Radiation

View factor

- According to Stefan-Boltzmann law, the energy emitted by a black body,

$$E_b = \sigma T^4$$

- If the body is non-black, the energy emitted can be measured as,

$$E = \varepsilon E_b = \varepsilon \sigma T^4$$

- or,

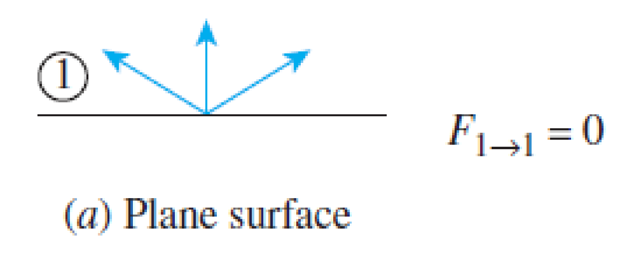
$$\frac{Q}{A} = \varepsilon \sigma T^4$$

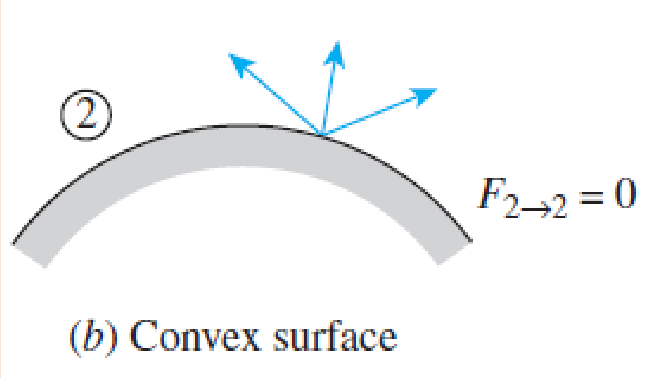
$\Rightarrow$ ***This is for a single body emitting energy***

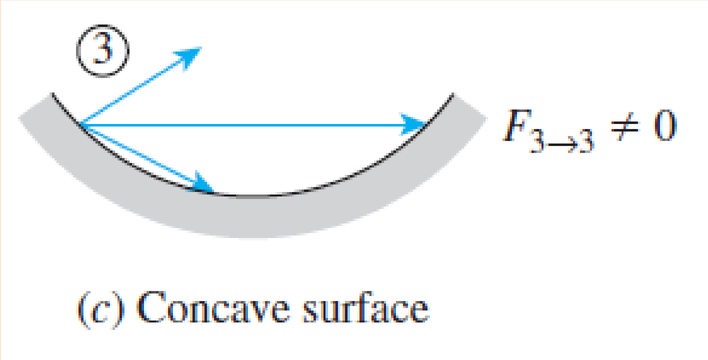
- However, when we have radiative heat transfer between two surfaces, apart from the temperature (T) and emissivities (ε), another factor becomes important – this is known as view factor or 'how well one surface can see the other'
- ***View factor (F_{ij}) is the fraction of the total radiant energy that is emitted by surface i which is intercepted by surface j***
- A diffuse radiating surface emits radiation in all directions so all the radiation emitted may not fall on another surface in the neighbourhood
- The radiation intercepted by surface j will be fully absorbed if j is a black body – otherwise, it will be partly absorbed and the rest reflected and/or transmitted

- The view factor from a surface to itself will be zero unless it can see itself

$F_{ii} = 0$ for plane and convex surfaces but not for a concave surface







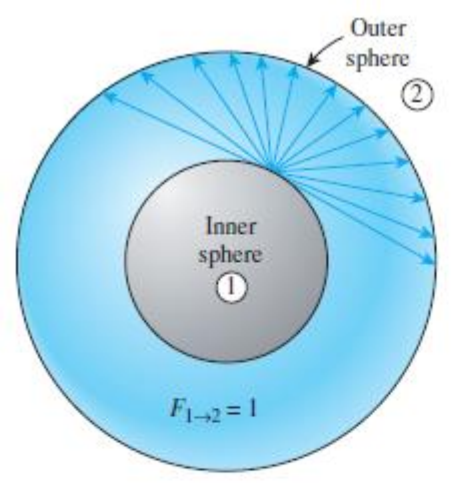
- The view factor ranges from 0 to 1

a) $F_{ij} = 0$ means the two surfaces do not have a direct view of each other

b) $F_{ij} = 1$ means the entire radiation emitted by i is intercepted by j

for eg., two concentric spheres, entire radiation leaving the surface of the inner sphere (1) will strike the larger sphere (2)

