Heat transfer with phase change

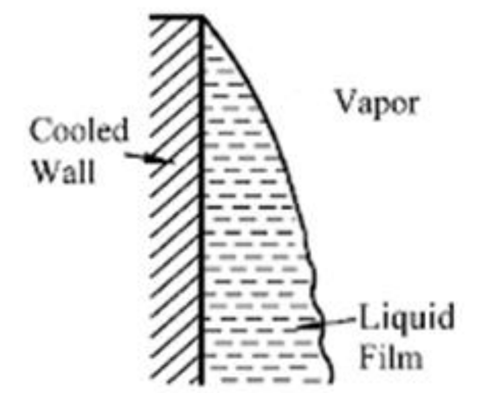
In this chapter, we shall study about two heat transfer activities involving phase change -

- **Condensation**
- **Boiling**

Condensation

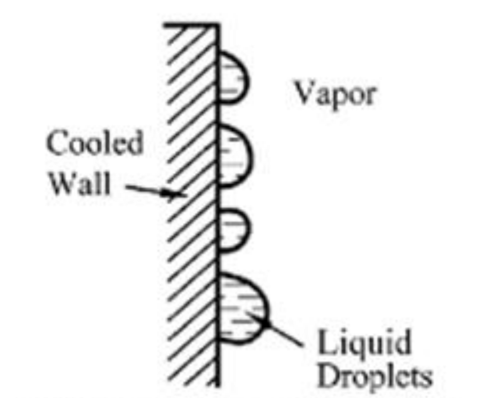
- Condensation means the *change of phase from vapour to liquid*
- If the temperature of the vapour is *reduced below the saturation temperature*, condensation occurs
- If a mixture of vapour and gas is cooled, vapour condenses to form minute droplets suspended in the carrier gas – this is called ***homogeneous condensation***
- However, if the vapour or a vapour-gas mixture comes in contact with a surface which is below the saturation temperature of the vapour, condensation occurs on the surface – this is called ***surface condensation***

- Two forms of condensation can be considered - ***film condensation*** and ***dropwise condensation***
- If the condensate wets the surface, a smooth film is formed and it flows down the surface as a continuous film under the influence of gravity – this is called ***film condensation***
- Here, the surface is blanketed by the film which grows in thickness as it moves down the plate, the *film provides thermal resistance to heat transfer*
- The latent heat is transferred through the liquid film and then conducted through the wall to the cooling fluid on the other side of the wall



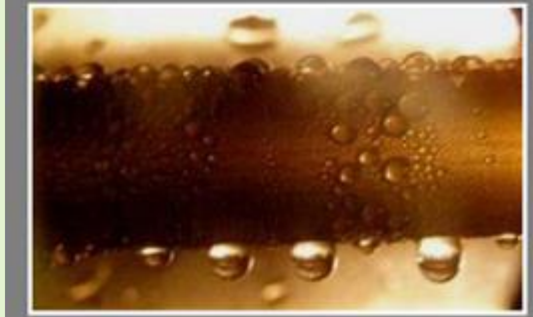
(a) Filmwise condensation

- If the surface is not wetted by the condensate, droplets are formed on the surface instead of a film
- The droplets grow in size and trickles down the surface in a random manner due to the action of gravity – this is known as ***dropwise condensation***
- In this case, a large portion of the surface is directly exposed to the vapour, and there is no liquid film barrier to resist heat flow
- This results in higher heat transfer rates
- *The average heat transfer coefficients for dropwise condensation is 5 to 10 times the coefficient for film condensation*
- Due to the higher heat transfer rates, dropwise condensation would be preferred over film condensation
- However, it is very difficult to maintain dropwise condensation, as most surfaces become wetted after exposure to a condensing vapour over an extended period of time
- Various surface coatings (such as silicone, octanoic acid) and vapour additives have been used in attempts maintain dropwise condensation but they have not been very successful
- Film condensation is more dependable and more common



(b) Dropwise condensation


Filmwise condensation


Dropwise condensation

Film condensation on a vertical surface

- Nusselt (1916) gave a theoretical analysis of laminar film condensation of a vapour
- Nusselt makes the following assumptions in his analysis:

