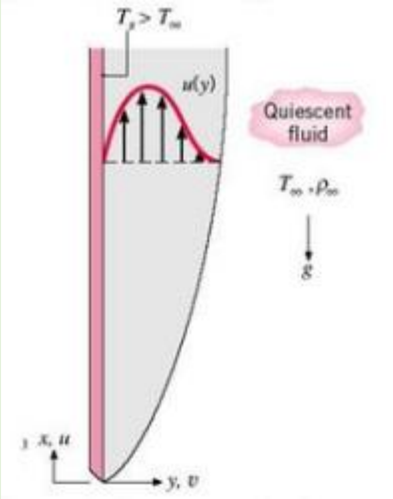
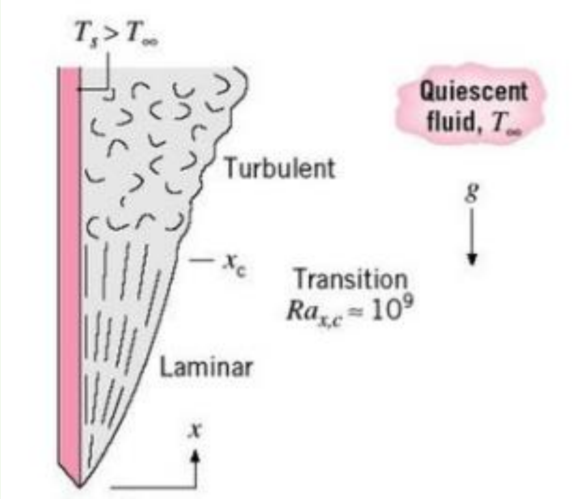
Natural Convection

- Natural or free convection is observed as a result of the motion of the fluid due to density changes arising from a heating process
- The movement of the fluid in free convection results from the buoyancy forces imposed on the fluid when its density in the proximity of the heat transfer surface is decreased as a result of the heating process
- A hot radiator used for heating a room is an important practical application which transfers heat by free convection





Free convection boundary layer along a heated plate

- In order to determine the rate of heat transfer by natural convection, it is essential to estimate the heat transfer coefficients under these conditions
- The **heat transfer coefficients** for natural convection involve **empirical correlations** which are made up of **Nusselt Number, Prandtl Number and Grashof Number**

- Grashof Number is the dimensionless group representing the ratio of the buoyancy forces to the viscous forces
- It plays the same role in natural convection as Reynolds Number does in forced convection

$$Gr_L = \frac{g\beta(T_s - T_o)L^3\rho^2}{\mu^2}$$

where $\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P = coefficient\ of\ volume\ expansion = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{T}$

$L = characterisitc\ length$ $\quad T_o = ambient\ fluid\ temp$ $\quad T_s = surface\ temperature$

$\rho = fluid\ density$ $\quad P = total\ pressure$

- When the fluid velocity is large enough, the free convection boundary layer changes from laminar to turbulent
- The condition at which the transition takes place is given by the value of the dimensionless number called Rayleigh Number $(Ra)_L$

$$Ra_L = (Gr_L)(Pr) = \frac{g\beta(T_s - T_o)L^3C_p\rho^2}{\mu k} = \frac{g\beta(T_s - T_o)L^3}{\nu\alpha}$$

$$\alpha = \frac{k}{\rho C_p}$$

$$\nu = \frac{\mu}{\rho}$$

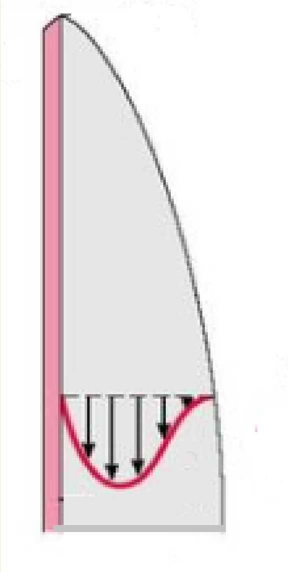
- For a vertical surface, critical $Ra_L = 10^9$

- The characteristic length used in Nusselt Number and Grashof Number depends on the geometry of the problem
  - For a vertical plate, it is the height of the plate L
  - For a horizontal cylinder it is the diameter d

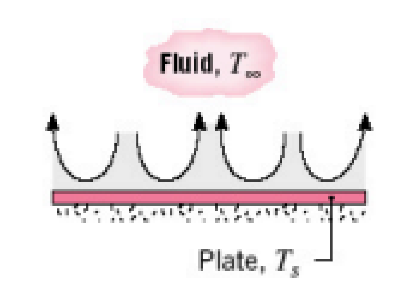
(A) Free convection from a flat surface

- The rate of free convection depends upon the orientation of the plate – vertical, horizontal or inclined
- The horizontal surface may be hot or cold and the hot or cold surface may face upwards or downwards

