Heat Exchangers

Effectiveness of a heat exchanger

- In heat exchanger design, the main design equation is $Q = UA(LMTD)$
- In order to estimate LMTD it is necessary to know the inlet and outlet temperatures of both the hot and cold liquids
- In case this is not known, the calculations for the heat exchanger then becomes a trial and error effort
- When only the inlet temperatures of the hot and cold fluid are known , it is preferable to use an alternate approach to heat exchanger design known as the effectiveness-NTU (or NTU) method
- The term ***effectiveness of the heat exchanger ,*** based on only the initial temperatures is defined as

$$Effectiveness, \eta = \frac{Actual\ heat\ transfer}{Maximum\ possible\ heat\ transfer}$$

- Actual heat transfer,

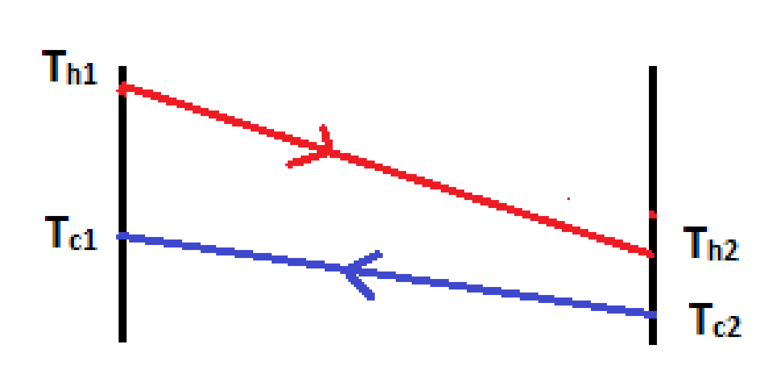
$$Q = m_c C_c (T_{c1} - T_{c2}) = m_h C_h (T_{h1} - T_{h2})$$

- The maximum value of the temperature difference is, $(T_{h1} - T_{c2})$ - the difference between the inlet temperatures of the hot and cold fluids
- Maximum heat transfer occurs when any fluid undergoes this temperature change $(T_{h1} - T_{c2})$
- However, since Q is constant, if ΔT is maximum, (mC) must be minimum
- Maximum possible heat transfer,

$$Q_{max} = (mC)_{min} (T_{h1} - T_{c2})$$

- As per the figure, if the maximum change in in the hot fluid, then the heat exchanger effectiveness is given by,

$$\eta_h = \frac{Q}{Q_{max}} = \frac{(mC)_h(T_{h1} - T_{h2})}{(mC)_{min}(T_{h1} - T_{c2})}$$



- Here as $(T_{h1} - T_{h2})$ is more than $(T_{c2} - T_{c1})$, $(mC)_h < (mC)_c$ Therefore, $(mC)_h = (mC)_{min}$

- Or,

$$Q = \eta_h Q_{max} = \eta_h (mC)_{min}(T_{h1} - T_{c2})$$

$$Q = \eta\ (mC)_{min}(T_{h1} - T_{c2})$$

- This is based only on the inlet temperatures

- If the maximum temperature change occurs in in the cold fluid, then $(mC)_c < (mC)_h$ and the heat exchanger effectiveness is given by,

$$\eta_c = \frac{Q}{Q_{max}} = \frac{(mC)_c(T_{c1} - T_{c2})}{(mC)_{min}(T_{h1} - T_{c2})}$$

- The values of η lies between 0 and 1

- For **counter-current flow**, the effectiveness is determined as,

$$\eta_{counter} = \frac{1 - exp\left[-\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right]}{1 - \frac{C_{min}}{C_{max}}\ exp\left[-\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right]}$$

where $C_{min} = (mC)_{min} = the\ minimum\ value\ either\ of\ the\ hot\ or\ cold\ fluid$

$C_{max} = (mC)_{max} = the\ maximum\ value\ either\ of\ the\ hot\ or\ cold\ fluid$

- For **co-current or parallel flow**

$$\eta_{co-current} = \frac{1 - exp\left[-\frac{UA}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}}$$

The above two expressions are for a double pipe heat exchanger in co-current and counter-current mode

Other expressions are available for shell and tube and cross flow exchangers (Incropera and Dewitt, Page 689)