Thus, $F_{12} = 1$



View factor relations

Reciprocity rule

- The energy leaving surface 1 and arriving at surface 2 = $E_{b1}A_1F_{12}$
- Energy leaving surface 2 and arriving at surface 1 is = $E_{b2}A_2F_{21}$
- Since the surfaces are black, all the incident radiation is absorbed and the net heat exchange is,

$$Q_{12} = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$$

- If both the surfaces are at the same temperature, there can be no heat exchange,

$$Q_{12} = 0$$

- Also for $T_1 = T_2$, $E_{b1} = E_{b2}$

and,

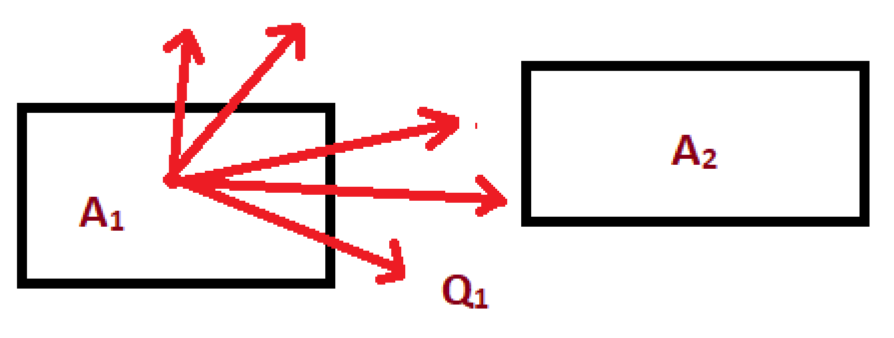
$$\boxed{A_1F_{12} = A_2F_{21}}$$

- The net heat exchange is

$$Q_{12} = A_1F_{12}(E_{b1} - E_{b2}) = A_2F_{21}(E_{b1} - E_{b2})$$

- Thus, reciprocity rule is given as ,

$$\boxed{A_iF_{ij} = A_jF_{ji}}$$



Two black bodies

Though this is derived for blackbodies, it holds for other surfaces as long as diffuse radiation is involved

Summation rule

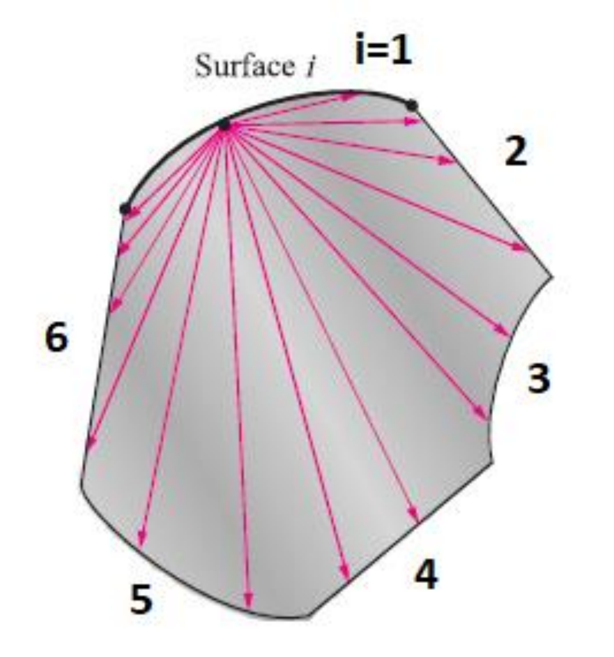
- The radiation leaving any surface i of an enclosure must be intercepted completely by the surfaces of the enclosure