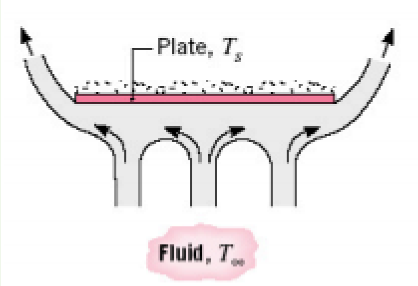
Flow patterns



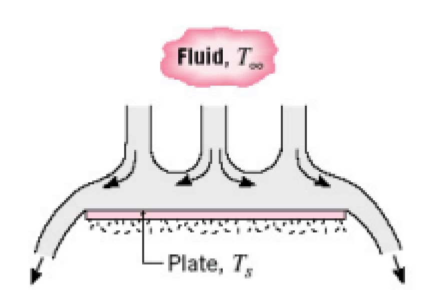
Cold vertical plate



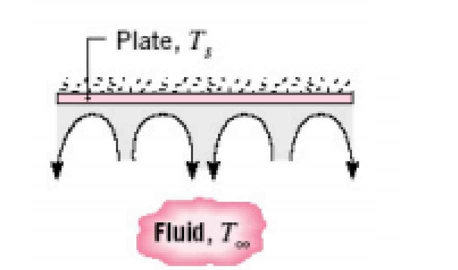
Hot surface facing up



Hot surface facing down



Cold surface facing up



Cold surface facing down

(a) Vertical plate (hot or cold)

Empirical equation by **Churchill and Chu (1975)**

$$Nu_L = \frac{hL}{k} = \left[0.825 + \frac{0.387(Ra_L)^{1/2}}{\left\{1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

for **entire range of $\boldsymbol{Ra_L}$**

For **laminar flow $\left(\boldsymbol{Ra_L < 10^9}\right)$**, the following equation gives better prediction

$$Nu_L = \frac{hL}{k} = 0.68 + \frac{0.67(Ra_L)^{1/4}}{\left\{1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{4/9}}$$

The above equations are also applicable if the plate is inclined and the angle of inclination with the vertical $\theta < 60^o$

(b) Horizontal plate (hot or cold)

(i) Heat transfer from upper surface of hot plate

$$Nu_L = \frac{hL}{k} = 0.54(Ra_L)^{1/4}$$

for **entire range of $\mathbf{10^4 \le Ra_L \le 10^7}$**

(ii) Heat transfer to the lower surface of a cold plate

$$Nu_L = \frac{hL}{k} = 0.15(Ra_L)^{1/3}$$

for **entire range of $\mathbf{10^7 \le Ra_L \le 10^{11}}$**

(iii) Heat transfer from the lower surface of a hot plate or to the upper surface of a cold plate

$$Nu_L = \frac{hL}{k} = 0.27(Ra_L)^{1/4}$$

for **entire range of $\mathbf{10^5 \le Ra_L \le 10^7}$**

In all the above cases, $L = \frac{Area\ of\ plate}{Perimeter\ of\ the\ plate}$

(B) Free convection from a cylinder

(a) Heat transfer from a horizontal cylinder

Morgan correlation

$$Nu = \frac{hd}{k} = C(Ra_d)^n$$

Ra_d	C	n
$10^{-10} – 10^{-2}$	0.675	0.058
$10^{-2} – 10^{2}$	1.02	0.148
$10^{2} – 10^{4}$	0.85	0.188
$10^{4} – 10^{7}$	0.48	0.250
$10^{7} – 10^{12}$	0.125	0.333

Another correlation by Churchill and Chu (1975)

$$Nu = \frac{hd}{k} = \left[0.60 + \frac{0.387(Ra_d)^{1/6}}{\left\{1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

$$\mathbf{10^{-5} \le Ra_d \le 10^{12}}$$

(b) Heat transfer from a vertical cylinder

If $\frac{d}{L} \ge \frac{35}{(Gr)^{1/4}}$ the equations for vertical flat plate can be used
(cylinder <-> vertical flat plate having breadth = circumference of cylinder)

