Forced Convection

Correlations for heat transfer coefficient

- Systems of different shapes and different flow conditions are encountered in practical heat transfer calculations
- The most common shapes are – cylindrical, spherical, flat
- Flow through circular, non-circular ducts, through packed beds, in an agitated vessel, across a bank of tubes are some common heat transfer problems
- The flow of fluid may be internal (through a pipe) or external (across outer surface of the pipe)
- Several empirical correlations are available for estimating the heat transfer coefficients for the different cases

(I) For internal flows

(a) Laminar flow through a circular pipe

- In industrial equipment used for heating and cooling a considerably viscous liquid, the flow is usually laminar ($Re < 2100$)
- The correlation given by **Sieder and Tate** (1936) is used for such a case

$$\boxed{\boldsymbol{Nu = 1.86\left[\frac{(Re)(Pr)}{L/d}\right]^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}}}$$

The correlation is applicable when

(a) $0.48 < Pr < 16700$

(b) $0.0044 < \frac{\mu}{\mu_w} < 9.75$

(c) $(Pe.\,{}^{d}/_{L}) > 10^8$

L = pipe length
d = pipe diameter
μ = viscosity of fluid at bulk temperature
μ_w = viscosity of fluid at wall temperature

Fluid properties are evaluated at mean bulk temperature of the fluid

(b) Turbulent flow through a circular pipe

- For fully developed turbulent flow through a pipe, the correlation used is the **Dittus-Boelter equation** (1930)

$$\boxed{\boldsymbol{Nu = 0.023(Re)^{0.8}(Pr)^{n}}}$$

$$\boxed{\begin{array}{l} n = 0.4 \text{ for heating } (T_w > T) \\ n = 0.3 \text{ for cooling } (T_w < T) \end{array}}$$

The correlation is applicable when

(a) $0.7 \le Pr \le 160$ (b) $^{d}/_{L} < 0.1$

(c) $(Re) \ge 10{,}000$ (d) $(T_w - T)$ is moderate

Fluid properties are evaluated at the average of the inlet and outlet temperatures of the fluid

- When the difference between T_w and T is substantial, its effect on fluid properties especially viscosity, needs to be accounted – the **Sieder-Tate equation** is used in such a case

$$\boxed{\boldsymbol{Nu = 0.027(Re)^{0.8}(Pr)^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14}}}$$

μ = viscosity of fluid at bulk temperature

μ_w = viscosity of fluid at wall temperature

$\left(\frac{\mu}{\mu_w}\right)$ = viscosity correction factor

The correlation is applicable when

(a) $0.7 \le Pr \le 16700$ (b)$^{d}/_{L} \le 0.1$

(c) $(Re) \ge 10{,}000$

(c) Flow through a circular pipe in the transition region (2100 < (Re)< 10,000)

$$\boxed{\boldsymbol{Nu = 0.116\left[(Re)^{2/3} - 125\right](Pr)^{1/3}\left[1 + \left(\frac{d}{L}\right)^{2/3}\right]\left(\frac{\mu}{\mu_w}\right)^{0.14}}}$$

(II) For external flows

(a) Flow over a flat plate

- Heat transfer in flow over a flat plate occurs through the boundary layer formed on the plate
- For ***heat transfer in laminar boundary layer flow***, the following correlation can be obtained for the local Nusselt No from approximate solution of the boundary equations of momentum and energy

$$\boxed{\boldsymbol{Nu_x = 0.332(Re_x)^{1/2}(Pr)^{1/3}}}$$

x = the distance from the leading edge of the plate

$$\boldsymbol{Nu_x = \frac{xh_x}{k}}$$

$$\boldsymbol{Re_x = \frac{xv\rho}{\mu}}$$

The correlation is applicable when

(a) $0.5 \le Pr \le 50$

(b) $Re_x < 3 \times 10^5$

- For ***heat transfer in turbulent boundary layer flow***, the following correlation is used

$$\boxed{\boldsymbol{Nu_x = 0.0296(Re_x)^{4/5}(Pr)^{1/3}}}$$

The correlation is applicable when

(a) $0.6 \le Pr \le 60$

(b) $5 \times 10^5 < Re_x < 10^7$

(b) Flow across a cylinder

- There are several correlations available for convective heat transfer in flow across a cylinder (cross flow of fluid over a cylinder and for average Nu)