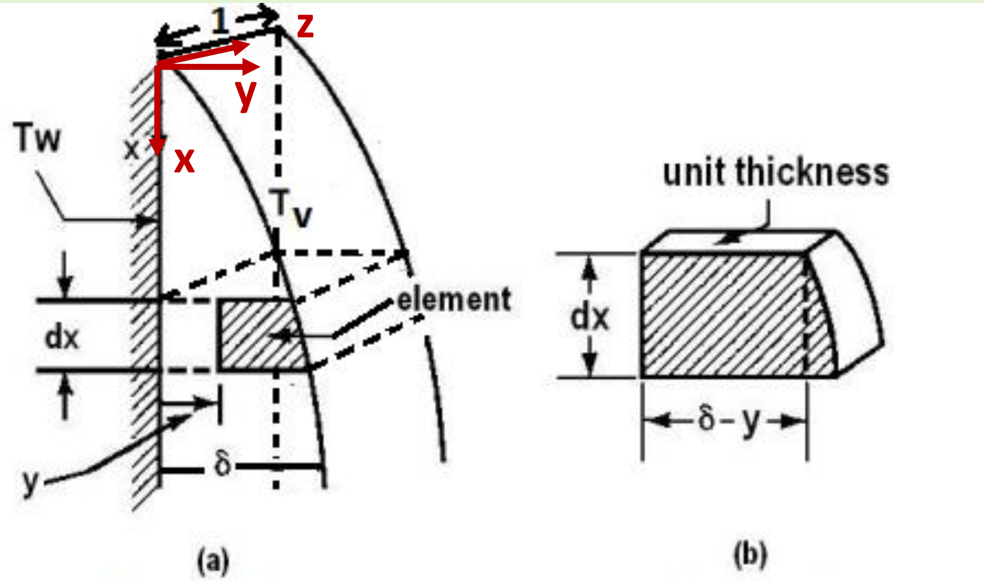
(a) the film flow is laminar

(b) the vapour is saturated

(c) heat transfer through the condensate film occurs by conduction only, temperature profile in film is linear

(d) There is no interfacial shear, i.e., viscous shear of the vapour and the liquid film is negligible

(e) Gravitational force is the only external force



Film condensation on a vertical plate: a) increase in film thickness with position, b) balance on element of condensate.

$\delta = f(x)$ and $u = f(y)$

The figure shows the vertical section of the film

1. A force balance is made on a liquid element (or control volume) of sides 'dx' and '$\delta - y$'
2. The breadth in the $z - direction$ is unity
3. T_w and T_v are the wall temperature and vapour temperature, respectively

The weight of the fluid element (F_2) of thickness dx between y and δ is balanced by the viscous shear force (F_1) at y and the buoyancy force (F_3)

The weight of the fluid element (F_2) of thickness dx between y and δ is balanced by the viscous shear force (F_1) at y and the buoyancy force (F_3)

By force balance, $F_2 = F_1 + F_3$

$$\rho_l g(\delta - y)dx = \mu \frac{du}{dy} dx + \rho_v g(\delta - y)dx$$

$$\rho_l g(\delta - y) = \mu \frac{du}{dy} + \rho_v g(\delta - y)$$

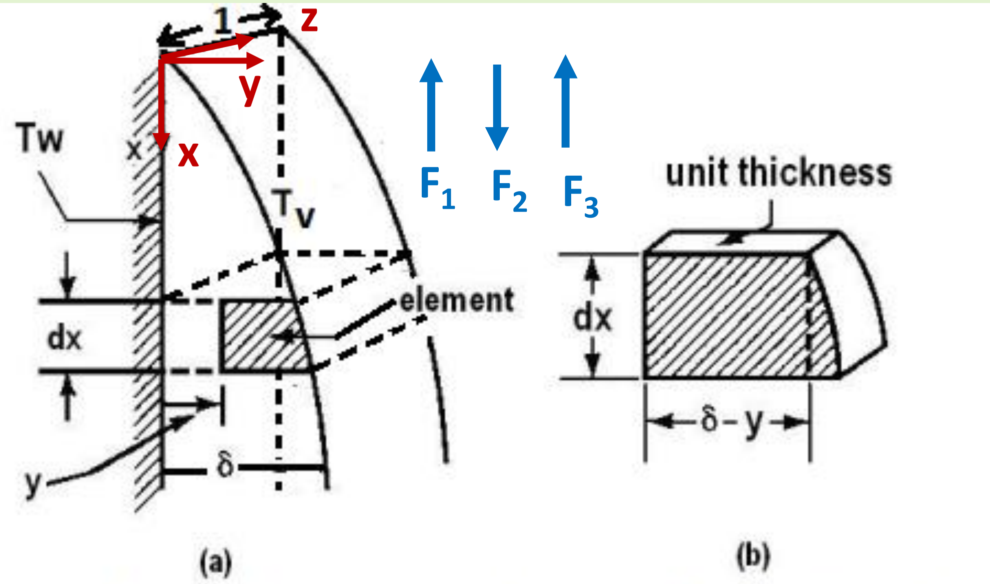
$$(\rho_l - \rho_v)g(\delta - y)dy = \mu du$$