- Therefore, the sum of the view factors from surface i of an enclosure to all surfaces, including itself, must be equal to unity

$$\sum_{j=1}^{N} F_{ij} = 1$$

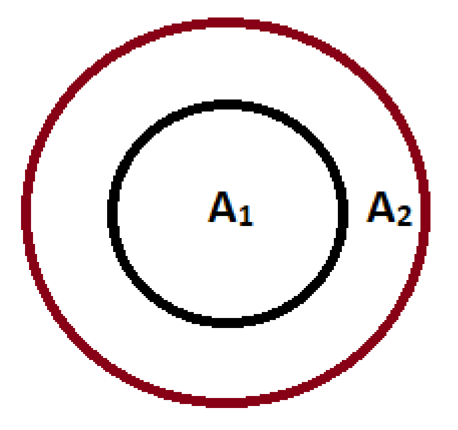
$$F_{11} + F_{12} + F_{13} + F_{14} + F_{15} + F_{16} = 1$$

- This is called the ***summation rule***



Problem

For the enclosure formed between two concentric spheres ($R_2 = 2R_1$), find the radiation leaving surface A_2 that strikes itself (F_{22})



As all the radiation leaving surface 1 (inner sphere) strikes surface 2 (outer sphere)

$$F_{12} = 1$$

Also, no radiation leaving surface 1 is intercepted by itself, hence,

$$F_{11} = 0$$

From the reciprocity rule, $A_1 F_{12} = A_2 F_{21}$

$$\therefore \quad F_{21} = \frac{A_1}{A_2} F_{12} = F_{12} \times \frac{4\pi R_1^2}{4\pi R_2^2}$$

$$F_{21} = F_{12} \times \left(\frac{R_1}{R_2}\right)^2 = F_{12} \times \left(\frac{R_1}{2R_1}\right)^2 = 1 \times \frac{1}{4} = \frac{1}{4}$$

From the summation rule, $F_{21} + F_{22} = 1$

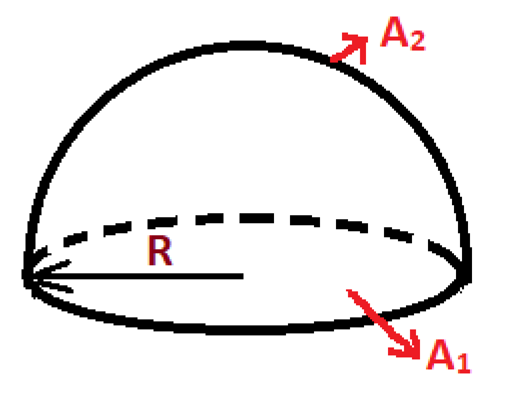
$$\therefore \quad F_{22} = 1 - F_{21}$$

$$F_{22} = 1 - \frac{1}{4}$$

$$\boldsymbol{F_{22} = \frac{3}{4}}$$

Problem

Determine the view factors between a plane A_1 covered by a hemisphere A_2



As all the radiation leaving surface 1 (plane) strikes surface 2 (hemisphere)

$$F_{12} = 1$$

From the reciprocity rule, $A_1 F_{12} = A_2 F_{21}$

Now $A_1 = \pi R^2$ and $A_2 = 2\pi R^2$

$$\therefore \quad F_{21} = \frac{A_1}{A_2} F_{12} = F_{12} \times \frac{\pi R^2}{2\pi R^2}$$

$$F_{21} = F_{12} \times \frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2}$$

For surface A_1, no radiation leaving the surface is intercepted by itself, hence,

$$F_{11} = 0$$

For surface A_2, $F_{21} + F_{22} = 1$

$$\therefore \quad F_{22} = 1 - F_{21}$$

$$F_{22} = 1 - \frac{1}{2} = \frac{1}{2}$$

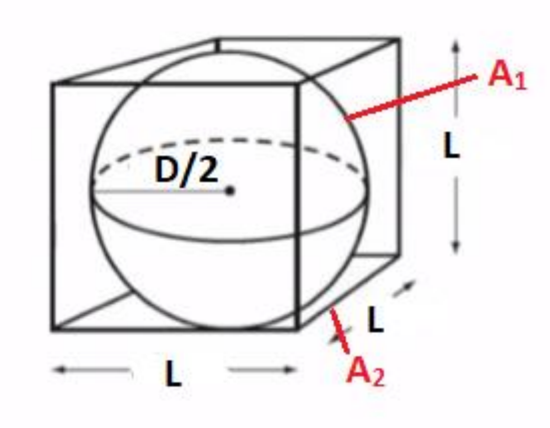
Therefore, $\boldsymbol{F_{11} = 0, F_{12} = 1, F_{22} = \frac{1}{2}, F_{21} = \frac{1}{2}}$

