Radiation

Laws of radiation

(i) Planck's law

- ***Planck's law or distribution is a one-parameter distribution function for the monochromatic emissive power of a black body, as a function of wavelength of radiation***
- The single parameter in this function of temperature

$$E_{b\lambda} = \frac{2\pi h c^2 \lambda^{-5}}{exp\left(^{hc}/_{\lambda kT}\right) - 1} = \frac{k_1 \lambda^{-5}}{exp\left(^{k_2}/_{\lambda T}\right) - 1}$$

where $E_{b\lambda}$ = monochromatic emissive power of a black body, $\frac{W}{m^2 \mu m}$

λ = wavelength of monochromatic radiation

T = absolute temperature of a black body

h = Planck's constant = 6.6256×10^{-34} J s

k = Boltzman's constant = 1.3805×10^{-23} J/K

c = velocity of light = 2.998×10^{8} m/s

$k_1 = 2\pi c^2 h$ = 3.743×10^{8} W(μm)^4 /m^2

$k_2 = hc/k$ = 1.4387×10^{4} μm K

$E_{b\lambda}$ or the monochromatic emissive power is the amount of radiant energy emitted by a surface per unit area per unit time and per unit wavelength, $\frac{W}{m^2 \mu m}$

The variation in blackbody emissive power with wavelength (λ) is plotted in the figure:

(a) At any temperature, the blackbody emits radiation covering a range of wavelengths

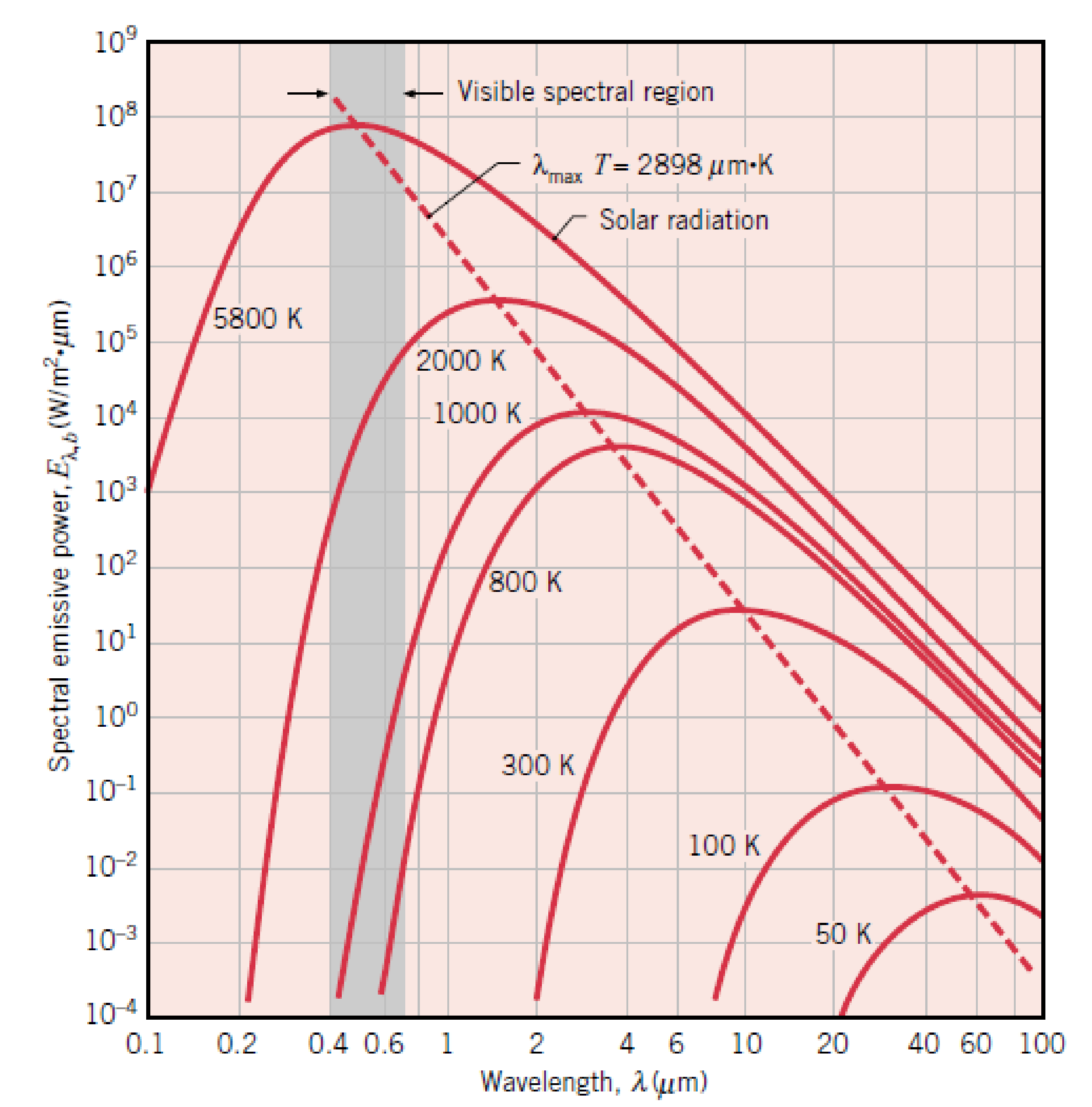
(b) The emitted radiation is a continuous function and has a maxima

