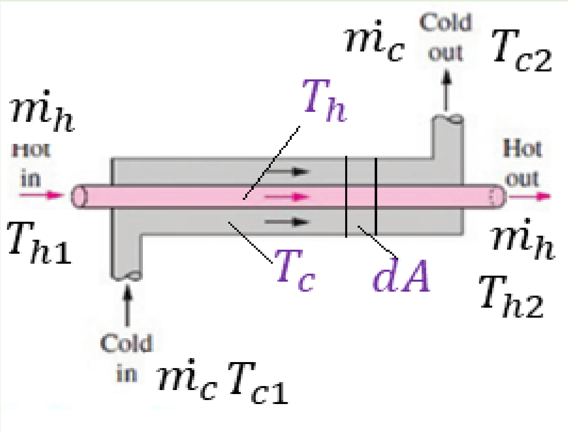
Forced Convection

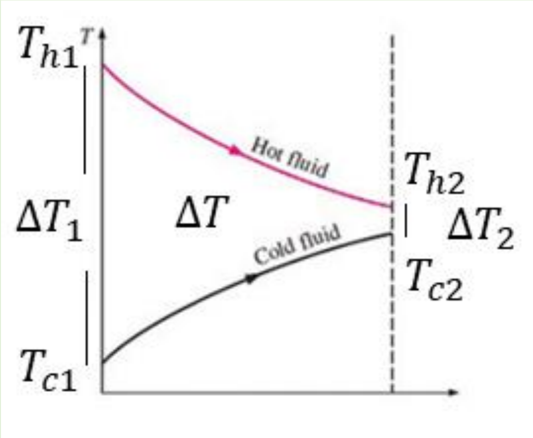
Heat transfer with a variable driving force

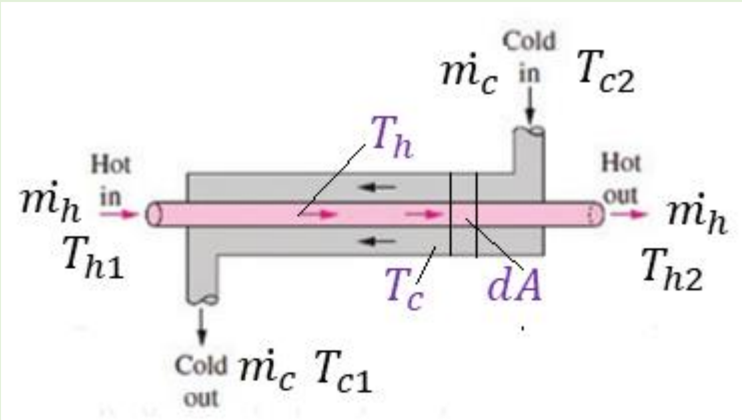
- The variation of the temperature driving force with position in a heat transfer equipment is quite common
- When a fluid flows along the axis of a tube and absorbs or transmits sensible heat, the temperature of the fluid varies over the entire length of the pipe and so does the driving force for heat transfer
- In case of a double pipe exchanger, both the hot and cold fluids can flow in the same direction (parallel or co-current flow) or in opposite directions (counter-current flow)
- To determine the heat flow we use the equation

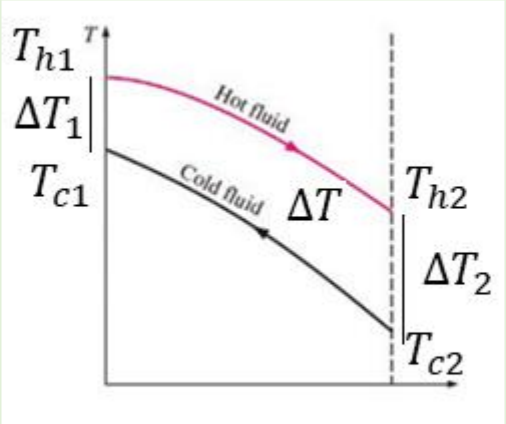
$$Q = UA\Delta T$$

- As the ΔT in case of a double pipe exchanger varies along the length of a pipe, an expression is estimated to determine the average driving force
- The temperature profiles for the two cases are:









- Now, heat transferred through an element dA

$$dQ = -\dot{m}_h C_h dT_h = \dot{m}_c C_c dT_c \quad .... (1)$$

- Heat transferred through an element dA can also be represented as, $dQ = UdA(T_h - T_c)$ .... (2)