Now at $y = 0, u = 0$ (no slip condition)

$$\int_0^u du = \frac{(\rho_l - \rho_v)g}{\mu} \int_0^y (\delta - y)dy$$

$$\boxed{u = \frac{(\rho_l - \rho_v)g}{\mu}\left[\delta y - \frac{y^2}{2}\right]}$$

This is the ***velocity profile in the freely flowing film***



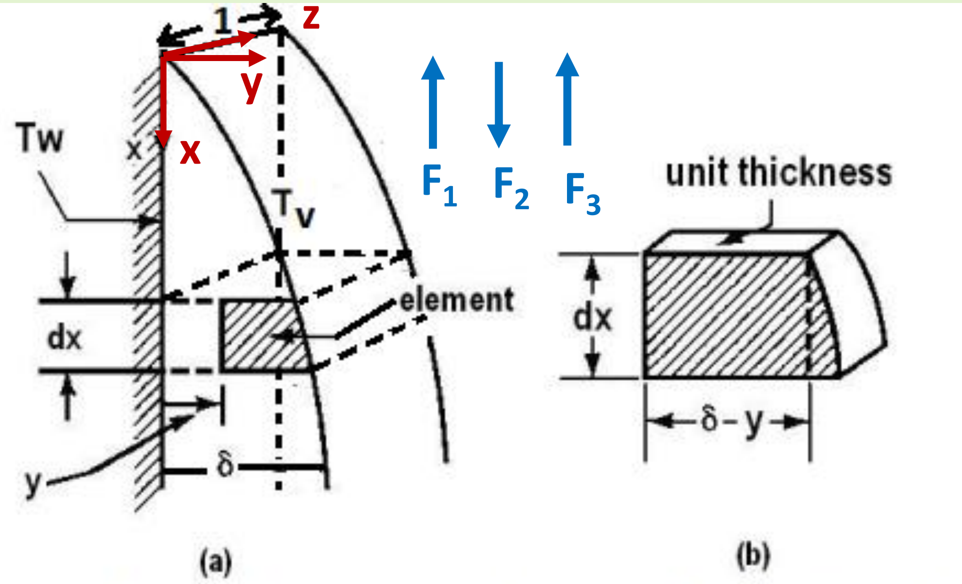
Film condensation on a vertical plate: a) increase in film thickness with position, b) balance on element of condensate.

Now, mass flow rate of condensate (per unit breadth of film) at any location x is,

$$\dot{m} = \int_0^\delta \rho_l u(dy.1) = \int_0^\delta \rho_l \left[\frac{(\rho_l - \rho_v)g}{\mu}\left(\delta y - \frac{y^2}{2}\right)\right] dy$$

$$\dot{m} = \frac{\rho_l(\rho_l - \rho_v)g}{\mu}\left\{\frac{\delta y^2}{2} - \frac{y^3}{6}\right\}_0^\delta = \frac{\rho_l(\rho_l - \rho_v)g}{\mu}\left\{\frac{\delta^3}{2} - \frac{\delta^3}{6}\right\}$$

$$\boxed{\dot{m} = \frac{\rho_l(\rho_l - \rho_v)g\delta^3}{3\mu}}$$



Film condensation on a vertical plate: a) increase in film thickness with position, b) balance on element of condensate.

The rate of heat transfer at the wall through the area $(dx \times 1)$ is

$$\boxed{q_x = -kdx\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{kdx.1(T_v - T_w)}{\delta}}$$

As the liquid flows from x to $x + dx$, the condensate film grows from δ to $\delta + d\delta$ due to addition of condensate

The amount of condensate added between x and $x + dx$ is $= \frac{d}{dx}(\dot{m})dx = \frac{d}{dx}\left[\frac{\rho_l(\rho_l - \rho_v)g\delta^3}{3\mu}\right]dx$

As δ is a function of x, the amount of condensate is given as,

$$= \frac{d}{d\delta}\left[\frac{\rho_l(\rho_l - \rho_v)g\delta^3}{3\mu}\right]\frac{d\delta}{dx}dx = \boxed{\frac{\rho_l(\rho_l - \rho_v)g\delta^2}{\mu}d\delta}$$