(c) At any wavelength, the amount of emitted radiation increases with increasing temperature

(d) The peak of the curve shifts to shorter wavelength as temperature increases

(e) The radiation emitted by the sun (considered to be a black body at ~ 5800 K) is maximum around the visible range

Earth at a temperature of 300 K emits radiation in the infra-red region



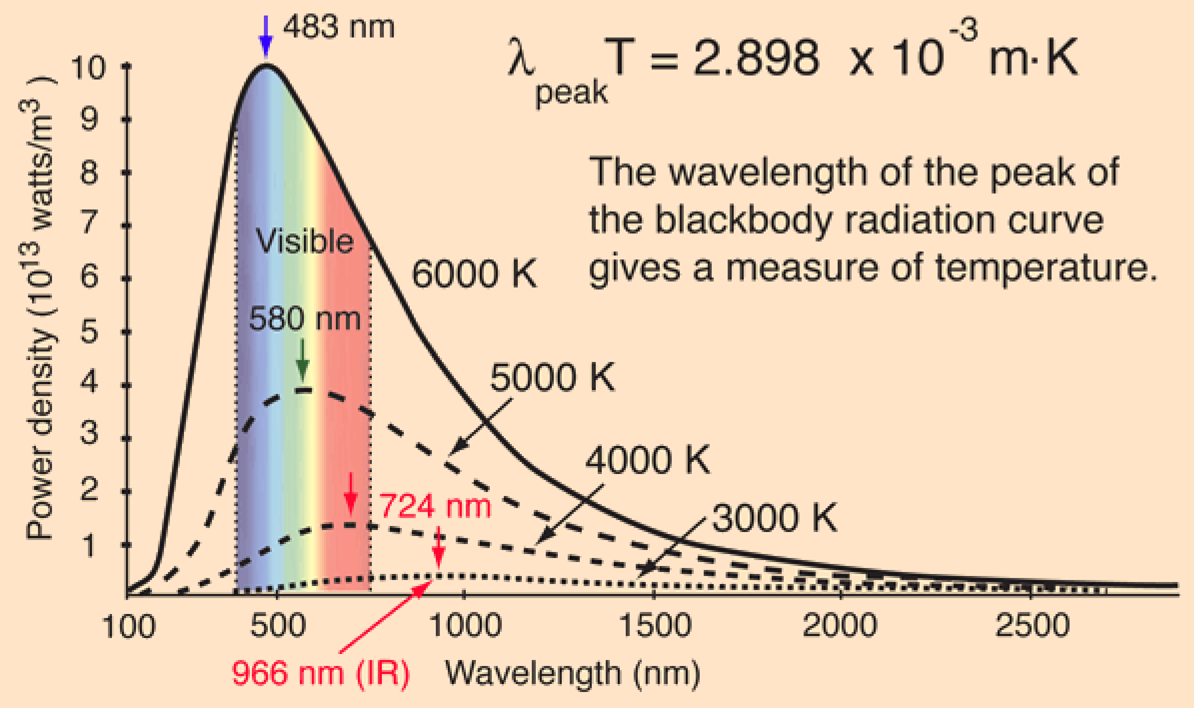
(ii) Wein's displacement law

- The plot of blackbody emissive power versus wavelength (λ) shows that λ_{max} (λ value corresponding to peak) is inversely proportional to the temperature of a black body and is related as,

$$\lambda_{max}T = constant = 2898\ \mu mK$$

- ***Wein's displacement law states that the product of wavelength of maximum value of monochromatic radiation and absolute temperature is a constant***

- When an electric radiation heater is plugged on, radiation energy starts emitting
- The radiation energy (heat) can be felt but cannot be seen as the radiation is in the infra-red region
- When the temperature reaches around 1000 K, the heater appears dull red as it starts emitting a detectable amount of visible red radiation at that temperature
- As the temperature further increases, the heater appears bright red and at around 1500 K, the heater emits radiation covering the entire visible range and appears white hot



(iii) Stefan – Boltzmann law

- ***The total emissive power of a blackbody*** (E_b) ***over the entire spectrum can be obtained by integrating the expression for monochromatic emissive power*** given by the Planck's law

