Forced Convection

Dimensionless groups in heat transfer

This is a list of different dimensionless groups that are frequently used in forced convective heat transfer and their significance

(1) Reynolds Number

$$Re = \frac{Lv\rho}{\mu} = \frac{\rho v^2}{\mu v / L} = \frac{inertial\ force\ per\ unit\ cross-sectional\ area}{viscous\ force\ per\ unit\ cross-sectional\ area}$$

- It is a measure of the relative magnitude of the **inertial force to the viscous force** occurring in a flow
- ρv^2 is the momentum flow through a tube of unit cross-sectional area (inertial force per unit area)
- v/L is the magnitude of the velocity gradient occurring and $\mu v/L$ is the shear stress due to action of viscosity

- The higher the Reynolds number the greater is the relative contribution of the inertial effect
- The smaller the number, greater will be the relative magnitude of the viscous stresses

$$Re = \frac{Lv\rho}{\mu} = \frac{\rho v^2 L^2}{\mu v L} = \frac{inertial\ force}{viscous\ force}$$

$$\left(\frac{kg}{m^3} \times \frac{m^2}{s^2} \times m^2 / \frac{kg}{m.s} \times \frac{m}{s} \times m\right)$$

L is the characteristic length for pipes it is diameter, D

(2) Prandtl Number

$$Pr = \frac{\mu C_p}{k} = \frac{\mu/\rho}{k/\rho C_p} = \frac{\nu}{\alpha} = \frac{momentum\ diffusivity}{thermal\ diffusivity}$$

- Prandtl Number depends on the fluid and its state
- The momentum diffusivity (or kinematic viscosity) indicates the impulse transport through molecular friction whereas thermal diffusivity is the heat energy transport through conduction
- Pr signifies the '**relative speed with which momentum and energy are propagated through a fluid**'
- It is the connecting link between velocity field and temperature field
- In heat transfer problems, Pr controls the relative thickness of the hydrodynamic and thermal boundary layers
- When Pr is small, heat diffuses faster compared to velocity

Thermal diffusivity, α

$$\alpha = \frac{k}{\rho C_p} = \frac{heat\ conducted}{heat\ stored}$$

α is the property of a material which represents how fast heat diffuses through a material

A material having high k or low C_p will have high α

A small value of α means heat is mostly absorbed by the material and only small amount of heat is conducted further

The larger the α, faster is the propagation of heat into the medium

(3) Nusselt Number

$$Nu = \frac{hL}{k} = \frac{h}{k/L} = \frac{convective\ heat\ flow\ rate\ through\ unit\ temperature\ gradient}{conductive\ heat\ flow\ rate\ through\ unit\ temperature\ gradient\ and\ a\ stationary\ thickness\ of\ L}$$

- Heat flux for convection = $q_{convection} = h\Delta T$
- Heat flux for conduction = $q_{conduction} = {}^{k}/_{L}\,\Delta T$
- A higher Nu indicates more effective convective heat transfer
- Nu is usually a function of Re and Pr

$$Nu = \frac{hL}{k} = \frac{L}{k/h} = \frac{characteristic\ length\ of\ the\ system}{equivalent\ conducting\ film\ thickness}$$

$$\delta = {}^{k}/_{h}$$

$$h = \frac{k}{\delta}$$

(4) Stanton Number

$$St = \frac{h}{v\rho C_p} = \frac{hA\Delta T}{(v\rho A)(C_p)(\Delta T)} = \frac{h\Delta T}{(\dot{m})(C_p)(\Delta T)} = \frac{heat\ tranfer\ due\ to\ convection}{rate\ of\ heat\ transfer\ by\ bulk\ flow}$$

- Rate of heat transfer by bulk flow = mass flow rate × specific heat x temperature change

$$St = \frac{\left({}^{hL}/_{k}\right)}{\left({}^{Lv\rho}/_{\mu}\right)\left({}^{C_p\mu}/_{k}\right)} = \frac{Nu}{(Re)(Pr)}$$