- Correlation proposed by **Fand** (1965)

$$\boxed{\boldsymbol{Nu = \left[0.35 + 0.56(Re)^{0.52}\right](Pr)^{0.3}}}$$

$$0.1 < Re < 10^5$$

- Correlation of **Churchill and Burnstein** (1977)

$$\boxed{\boldsymbol{Nu = 0.3 + \frac{0.62(Re)^{1/2}(Pr)^{1/3}}{\left[1 + \left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re}{2.82 \times 10^5}\right)^{1/2}\right]}}$$

(a) $2 \times 10^4 < Re < 4 \times 10^5$
(b) $(Re)(Pr) > 0.2$

- Correlation by **Whitaker** (1972)

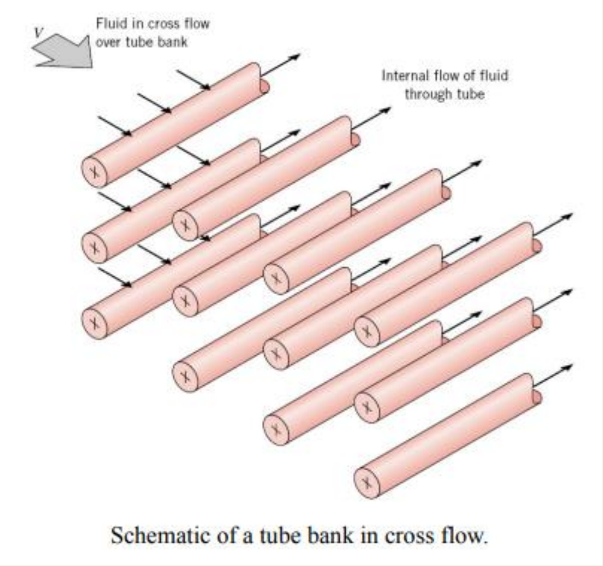
$$\boxed{\boldsymbol{Nu = \left[0.40(Re)^{1/2} + 0.06(Re)^{2/3}\right](Pr)^{0.4}\left(\frac{\mu}{\mu_w}\right)^{0.25}}}$$

(a) $0.65 < Pr < 300$

(b) $40 < Re < 105$

(c) $0.25 < \frac{\mu}{\mu_w} < 5.2$

(c) Flow across a bank of tubes



Schematic of a tube bank in cross flow.

- Cross flow of a fluid (such as air) over a tube bank (collection of tubes) is seen in applications like waste heat recovery from hot flue gases, flow of hot combustion gases across the tube bank in a waste heat boiler etc
- The tube banks may be **aligned** or **staggered**
- Correlation proposed by **Zhukauskas** (1972)

