrow \qquad 0 + 0 - i + 0 + 0 = 0$$

Solving the equations, we have, $\quad i = 0,\ j = -1,\ g = 1,\ f = 1$

Substituting these values, the first dimensionless group becomes, $\quad \pi_2 = d^1 v^1 k^0 \mu^{-1} \rho$

$$\boldsymbol{\pi_2 = \frac{d v \rho}{\mu} = Reynolds\ Number = Re}$$

Considering the third group (π_3)

$$\pi_3 = d^l v^m k^n \mu^p C_p$$

$$M^o L^o T^o \theta^o = L^l \left(\frac{L}{T}\right)^m \left(\frac{ML}{T^3\theta}\right)^n \left(\frac{M}{LT}\right)^p \left(\frac{L^2}{T^2\theta}\right)$$

Equating the exponents, we get,

$$For \quad L \Rightarrow \qquad l + m + n - p + 2 = 0$$

$$For \quad M \Rightarrow \qquad 0 + 0 + n + p + 0 = 0$$

$$For \quad T \Rightarrow \qquad 0 - m - 3n - p - 2 = 0$$

$$For \quad \theta \Rightarrow \qquad 0 + 0 - n + 0 - 1 = 0$$

Solving the equations, we have, $n = -1,\ p = 1,\ m = 0,\ l = 0$

Substituting these values, the first dimensionless group becomes, $\pi_3 = d^0 v^0 k^{-1} \mu^1 C_p$

$$\boldsymbol{\pi_3 = \frac{\mu C_p}{k} = Prandtl\ Number = Pr}$$

Putting the three groups together we have,

$$\pi_1 = f(\pi_2 , \pi_3)$$

$$\left(\frac{hd}{k}\right) = f\left[\left(\frac{dv\rho}{\mu}\right)\left(\frac{\mu C_p}{k}\right)\right]$$

This may be written as,

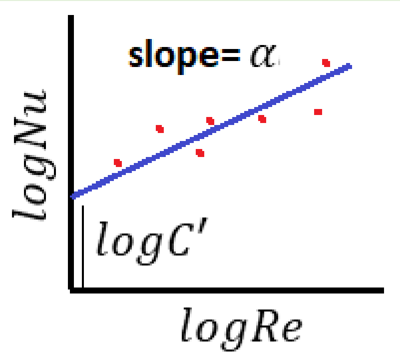
$$\left(\frac{hd}{k}\right) = C\left(\frac{dv\rho}{\mu}\right)^{\alpha}\left(\frac{\mu C_p}{k}\right)^{\beta}$$

or, $\boldsymbol{Nu = C(Re)^{\alpha}(Pr)^{\beta}}$

where $\boldsymbol{C}$ is a constant and $\boldsymbol{\alpha, \beta}$ are exponents

These are determined experimentally

- $\boldsymbol{Pr}$ is kept constant and $\boldsymbol{Re}$ is changed (by changing say, d) and the change in $\boldsymbol{Nu}$ is observed
- $\boldsymbol{Nu = C(Re)^{\alpha}(Pr)^{\beta} = C'(Re)^{\alpha}}$ or $\boldsymbol{logNu = logC' + \alpha logRe}$
- $\boldsymbol{C'}$ (and hence $\boldsymbol{C}$) and $\boldsymbol{\alpha}$ are determined from slope and intercept
- Similarly, by altering $\boldsymbol{Pr}$, keeping $\boldsymbol{Re}$ constant, one can find $\boldsymbol{\beta}$



- **The values of C, α and β depend on the type of flow conditions and the system (geometry) through which the fluid flows**