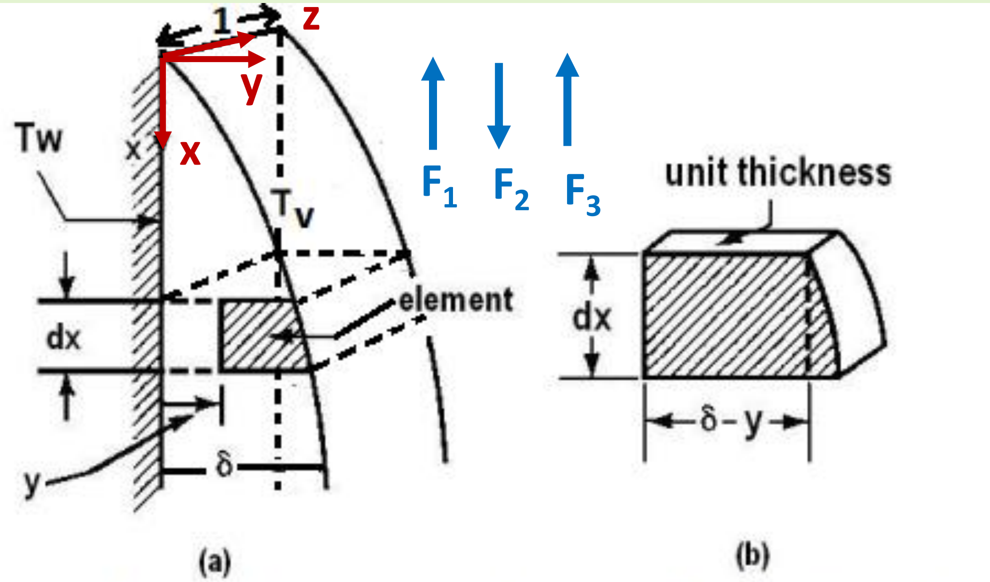
The heat removed at the wall is equal to the heat given out by the condensing liquid mass (between x to $x + dx$)

$$\frac{\rho_l(\rho_l - \rho_v)g\delta^2}{\mu} d\delta.\lambda = \frac{kdx.1(T_v - T_w)}{\delta}$$

$$\int_0^{\delta} \delta^3\, d\delta = \frac{k\mu(T_v - T_w)}{\rho_l(\rho_l - \rho_v)g\lambda} \int_0^{x} dx$$

$$\frac{\delta^4}{4} = \frac{k\mu(T_v - T_w)x}{\rho_l(\rho_l - \rho_v)g\lambda}$$

$$\boxed{\delta = \left[\frac{4k\mu x(T_v - T_w)}{\rho_l(\rho_l - \rho_v)g\lambda}\right]^{1/4}}$$



Film condensation on a vertical plate: a) increase in film thickness with position, b) balance on element of condensate.

The heat transfer coefficient is now written as

$$hdx(T_w - T_v) = -\frac{kdx(T_v - T_w)}{\delta}$$

Now,

$$h = \frac{k}{\delta}$$

Therefore,

$$\boxed{h_x = \left[\frac{\rho_l(\rho_l - \rho_v)g\lambda k^3}{4\mu x(T_v - T_w)}\right]^{1/4}}$$

The average heat transfer coefficient over a length L is,

$$\bar{h} = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\left[\frac{\rho_l(\rho_l - \rho_v)g\lambda k^3}{4\mu(T_v - T_w)}\right]^{1/4}\left(\frac{4}{3}\right)L^{3/4}$$

$$\boxed{\bar{h} = 0.943\left[\frac{g\lambda\rho_l(\rho_l - \rho_v)k^3}{\mu L(T_v - T_w)}\right]^{1/4}}$$

The liquid properties are evaluated at the mean film temperature $T_m = \frac{T_{sat}+T_w}{2}$

The vapor density and latent heat of vaporization, λ are evaluated at T_{sat}

This is the ***heat transfer coefficient for condensation on the vertical surface***

In case the surface is not vertical but inclined, g in the above equation is replaced by $g\cos\theta$

When the surface on which the condensation occurs is sufficiently large or there is a sufficient amount of condensate flow, turbulence may appear in the condensate

The criterion to determine whether the flow is laminar or turbulent is the Reynolds Number