- From equation (1) $\qquad dT_h = \frac{-dQ}{\dot{m}_h C_h}$

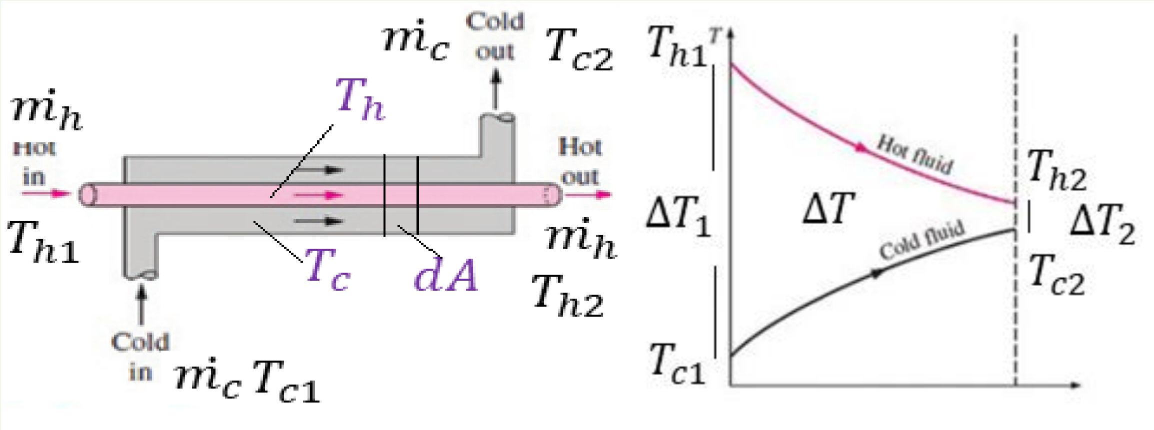
  and $\qquad dT_c = \frac{dQ}{\dot{m}_c C_c}$

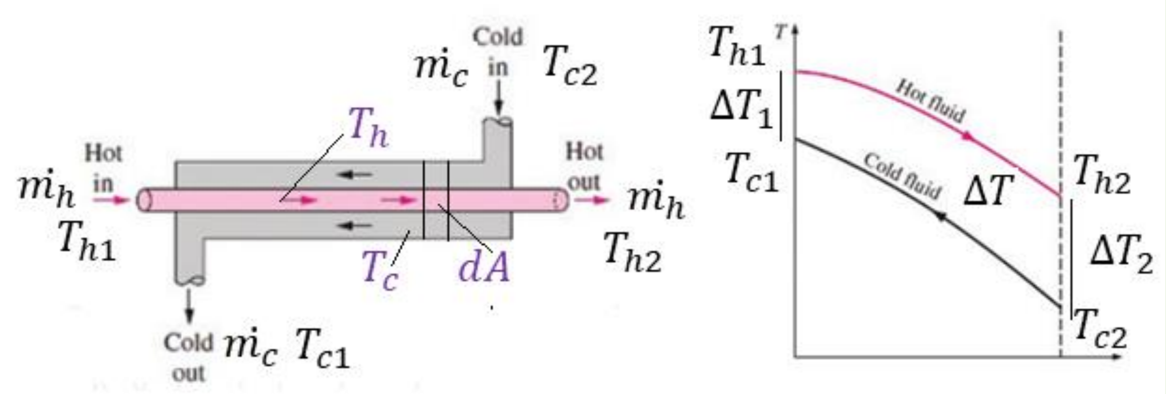
- Also, $dT_h - dT_c = d(T_h - T_c) = -dQ\left[\frac{1}{\dot{m}_h C_h} + \frac{1}{\dot{m}_c C_c}\right]$ .....(3)

- Putting the value of dQ from (2) into (3) we have,

$$d(T_h - T_c) = -UdA(T_h - T_c)\left[\frac{1}{\dot{m}_h C_h} + \frac{1}{\dot{m}_c C_c}\right]$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -UdA\left[\frac{1}{\dot{m}_h C_h} + \frac{1}{\dot{m}_c C_c}\right] \quad .....(4)$$





Integrating equation (4) between conditions 1 and 2 we have,

$$ln\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} = -UA\left[\frac{1}{\dot{m}_h C_h} + \frac{1}{\dot{m}_c C_c}\right] \quad .... (5)$$

- Equation (1) can be rewritten as

$$Q = -\dot{m}_h C_h (T_{h2} - T_{h1}) = \dot{m}_c C_c (T_{c2} - T_{c1}) \quad \text{........ (6)}$$

- Replacing values of $\dot{m}_h C_h$ and $\dot{m}_c C_c$ from equation (6) into equation (5)

$$ln\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} = -UA\left[\frac{-(T_{h2} - T_{h1})}{Q} + \frac{(T_{c2} - T_{c1})}{Q}\right]$$

