Forced Convection

Momentum and Heat Transfer Analogies

- There has been a substantial amount of fairly accurate theoretical studies on fluid friction and heat transfer in the laminar flow regime
- Studies in turbulent flow , however, is mostly empirical
- Reynolds in 1874, pointed out that there was probably a relationship between the transfer of heat between a hot fluid and a surface and fluid friction
- There are some advantages in having an equation relating heat transfer and fluid friction
- For most cases, fluid friction experiments are simpler than heat transfer experiments

- It is a well known fact that the ***basic laws of transport of momentum and heat can be expressed in a similar form***
- Let us consider the flow of a fluid in laminar motion through a circular pipe – the wall of the pipe is at a higher temperature and the fluid gets heated as it flows through the pipe
- For transport of momentum,

According to Newton's law of viscosity, $\tau = -\mu\left(\frac{du}{dr}\right)$ where μ is the shear stress (momentum flux)

$$\tau = -\frac{\mu}{\rho}\frac{d}{dr}(\rho u) = -\nu\frac{d}{dr}(\rho u)$$

Momentum flux = (Momentum diffusivity)(Gradient of momentum concentration)

- For transport of heat,

According to Fourier's law, $q = k\left(\frac{dT}{dr}\right)$

$$q = \frac{k}{\rho C_p}\frac{d}{dr}(\rho C_p T) = \alpha\frac{d}{dr}(\rho C_p T)$$

Heat flux = (Thermal diffusivity)(Gradient of concentration of heat energy)

- However, the transport of momentum or heat (or mass) in a 'turbulent medium' are not governed by the above simple laws – a turbulent medium has randomly moving fluid elements called 'eddies'

- Reynolds suggested a simple way of expressing the momentum or heat flux in a turbulent medium

Turbulent transport of momentum, $\tau = -(\nu + \varepsilon_M)\frac{d}{dr}(\rho u)$

Turbulent transport of heat, $q = -(\alpha + \varepsilon_H)\frac{d}{dr}(\rho C_p T)$

At the wall $(r = R)$, shear stress is the wall shear stress

$$-(\nu + \varepsilon_M)\left[\frac{d(\rho u)}{dr}\right]_{r=R} = \tau_w$$

and $\tau_w = \frac{1}{2} f \rho v^2$

f = Fanning friction factor

$$-\left[\frac{d(\hat{u})}{dr}\right]_{r=R} = \frac{\tau_w}{\rho v(\nu + \varepsilon_M)} = \frac{fv}{2(\nu + \varepsilon_M)}$$

where $\hat{u} = \frac{u}{v}$ = dimensionless velocity

v = mean fluid velocity

For the wall heat flux and wall temperature profile

$$(\alpha + \varepsilon_H)\left[\frac{d(\rho C_p T)}{dr}\right]_{r=R} = q_w = h(T_w - T_m)$$

$$\left[\frac{d(\hat{T})}{dr}\right]_{r=R} = \frac{h}{\rho C_p(\alpha + \varepsilon_H)}$$

where $\hat{T} = \frac{T}{T_w - T_m}$ = dimensionless temp

T_m= mean fluid temperature

Reynolds assumed:

(a) Gradients of dimensionless velocity and dimensionless temperature at the wall are equal

(b) Heat and momentum are transported at the same rate [$(\alpha = \nu)$ and $(\varepsilon_m = \varepsilon_H)$]

$$\therefore\ (\nu + \varepsilon_m) = (\alpha + \varepsilon_H)$$

$$\frac{fv}{2(\nu + \varepsilon_M)} = \frac{h}{\rho C_p(\alpha + \varepsilon_H)} \qquad \text{or,} \qquad \frac{fv}{2} = \frac{h}{\rho C_p}$$

$$\frac{f}{2} = \frac{\left(hd/_k\right)}{\left(\mu C_p/_k\right)\left(dv\rho/_\mu\right)}$$

$$\frac{(Nu)}{(Re)(Pr)} = St = \frac{f}{2} = \frac{h}{\rho C_p v}$$

This is the Reynolds Analogy

- In 1910, Prandtl described the turbulent transport by assuming that momentum and heat transfer occur through eddy transport in the 'turbulent core' and diffusive transport in the 'laminar sublayer', near the wall
- Based on the assumption, Prandtl developed the equation relating Stanton number and Fanning's friction factor

$$St = \frac{f/_2}{1 + 5\sqrt{f/_2}\,(Pr - 1)}$$

- This **equation is called the Prandtl Analogy**
- It becomes equal to Reynolds Analogy when $Pr = 1$

- In 1934. Chilton and Colburn observed that experimental data could be better correlated if $1 + 5\sqrt{f/_2}\,(Pr - 1)$ of the Prandtl Analogy is replaced by $Pr^{2/3}$

$$\therefore \quad St = \frac{f/_2}{(Pr)^{2/3}}$$

Now,

$$\frac{(Nu)}{(Re)(Pr)} = St = \frac{f/_2}{(Pr)^{2/3}}$$

$$\frac{(Nu)}{(Re)(Pr)^{1/3}} = f/2$$

- $\frac{(Nu)}{(Re)(Pr)^{1/3}}$ is also called the Colburn j-factor (j_H)

$$\therefore \frac{(Nu)}{(Re)(Pr)^{1/3}} = j_H = f/2$$

- This **equation is called the Chilton-Colburn Analogy**

The heat transfer coefficients can be estimated using these analogies

- Reynolds Analogy assumes that turbulent diffusivities are all equal and molecular diffusivities are negligible - this analogy breaks down when viscous sub-layer becomes important
- Prandtl Analogy considers molecular diffusion equation for viscous sublayer and Reynolds Analogy equation for turbulent region
- Von Karman further improved this by considering a buffer region in addition to viscous sublayer and turbulent core

$$Nu = \frac{\left(f/2\right)(Re)(Pr)}{1 + 5\sqrt{f/2}\left\{(Pr-1) + ln\left[1 + \left(5/6\right)(Pr-1)\right]\right\}}$$

This **equation is called the Von Karman Analogy**