For condensation systems,

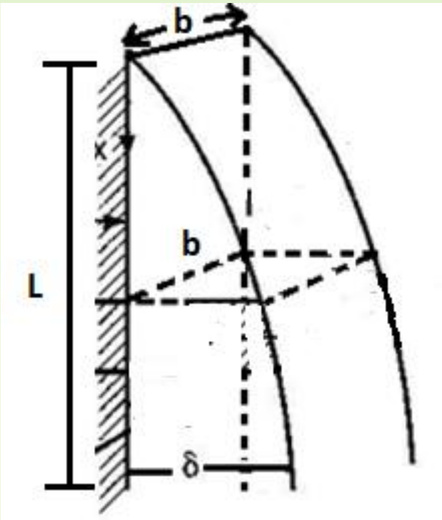
$$Re = \frac{D_e \rho v}{\mu}$$

$$D_e = 4 \times \frac{b \times L}{b} = \frac{4A}{P}$$

$$Re = \frac{4A\rho v}{\mu P}$$

Now, $\dot{m} = A\rho v$

$$\therefore \boxed{Re = \frac{4\dot{m}}{\mu P}}$$



P = perimeter

For a vertical plate,
$$P = b$$

For a vertical tube,
$$P = \pi d$$

- The critical Reynolds number (Re) is approximately 30
- Between 30 and 1800 waves and ripples appear in the condensate though fluid flow is still laminar
- For turbulent flow, $Re > 1800$

$$Nu = \frac{hL}{k} = 0.0077 \left(\frac{g\rho_l^2 L^3}{\mu^2} \right)^{1/3} (Re)^{0.4}$$

- The Re can be calculated by the following:

$$Q = hA(T_{sat} - T_w) = \dot{m}\lambda$$

$$\dot{m} = \frac{Q}{\lambda} = \frac{hA(T_{sat} - T_w)}{\lambda}$$

$$\boxed{Re = \frac{4hA(T_{sat} - T_w)}{\mu P \lambda}}$$

Film condensation outside a horizontal tube or a tube bank

The correlation for condensation on a single horizontal tube is,

$$\boxed{\bar{h} = 0.728\left[\frac{g\lambda\rho_l(\rho_l - \rho_v)k^3}{\mu d(T_v - T_w)}\right]^{1/4}}$$

where d is the tube diameter

The correlation for condensation on a vertical tier of N horizontal tubes is,

$$\boxed{\bar{h} = 0.728\left[\frac{g\lambda\rho_l(\rho_l - \rho_v)k^3}{N\mu d(T_v - T_w)}\right]^{1/4}}$$

Film condensation inside a horizontal tube

The heat transfer coefficient for condensation inside a horizontal tube is given by,

$$\boxed{\bar{h} = 0.555\left[\frac{\lambda'' g\rho^l(\rho_l - \rho_v)k^3}{\mu d_i(T_v - T_w)}\right]^{1/4}}$$

$$\lambda'' = \lambda\left[1 + \frac{3}{8}Ja\right]$$

$$Ja = Jacob\ No = \frac{C_{pl}(T_v - T_w)}{\lambda}$$

For heat exchanger design, where one fluid undergoes condensation either inside or outside the tubes (in the shell), the above mentioned equations are used to estimate the heat transfer coefficient

Problem

A vertical square plate, 30 cm by 30 cm, is exposed to steam at atmospheric pressure. The plate temperature is 98°C. Calculate the rate of heat transfer and mass of steam condensed.

The properties of steam is evaluated at the mean temperature, $T_m = \frac{98+100}{2} = 99°\text{C}$

$k = 0.68\ W/m°\text{C}, \quad \rho_l = 960 \frac{kg}{m^3}, \quad \mu = 2.82 \times 10^{-4} \frac{kg}{ms}, \quad \lambda = 2255 \frac{kJ}{kg}, \quad T_v = T_{sat} = 100°\text{C}$

Assuming that the condensate flow is laminar, (this has to be verified later)

$$\bar{h} = 0.943 \left[\frac{g\lambda\rho_l(\rho_l - \rho_v)k^3}{\mu L(T_v - T_w)}\right]^{1/4} = 0.943 \left[\frac{9.8 \times 2255 \times 1000 \times 960^2 \times (0.68)^3}{2.82 \times 10^{-4} \times 0.3(100 - 98)}\right]^{1/4} = 13152.92\ W/m^2°\text{C}$$

$$Re = \frac{4hA(T_{sat} - T_w)}{\mu P\lambda} = \frac{4hL(T_{sat} - T_w)}{\mu\lambda} = \frac{4 \times 13152.92 \times 0.3(100 - 98)}{2.82 \times 10^{-4} \times 2255 \times 1000} = 49.64\ (laminar)$$

$$Q = hA(T_{sat} - T_w) = 13152.92 \times 0.3 \times 0.3(100 - 98) = 2367.53\ W$$

$$\dot{m} = \frac{Q}{\lambda} = \frac{2367.53}{2255 \times 1000} = 1.0499 \times 10^{-3}\ kg/s$$

$$\dot{m} = 3.78 kg/h$$

Problem

Saturated steam at 68.9 kPa is condensing on a vertical tube 0.305 m long having an OD of 0.0254 m and a surface temperature of 86.11°C. Calculate the average heat transfer coefficient and mass of steam condensed.