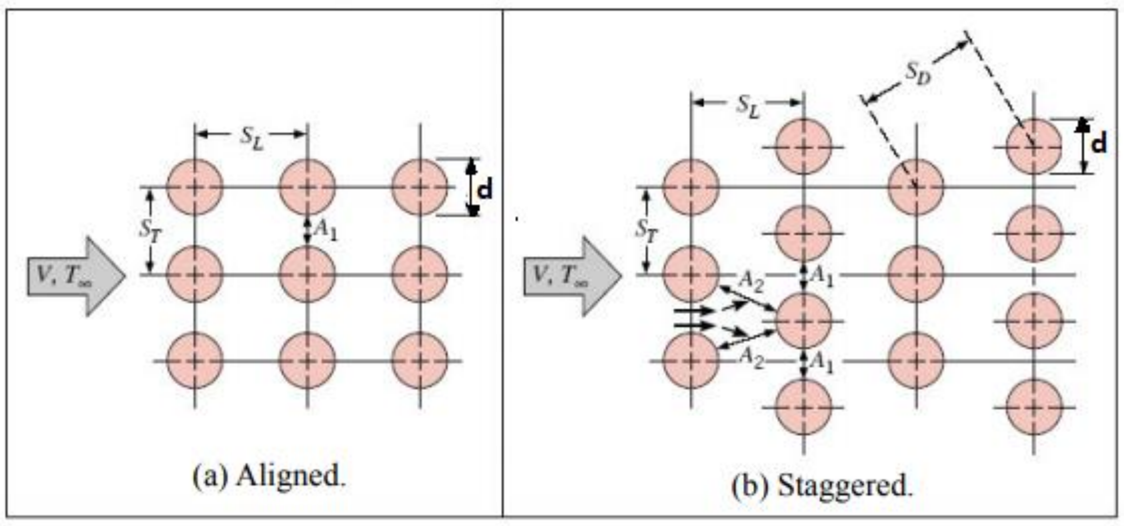
$$\boxed{\boldsymbol{Nu = C\left(Re_{d,max}\right)^{m}(Pr)^{0.36}\left(\frac{Pr}{Pr_w}\right)^{0.25}}}$$

$$Re_{d,max} = \frac{(d\rho V_{max})}{\mu}$$

Pr is the bulk Prandtl No, Pr_w is the wall Prandtl No

$$V_{max} = \frac{S_T}{S_T - d}V \qquad \text{(for aligned arrangement)}$$

$$V_{max} = \frac{S_T}{2(S_T - d)}V \qquad \text{(for staggered arrangement)}$$



(a) Aligned.
(b) Staggered.

Aligned : Re: $10^3 - 2 \times 10^5$, C: 0.27, $m = 0.63$
Staggered : Re: $2 \times 10^5 - 2 \times 10^6$, C: 0.21, $m = 0.84$

(d) Flow past a sphere

- Following correlations are available for flow over a single sphere

  (i) Flow of liquid past a sphere, **Krammer's correlation**

$$\boxed{\boldsymbol{Nu = \left[0.97 + 0.68(Re)^{0.5}\right](Pr)^{0.3}}}$$

  (ii) Correlation by **Whitaker**

$$\boxed{\boldsymbol{Nu = 2 + \left[0.4(Re)^{1/2} + 0.06(Re)^{2/3}\right](Pr)^{0.4}\left(\frac{\mu}{\mu_w}\right)^{0.25}}}$$

(e) Heat transfer coefficient in a packed and a fluidized bed

- Heat transfer to or from a gas flowing through a packed bed of solid has application like heat generated in an exothermic reaction in a gas-solid catalytic reactor, drying moist solid etc

- For gas flow through a **packed bed**, heat transfer correlation is given as,

$$\boxed{\boldsymbol{\varepsilon(St)(Pr)^{2/3} = 2.06(Re)_{d_p}^{-0.575}}}$$

(a) $Pr \approx 0.7$
(b) $90 \leq (Re)_{d_p} \leq 4000$

$$St = \frac{Nu}{(Re)(Pr)} \qquad (Re)_{d_p} = \frac{(d_p v_o \rho)}{\mu}$$

where v_o = superficial fluid velocity d_p = particle diameter ε = bed porosity

- For a **fluidized bed**, heat transfer between the fluid and the solid (as in fluidized bed reactors, dryers etc), the following correlation is used,

$$\boxed{\left(\frac{hd_p}{k}\right) = 2 + 0.6\left(\frac{d_p v_o \rho}{\mu}\right)^{1/2}\left(\frac{C_p \mu}{k}\right)^{1/3}}$$

where v_o = superficial fluid velocity d_p = particle diameter ε = bed porosity