TABLE 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation	
Concentric tube		
Parallel flow	$\varepsilon = \dfrac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$	
Counterflow	$\varepsilon = \dfrac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}$	$(C_r < 1)$
	$\varepsilon = \dfrac{NTU}{1 + NTU}$	$(C_r = 1)$
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$	
n Shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$	
Cross-flow (single pass)		
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(NTU)^{0.22}\{\exp[-C_r(NTU)^{0.78}] - 1\}\right]$	
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-NTU)]\})$	
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(NTU)]\})$	
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-NTU)$	

$$C_r \equiv C_{min}/C_{max}$$

$$\frac{C_{min}}{C_{max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}}$$

- Now, the term $\frac{UA}{C_{min}}$ is known as **Number of Transfer Units (NTU)** and, $\boldsymbol{NTU = f\left(\eta, \frac{C_{min}}{C_{max}}\right)}$

- For a double-pipe exchanger **counter current flow**,

$$NTU = \frac{1}{\frac{C_{min}}{C_{max}} - 1} ln\left(\frac{\eta - 1}{\eta \frac{C_{min}}{C_{max}} - 1}\right) \qquad \text{For } \frac{C_{min}}{C_{max}} < 1$$

$$NTU = \frac{\eta}{1 - \eta} \qquad \text{For } \frac{C_{min}}{C_{max}} = 1$$

- For a double-pipe exchanger **co current flow**,

$$NTU = -\frac{ln\left[1 - \eta\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}}$$

- These expressions are for a double pipe heat exchanger. Other such expressions are available for other exchanger configurations (Incropera and Dewitt, Page 690)

Table 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation
Concentric tube	
Parallel flow	$\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$
Counterflow	$\mathrm{NTU} = \dfrac{1}{C_r-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon C_r-1}\right) \quad (C_r<1)$ $\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon} \quad (C_r=1)$
Shell-and-tube	
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$ $E = \dfrac{2/\varepsilon_1-(1+C_r)}{(1+C_r^2)^{1/2}}$
n Shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r-1}{\varepsilon-1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$
Cross-flow (single pass)	
C_{max} (mixed), C_{min} (unmixed)	$\mathrm{NTU} = -\ln\left[1+\left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$
C_{min} (mixed), C_{max} (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1-\varepsilon)$

$$C_r \equiv C_{min}/C_{max}.$$

$$\frac{C_{min}}{C_{max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}}$$

Effective-NTU Method Example

A counter-current flow double pipe heat exchanger is to heat water from 20^{o}C to 80^{o}C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160^{o}C at a flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. The overall heat transfer coefficient of the heat exchanger is 640 W/m^{2}K. Using the effectiveness-NTU method, determine the length of the heat exchanger required for desired heating.

$C_h = \dot{m}_h \, C_{ph} = 2 \times 4.31 = 8.62$ kW/K

$C_c = \dot{m}_c \, C_{pc} = 1.2 \times 4.18 = 5.02$ kW/K

$C_{min} = C_c = 5.02$ kW/K and $C_{max} = C_h = 8.62$ kW/K

$Q_{max} = C_{min}\left(T_{h,in} - T_{c,in}\right) = 5.02(160 - 20) = 702.8$ kW

$\rightarrow Maximum\ possible\ heat\ transfer\ rate\ in\ the\ exchanger$

$Q = \left[\dot{m} C_p (T_{out} - T_{in})\right]_{water} = 1.2 \times 4.18 \times (80 - 20) = 301.0$ kW

$$\eta = \frac{Q}{Q_{max}} = \frac{301}{702.8} = 0.428$$

For a double-pipe exchanger **counter current flow**,

$$NTU = \frac{1}{\frac{C_{min}}{C_{max}} - 1} \ln\left(\frac{\eta - 1}{\eta \frac{C_{min}}{C_{max}} - 1}\right)$$

$$NTU = \frac{1}{\frac{C_{min}}{C_{max}} - 1} ln\left(\frac{\eta - 1}{\eta \frac{C_{min}}{C_{max}} - 1}\right)$$

$$NTU = \frac{1}{\frac{5.02}{8.62} - 1} ln\left(\frac{0.428 - 1}{0.428 \times \frac{5.02}{8.62} - 1}\right) = 0.651$$

Now,

$$NTU = \frac{UA}{C_{min}}$$

Therefore,

$$A = \frac{NTU \times C_{min}}{U} = \frac{0.651 \times 5.02 \times 1000}{640} = 5.11\ m^2$$

Since,

$$A = \pi DL$$

$$L = \frac{A}{\pi D} = \frac{5.11}{\pi \times 1.5 \times 10^{-2}} = 108\ m$$