$T_{sat} = 89.47°\mathrm{C} \qquad T_w = 86.11\ °\mathrm{C}$

The properties of steam is evaluated at the mean temperature, $T_m = \frac{89.47+86.11}{2} = 87.8°\mathrm{C}$

$k = 0.675\ W/m°\mathrm{C}, \quad \rho_l = 966.7 \frac{kg}{m^3}\ ,\ \rho_v = 0.391 \frac{kg}{m^3}\ ,\ \mu = 3.24 \times 10^{-4} \frac{kg}{ms}, \qquad \lambda = 2283.2 \frac{kJ}{kg},$

The heat transfer coefficient for condensation outside a vertical tube is given by ,

$$Nu = 1.13 \left[\frac{g\lambda\rho_l(\rho_l - \rho_v)L^3}{\mu k_l(T_v - T_w)}\right]^{1/4} = 1.13 \left[\frac{9.8 \times 2283.2 \times 1000 \times 966.7(966.7 - 0.391) \times (0.305)^3}{3.24 \times 10^{-4} \times 0.675(89.47 - 86.11)}\right]^{1/4} = 6038.5$$

$$h = Nu \times \frac{k}{d} = 6038.5 \times \frac{0.675}{0.305} = 13363.9\ W/m^2°\mathrm{C}$$

$$Q = UA(T_{sat} - T_w) = \dot{m}\lambda$$

$$13363.9 \times \pi \times 0.305 \times 0.0254 \times (89.47 - 86.11) = \dot{m} \times 2283.2 \times 1000$$

$$\dot{m} = 4.74 \times 10^{-4}\ kg/s$$

$$Re = \frac{4\dot{m}}{\mu\pi D} = \frac{4.74 \times 10^{-4}}{3.24 \times 10^{-4} \times \pi \times 0.0254} = 73.33$$

		Specific volume m³/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/kg-K		
Temp. T °C	Sat. Press. P_{sat} kPa	Sat. Liquid v_f	Sat. Vapor v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.2469	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.6291	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.5953	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.352	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.763	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.043	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.202	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.597	0.001026	4.1291	313.99	2161.3	2475.3	314.03	2320.6	2634.6	1.0158	6.6655	7.6812
80	47.416	0.001029	3.4053	334.97	2146.6	2481.6	335.02	2308.0	2643.0	1.0756	6.5355	7.6111
85	57.868	0.001032	2.8261	355.96	2131.9	2487.8	356.02	2295.3	2651.4	1.1346	6.4089	7.5435
90	70.183	0.001036	2.3593	376.97	2117.0	2494.0	377.04	2282.5	2659.6	1.1929	6.2853	7.4782
95	84.609	0.001040	1.9808	398.00	2102.0	2500.1	398.09	2269.6	2667.6	1.2504	6.1647	7.4151
100	101.42	0.001043	1.6720	419.06	2087.0	2506.0	419.17	2256.4	2675.6	1.3072	6.0470	7.3542
105	120.90	0.001047	1.4186	440.15	2071.8	2511.9	440.28	2243.1	2683.4	1.3634	5.9319	7.2952
110	143.38	0.001052	1.2094	461.27	2056.4	2517.7	461.42	2229.7	2691.1	1.4188	5.8193	7.2382
115	169.18	0.001056	1.0360	482.42	2040.9	2523.3	482.59	2216.0	2698.6	1.4737	5.7092	7.1829
120	198.67	0.001060	0.89133	503.60	2025.3	2528.9	503.81	2202.1	2706.0	1.5279	5.6013	7.1292

57.866 kPa → 85°C

70.183 kPa → 90°C

(70.183 − 57.866) → (90 − 85)

12.317 → 5°C

(70.183 − 68.9) →

1.283

$$\frac{1.283}{12.317} \times 5 = 0.5208\,°C$$

(90 − x)

90 − x = 0.5208°C

x = 89.47°C