If the cylinder diameter is small, then other correlations have to be used

(C) Free convection from a sphere

- Correlation given by Churchill (1983)

$$Nu = \frac{hd}{k} = 2 + \frac{0.589(Ra_d)^{1/4}}{\left\{1 + \left(\frac{0.469}{Pr}\right)^{9/16}\right\}^{4/9}}$$

$$\boldsymbol{Ra_d \le 10^4}$$
$$\boldsymbol{Pr \ge 0.7}$$

(D) Free convection in an enclosure

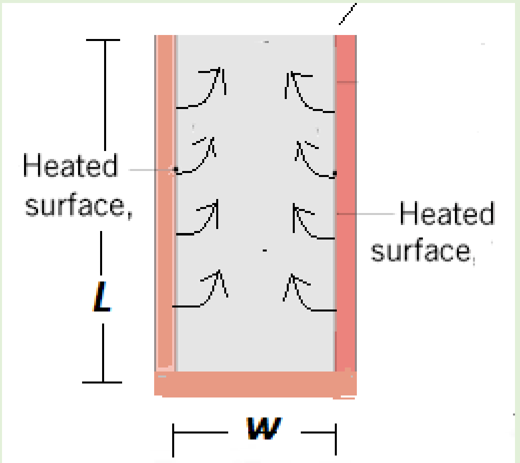
(a) Vertical rectangular cavity

$$Nu = 0.42(Ra_w)^{0.25}(Pr)^{0.012}\left(\frac{L}{w}\right)^{-0.3}$$

For $\boldsymbol{10^4 < Ra_w \le 10^7,\quad 1 < Pr < 2 \times 10^4,\quad 10 < \frac{L}{w} < 40}$

$$Nu = 0.046(Ra_w)^{1/3}$$

For $\boldsymbol{10^6 < Ra_w < 10^9,\quad 1 < Pr < 20,\quad 1 < \frac{L}{w} < 40}$



$$\frac{w}{L} = aspect\ ratio$$
$$L = characterisitc\ length$$

(b) Heat transfer in enclosure formed by two concentric horizontal cylinders

$$q = \frac{2\pi k_{eff}(T_i - T_o)}{ln\left({}^{d_o}/_{d_i}\right)}$$

$$\frac{k_{eff}}{k} = 0.386\left[\frac{Pr}{0.861 + Pr}\right]^{0.25}(Ra_e)^{0.25}$$

$$Ra_e = \frac{\left[ln\left({}^{d_o}/_{d_i}\right)\right]^4}{L^3\left(d_i^{-0.6} + d_o^{-0.6}\right)^5} Ra_L$$

$$\boldsymbol{10^2 < Ra_e \leq 10^7}$$

$d_i = outer\ diameter\ of\ inner\ tube$
$d_o = inner\ diameter\ of\ outer\ tube$
$L = d_o - d_i = characterisitc\ length\ of\ enclosure$

Combined free and forced convection

- If $\frac{Gr_L}{Re_L^2} \ll 1$, forced convection regime
- If $\frac{Gr_L}{Re_L^2} \gg 1$, free convection regime
- If $\frac{Gr_L}{Re_L^2} \approx 1$, mixed convection regime

If heat transfer is in the mixed convection regime,

$$Nu^m = \left(Nu_{forced}\right)^m \pm \left(Nu_{free}\right)^m$$

m is usually 3

the $\pm$ sign depends on whether free and forced convection is in the same or opposite direction

Problem

A vertical plate 4 m high is maintained at 60°C and exposed to atmospheric air at 10°C . Calculate the rate of heat transfer if the plate is 10 m wide.

Mean film temperature, $T_f = \frac{60+10}{2} = 35°\text{C} = 308\ K$

Air properties at 308 K, $k = 0.02685\ W/m°\text{C}$, $\beta = \frac{1}{T} = \frac{1}{308} = 3.25 \times 10^{-3} K^{-1}$, $\rho = 1.1614 \frac{kg}{m^3}$,

$\mu = 1.91631 \times 10^{-5} \frac{kg}{ms}$, $C_p = 0.9808 \frac{kJ}{kgK}$

$$Gr_L = \frac{g\beta(T_s - T_o)L^3\rho^2}{\mu^2} = \frac{9.8 \times 3.25 \times 10^{-3} \times (333 - 283) \times 4^2 \times (1.1614)^2}{(1.91631 \times 10^{-5})^2} = 3.7436 \times 10^{11}$$

$$Pr = \frac{\mu C_p}{k} = \frac{1.91631 \times 10^{-5} \times 0.9808}{0.02685} = 0.7$$

$$Ra_L = (3.7436 \times 10^{11})(0.7) = (Gr_L)(Pr) = 2.62 \times 10^{11}$$

$$Nu_L = \frac{hL}{k} = \left[0.825 + \frac{0.387(Ra_L)^{1/2}}{\left\{1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2 = \left[0.825 + \frac{0.387(2.62 \times 10^{11})^{1/2}}{\left\{1 + \left(\frac{0.492}{0.7}\right)^{9/16}\right\}^{8/27}}\right]^2 = 715.49$$

$$h = \frac{Nu_L k}{L} = \frac{715.49 \times 0.02685}{4} = 4.80\ W/m^2°\text{C}$$

$$Q = hA(T_s - T_o) = 4.80 \times 4 \times 10 \times (333 - 283) = 9600\ W = 9.6\ kW$$