Or,

$$Q = UA\frac{[(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})]}{ln\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})}} \quad \text{........ (7)}$$

Comparing this with, $Q = UA(\Delta T_m)$

$$\Delta T_m = \frac{[(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})]}{ln\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})}}$$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\frac{\Delta T_2}{\Delta T_1}}$$

This temperature difference is called the log mean temperature difference (LMTD)

- If ΔT_1 and ΔT_2 are nearly equal, their arithmetic average can be used as ΔT_m
- If one of the fluids is at a constant temperature (as in a phase change condition), there is no difference between a counter-current and a co-current flow condition

Problem

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg°C in a double pipe exchanger. The oil enters the exchanger at 110°C and leaves at 85°C. The overall heat transfer coefficient is 320 W/m^2°C. Calculate the exchanger area for (a) counter-current and (b) co-current flow

Also, estimate the flow rate of oil ($C_p$of H_2O = 4.18 kJ/kg°C)

Total heat transfer, $Q = -\dot{m}_h C_h (T_{h2} - T_{h1}) = \dot{m}_c C_c (T_{c2} - T_{c1})$

$$Q = \frac{68}{60}\left(\frac{kg}{s}\right) \times 4.18 \left(\frac{kJ}{kg°C}\right) \times (75 - 35) = 189.49 \text{ kW}$$

$$189.49 \times 1000 = \dot{m}_h \times 1.9 \times (110 - 85)$$

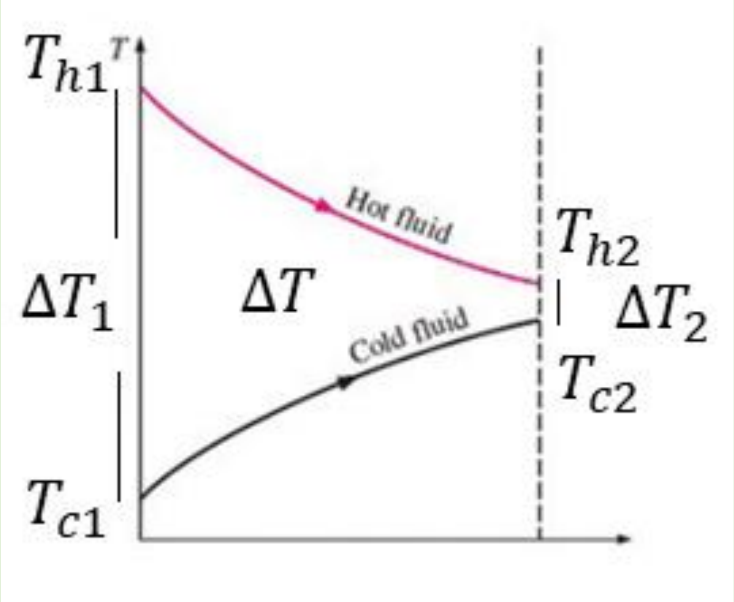
$$\dot{m}_h = 3989.26 \frac{kg}{s} = 66.89\ kg/min$$

(a) Co-current flow

T_{h1} = 110°C T_{h2}= 85°C T_{c1}= 35°C T_{c2}= 75°C

$\Delta T_1 = 110 - 35 = 75°C$ $\Delta T_2 = 85 - 75 = 10°C$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\frac{\Delta T_2}{\Delta T_1}} = \frac{10 - 75}{ln\frac{10}{75}} = 32.26°C$$



(a) Co-current flow

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\frac{\Delta T_2}{\Delta T_1}} = \frac{10 - 75}{ln\frac{10}{75}} = 32.26°C$$

Now, $Q = UA(\Delta T_m)$

$$\boldsymbol{A_{co-current}} = \frac{Q}{U(\Delta T_m)} = \frac{189.49 \times 1000}{320 \times 32.26} = \boldsymbol{18.36\ m^2}$$