(5) Peclet Number

$$Pe = \frac{\rho v L C_p}{k} = \frac{heat\ ransfer\ by\ bulk\ flow}{rate\ of\ heat\ transfer\ by\ conduction}$$

$$Pe = \frac{(\dot{m})(C_p)(\Delta T)}{\left(kA\Delta T/L\right)} = \frac{(\dot{m})(C_p)(L)}{(kA)} = \frac{(v\rho A)(C_p)(L)}{(kA)} = \frac{\rho v L C_p}{k}$$

$$Pe = \frac{\rho v L/\mu}{k/\mu C_p} = Re.Pr$$

(6) Graetz Number

$$Gr \propto Pe.\frac{d}{l} \propto \frac{\rho v d C_p}{k}.\frac{d}{l}$$

Graetz number characterizes laminar flow in a pipe

$$Gr = \frac{heat\ capacity\ of\ the\ fluid\ flowing\ through\ the\ pipe\ per\ unit\ length}{conductivity\ of\ the\ pipe}$$

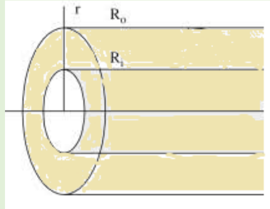
$$Gr = \frac{\left(\rho v A C_p/l\right)}{k} = \frac{\pi d^2}{4}\frac{\left(\rho v C_p/l\right)}{k} = \frac{\pi}{4}.\frac{(d^2 \rho v C_p)}{k.l} = \frac{\pi}{4}\left(\frac{dv\rho}{\mu}\right)\left(\frac{C_p\mu}{k}\right)\left(\frac{d}{l}\right) = \frac{\pi}{4}(Re)(Pr)\left(\frac{d}{l}\right)$$

Two other dimensionless numbers. Grashoff No and Biot No are related to natural convection and unsteady state heat transfer, respectively

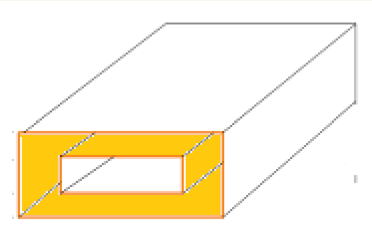
Characteristic length, Equivalent diameter

- In several of the dimensionless numbers discussed earlier such as Re, Nu, Pe etc, we come across the term ***characteristic length***
- Characteristic length is defined based on the geometry of the system
- For a circular pipe, the pipe diameter is taken as the characteristic length
- For flow past a flat plate, the distance from the leading edge is the characteristic length
- However, in case of geometries such as a rectangular duct, annulus between two pipes, rectangular annulus, the ***equivalent diameter*** is used as the characteristic length

Square/rectangular duct



annulus between two pipes



rectangular annulus

- For such non-circular geometries, equivalent diameter (d_e) is four times the hydraulic radius (r_H)

$$d_e = 4 \times r_H$$

$$r_H = \frac{cross - sectional\ area\ of\ flow}{wetted\ perimeter}$$

- **Equivalent diameter is the *diameter of a circular duct which would have the same resistance against flow (same pressure drop) or would have the same heat transfer as the non-circular duct under equal and comparable conditions***

- In a rectangular duct of length L and breadth B,

$$d_e = 4 \times r_H = 4 \times \frac{cross\text{–}sectional\ area\ of\ flow}{wetted\ perimeter} = 4 \times \frac{L \times B}{2(L+B)} = \frac{2LB}{(L+B)}$$

- It is important to remember that the *estimation of equivalent diameter is sometimes dependent on the process (or where it is applied or used)*
- For eg., in case of two concentric pipes forming a double pipe heat exchanger, the hot fluid flows through the inner tube and the cold fluid through the annulus
- If we need to *calculate the pressure drop for flow through the annulus*, the equivalent diameter can be estimated as,

$$d_e = 4 \times \frac{\pi/4\left(D_i^2 - d_o^2\right)}{\pi(D_i + d_o)} = (D_i - d_o)$$

Where d_i= inner diameter of inner tube

d_o= outer diameter of inner tube

D_i= inner diameter of outer tube

D_o= outer diameter of outer tube

- However, if *estimation of heat transfer is the objective*, then the wetted perimeter becomes different, as transfer of heat takes place only from the outer surface of the inner pipe
- Thus, equivalent diameter in this case is,

$$d_e = 4 \times \frac{\pi/4\left(D_i^2 - d_o^2\right)}{\pi d_o} = \frac{\left(D_i^2 - d_o^2\right)}{d_o}$$