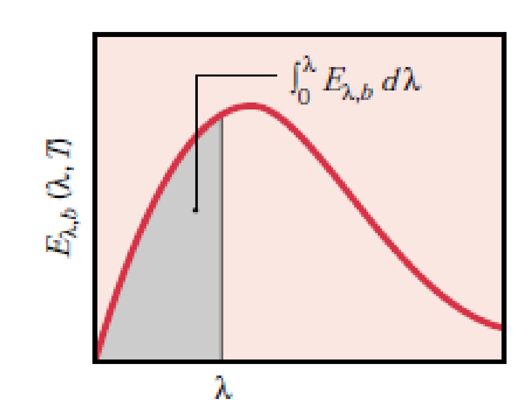
$$E_b = \int_0^\infty E_{b\lambda} d\lambda = \int_0^\infty \frac{k_1 \lambda^{-5}}{exp\left({}^{k_2}/_{\lambda T}\right) - 1} d\lambda$$

$$\boldsymbol{E_b = \sigma T^4}$$

where $\sigma = \left(\frac{\pi^4}{15}\right)(k_1 k_2^4) = 5.729 \times 10^{-8}\ W/m^2 K^4$

- This law helps to calculate the total amount of radiation of all wavelengths emitted in all possible directions, by a black body

- The fraction of the total energy emission from a blackbody on a given range of wavelength $(0 - \lambda)$ is given by,



$$F_{b,(0-\lambda)} = \frac{\int_0^\lambda E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{1}{\sigma T^4} \int_0^\lambda \frac{k_1 \lambda^{-5}}{exp\left({}^{k_2}/_{\lambda T}\right) - 1} d\lambda$$

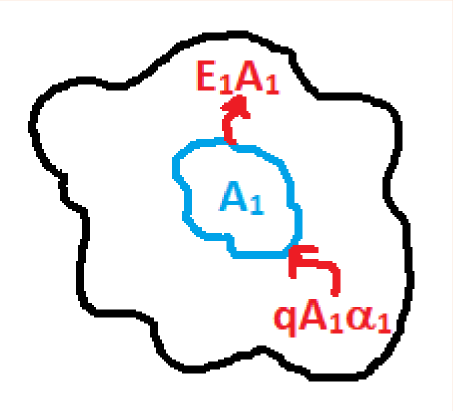
(iv) Kirchoff's law

- An important property in radiation is the emissivity of a surface
- The *emissivity of a surface is defined as the ratio of the radiation emitted by the surface to the radiation emitted by a black body*
- The emissivity (ε) varies between 0 and 1
- The emissivity of a real surface varies with the temperature of the surface, wavelength and direction of emitted radiation

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T^4}$$

Next, a relation is derived between absorptivity (α) and emissivity (ε)

- Let us consider a large enclosure of surface temperature T_S, which behaves like a black body
- The radiant heat flux incident on any surface in the enclosure is q
- A small body of area A_1 is present inside the enclosure
- If E_1 denotes the emissive power of the body, and α_1 is absorptivity



Rate at which body 1 emits radiant energy = $E_1 A_1$

Rate at which body 1 receives radiant energy = $\alpha_1 q A_1$

- If body 1 is in thermal equilibrium with the enclosure,

$$rate\ of\ emission = rate\ of\ absorption$$

$$E_1 A_1 = \alpha_1 q A_1$$

$$E_1 = \alpha_1 q$$

- Now, if body 1 is replaced by a black body, $E_1 = E_b$ and $\alpha_1 = \alpha_b = 1$
- For the black body,

$$E_b = q$$

- Dividing the two expressions we get,

$$\frac{E_1}{E_b} = \frac{\alpha_1 q}{q} = \alpha_1$$

- According to the definition of emissivity,

$$\varepsilon_1 = \frac{E_1}{E_b}$$

- Therefore, $\boxed{\boldsymbol{\varepsilon_1 = \alpha_1}}$

Kirchoff's law states that the emissivity of a body which is in thermal equilibrium with its surroundings is equal to its absorptivity

- Real substances emit less radiation than ideal black surfaces as measured by the emissivity of the material
- The emissivity and absorptivity of a real substance may vary with its temperature or wavelength of radiation emitted or absorbed
- A **gray body** is defined as a substance whose emissivity and absorptivity are independent of wavelength
- Thus, a gray body is also an ideal body, but its ε *and* α *values are both less than unity*
- Gray bodies do not exist in practise and the concept is an idealized one
- For gray bodies, $\varepsilon_\lambda = constant$ and $\alpha_\lambda = constant$ (*the monochromatic properties are constant over all wavelengths)*
- Total absorptivity α and monochromatic absorptivity α_λ are equal, as are ε and ε_λ

$$\boxed{\varepsilon = \varepsilon_\lambda} \qquad \boxed{\alpha = \alpha_\lambda}$$

- Applying Kirchoff's law to a grey body, $\alpha_\lambda = \varepsilon_\lambda$ and $\alpha = \varepsilon$

ε_λ is the spectral emissivity and is defined as the ratio of intensity of radiation emitted by a body at wavelength λ to that of the radiation emitted by a blackbody at the same temperature and wavelength

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$