(b) Counter-current flow

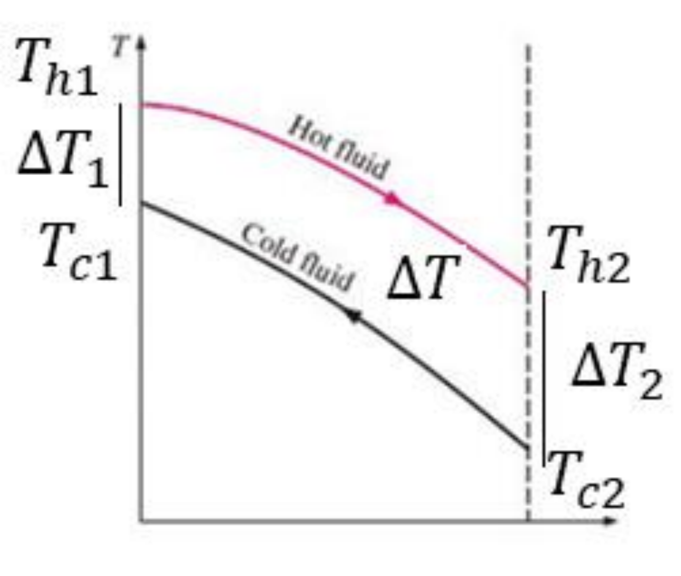
T_{h1} = 110°C T_{h2}= 85°C T_{c1}= 75°C T_{c2}= 35°C

$\Delta T_1 = 110 - 75 = 35°C$ $\Delta T_2 = 85 - 35 = 50°C$

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\frac{\Delta T_2}{\Delta T_1}} = \frac{50 - 35}{ln\frac{50}{35}} = 42.06°C$$

Now, $Q = UA(\Delta T_m)$

$$\boldsymbol{A_{counter-current}} = \frac{Q}{U(\Delta T_m)} = \frac{189.49 \times 1000}{320 \times 42.06} = \boldsymbol{14.08\ m^2}$$



Area requirement for counter-current exchanger is lower

Problem

A 41 mm inner diameter pipe with pipe thickness = 3.5 mm, carries water flowing at a rate of 1 kg/s. Water enters the pipe at 28°C and is heated by a stream of hot flue gas in cross-flow over the pipe. The gas velocity is 10 m/s. The arrangement essentially aims at recovering some of the waste heat of the gas stream which has a bulk temperature of 250°C. The length of the pipe is 20 m. At what temperature does the water leave the pipe?

The properties of the flue gas are about the same as those of air. Thermal conductivity of the pipe material = 45 kcal/h m°C

In this problem, the driving force for heat transfer is variable

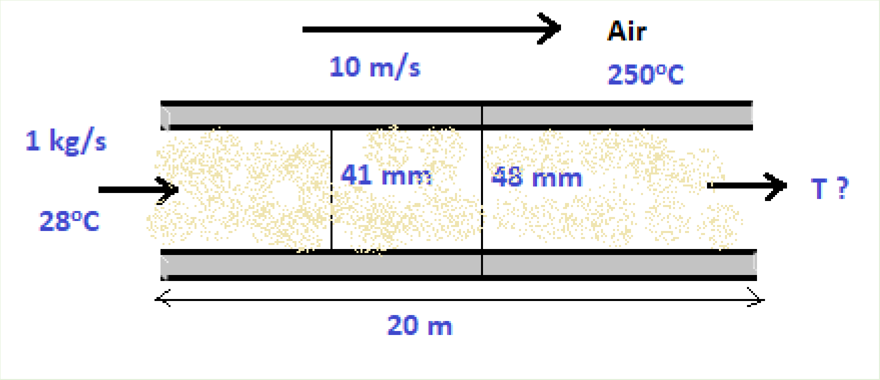
It is maximum at the inlet of the pipe and decrease along the length as the temperature of water rises

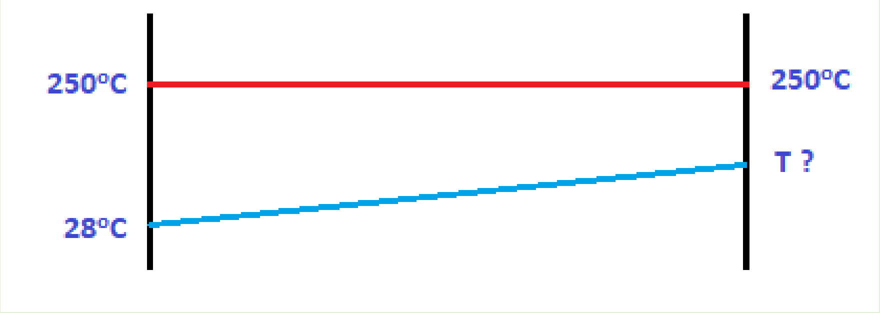
In order to calculate the outlet temperature, it is necessary to calculate the heat transfer coefficient of the water side in the pipe and flue gas (or air) outside the pipe

The physical properties of water and air used for this purpose is:

Water: $density\ (\rho) = 996\frac{kg}{m^3},\quad C_p = 1\frac{kcal}{kg}°C,\quad \mu = 8.6\times10^{-4}\frac{kg}{ms},$
$k = 0.528\ kcal/hm°C$