Problem

A sulphur dioxide production plant has a cylindrical reactor 10.5 m high and 4 m in diameter. It is insulated by a layer of mineral wool ($k_i = 0.0602\ kcal/hm°C$). An average reactor wall temperature of 460 °C may be assumed. If heat loss from the insulated reactor occurs only by free convection, what thickness of insulation should be used so that the insulation skin temperature does not exceed 65 °C? Temperature of the atmosphere can be assumed to be 30°C

The reactor is a vertical cylinder with an insulation

At steady state, heat transfer through the insulation (Q_i) = heat transfer from the surface by free convection (Q_c)

In case of a vertical cylinder, the equation for a vertical plate can be used if $\frac{d}{L} \geq \frac{35}{(Gr)^{1/4}}$

Mean film temperature, $T_f = \frac{65+30}{2} = 47.5°C = 320.5\ K$

Air properties at 308 K, $k = 0.0241\ W/m°C$, $\beta = \frac{1}{T} = \frac{1}{320.5} = 3.12 \times 10^{-3} K^{-1}$, $\nu = 1.84 \times 10^{-5} \frac{m^2}{s}$, $Pr = 0.705$,

Also, $T_s = 65 + 273 = 338K$, $T_o = 30 + 273 = 303K$ $L = 10.5\ m$ $d_i = 4\ m$

$$Gr_L = \frac{g\beta(T_s - T_o)L^3\rho^2}{\mu^2} = \frac{g\beta(T_s - T_o)L^3}{\nu^2} = \frac{9.8 \times 3.12 \times 10^{-3} \times (338 - 303) \times (10.5)^2}{(1.84 \times 10^{-5})^2} = 3.66 \times 10^{12}$$

Now, $\frac{d}{L} = \frac{4}{10.5} = 0.38$ and $\frac{35}{(Gr)^{1/4}} = \frac{35}{(3.66\times10^{12})^{1/4}} = 0.025$

Therefore, $\frac{d_i}{L} \geq \frac{35}{(Gr)^{1/4}}$

$$Ra_L = (3.66 \times 10^{12})(0.705) = (Gr_L)(Pr) = 2.5803 \times 10^{12}$$

$$Nu_L = \frac{hL}{k} = \left[0.825 + \frac{0.387(Ra_L)^{1/6}}{\left\{1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2 = \left[0.825 + \frac{0.387(2.5803 \times 10^{12})^{1/2}}{\left\{1 + \left(\frac{0.492}{0.705}\right)^{9/16}\right\}^{8/27}}\right]^2 = 1504$$

$$h = \frac{Nu_L k}{L} = \frac{1504 \times 0.0241}{10.5} = 3.45 \ kcal/hm^2{}^\circ\mathrm{C}$$

Heat transfer from the surface by free convection (Q_c)

$$Q_c = hA(T_s - T_o) = h(\pi d_o L)(T_s - T_o)$$

Let thickness of insulation be x, then $d_o = d_i + 2x = 4 + 2x$

Thus, $Q_c = h(\pi d_o L)(T_s - T_o) = 3.45 \times \pi \times (4 + 2x) \times 10.5 \times (338 - 303) = 3983.15(4 + 2x)$

- Heat transfer (conductive) through the cylinder covered with insulation (Q_i)

$$Q_i = \frac{2\pi Lk(T_s - T_o)}{ln\left(\frac{d_o}{d_i}\right)} = \frac{2 \times \pi \times 10.5 \times 0.0602(338 - 303)}{ln\left(\frac{4+2x}{4}\right)} = \frac{1568.783}{ln(1+0.5x)}$$

At steady state, $Q_c = Q_i$

$$3983.15(4+2x) = \frac{1568.783}{ln(1+0.5x)}$$

The value for x has been found by trial and error

The thickness of insulation that would lead to the insulation skin temperature not more than 65 ℃ is 0.188 m = 18.8 cm

x	LHS	RHS
0.5	19915.75	7030.375
0.1	16729.23	32153.673
0.2	17525.86	16459.76
0.19	17446.197	17286.339
0.188	**17430.264**	**17461.830**
0.187	17422.298	17551.136