Air(Flue gas): $density\ (\rho) = 0.891\frac{kg}{m^3},\quad C_p = 0.243\frac{kcal}{kg}°C,$
$\mu = 2.33\times10^{-5}\frac{kg}{ms},\quad k = 0.0292\ kcal/hm°C$





The **heat transfer coefficient of the water side** is determined in the following manner:

The flow of water is through a pipe; the Reynolds No is first calculated to determine whether it is laminar or turbulent

Cross-sectional area of pipe $= \frac{\pi}{4} \times d_i^2 = \frac{\pi}{4} \times (41 \times 10^{-3})^2 = 1.32 \times 10^{-3}\ m^2$

Velocity of water in pipe $= \frac{\dot{m}}{\rho A} = \frac{1\ kg/s}{996 \frac{kg}{m^3} \times 1.32 \times 10^{-3}\ m^2} = 0.761\ m/s$

Reynolds No $= Re = \frac{d_i v \rho}{\mu} = \frac{41 \times 10^{-3} \times 0.761 \times 996}{8.6 \times 10^{-4}} = 36135\ (turbulent)$

The Dittus-Boelter equation can be used to determine h_i

$$\boldsymbol{Nu = 0.023 (Re)^{0.8} (Pr)^{0.4}}$$

Prandtl No $= Pr = \frac{\mu C_p}{k} = \frac{8.6 \times 10^{-4} \times 1}{0.528/3600} = 5.86$

$$\boldsymbol{Nu = \frac{h_i d_i}{k} = 0.023 (Re)^{0.8} (Pr)^{0.4}}$$

$$\boldsymbol{\frac{h_i d_i}{k} = 0.023 (36135)^{0.8} (5.86)^{0.4} = 206.65}$$

$$\boldsymbol{h_i = \frac{0.528}{41 \times 10^{-3}} \times 206.65 = 2661.25\ kcal/hm^2 {}^\circ C}$$

The **heat transfer coefficient of the air (flue gas) side** is determined as:

For the gas side, a correlation is chosen flow past a cylinder;

The Reynolds No (based on the outer diameter of tube) is first calculated to determine the correct equation to use

Reynolds No $= Re = \frac{d_o v \rho}{\mu} = \frac{48\times10^{-3}\times10\times0.891}{2.33\times10^{-5}} = 18355.36$

Prandtl No $= Pr = \frac{\mu C_p}{k} = \frac{2.33\times10^{-5}\times0.243}{0.0292/3600} = 0.698$

As $Re < 10^5$, the **Fand correlation** seems to be the appropriate one to use to calculate h_o

$$\boldsymbol{Nu = \left[0.35 + 0.56(Re)^{0.52}\right](Pr)^{0.3}}$$

$$\boldsymbol{Nu = \frac{h_o d_o}{k} = \left[0.35 + 0.56(18355.36)^{0.52}\right](0.698)^{0.3} = 83.203}$$

$$\boldsymbol{h_o = \frac{0.0.0292}{48 \times 10^{-3}} \times 83.203 = 50.62\ kcal/hm^2}°\mathrm{C}$$

Now, overall heat transfer coefficient is calculated using the following equation,

$$\boldsymbol{\frac{1}{U_o} = \frac{r_o}{r_i h_i} + \frac{r_o ln\left({r_o}/{r_i}\right)}{k_w} + \frac{1}{h_o}}$$

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$$\frac{1}{U_o} = \frac{48 \times 10^{-3}}{41 \times 10^{-3} \times 2661.25} + \frac{48 \times 10^{-3} ln\left({48 \times 10^{-3}}/{41 \times 10^{-3}}\right)}{45} + \frac{1}{50.62}$$

$$\boldsymbol{U_o = 49.314\ \ kcal/hm^2℃}$$

Rate of heat transfer, $Q = \left(mC_p\Delta T\right)_{water} = 1 \times 1 \times (T - 28)\ kcal/s℃ = 3600(T_2 - 28)\ kcal/h℃$

Rate of heat transfer, $Q = U_o A_o(LMTD) = 49.314 \times \pi \times 48 \times 10^{-3} \times 20 \times \left[\frac{(250-28)-(250-T_2)}{ln\left(\frac{250-28}{250-T_2}\right)}\right]$

$$(T_2 - 28) = 0.041313 \times \left[\frac{(250 - 28) - (250 - T_2)}{ln\left(\frac{250 - 28}{250 - T_2}\right)}\right]$$

T_2	LHS	RHS
40	12	8.9213
38	10	8.96
37	9	8.984 ~ 9

$$\boldsymbol{T_2 = 37}℃$$