Problem

Determine the view factors F_{12} and F_{21} for the following geometries,

(i) Sphere of diameter D inside a cubical box of length = D

(ii) Diagonal partition within the long, square duct



(i) For the sphere inside the box, $L = D$

$F_{12} = 1$ and $F_{11} = 0$ since $F_{11} + F_{12} = 1$

From the reciprocity rule, $A_1F_{12} = A_2F_{21}$

Now $A_1 = \pi D^2$ and $A_2 = 6L^2$

$$\therefore \quad F_{21} = \frac{A_1}{A_2}F_{12} = F_{12} \times \frac{\pi D^2}{6L^2} = F_{12} \times \frac{\pi D^2}{6D^2}$$

$$F_{21} = F_{12} \times \frac{\pi}{6} = 1 \times \frac{\pi}{6} = \frac{\pi}{6}$$

For surface A_2, $F_{21} + F_{22} = 1$

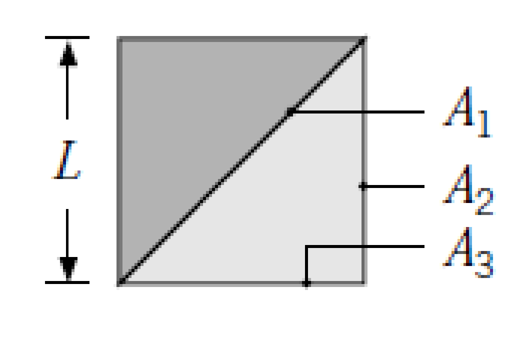
$$\therefore \quad F_{22} = 1 - F_{21}$$

$$F_{22} = 1 - \frac{\pi}{6} = 0.4764$$

Therefore, $\boldsymbol{F_{12} = 1, , F_{21} = \frac{\pi}{6}}$

Determine the view factors F_{12} and F_{21} for the following geometries,

(ii) Diagonal partition within the long, square duct



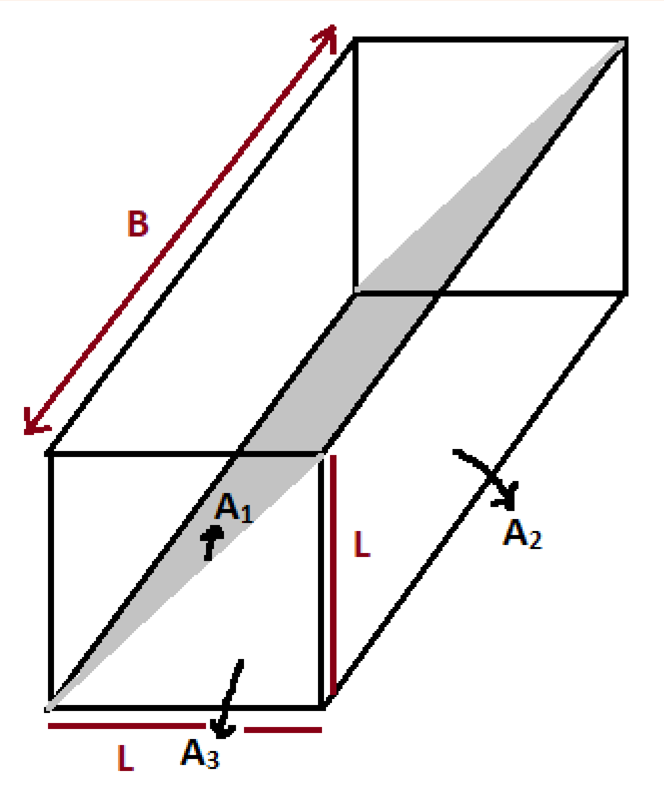
For the long square duct , $F_{11} + F_{12} + F_{13} = 1$

Now, $F_{11} = 0$ and $F_{12} = F_{13}$ (symmetry)

From the summation rule, $F_{11} + 2F_{12} = 1$

$$2F_{12} = 1$$

$$F_{12} = 0.5$$



Now $A_1 = \sqrt{2}L \times B$ and $A_2 = L \times B$

$$\therefore \quad F_{21} = \frac{A_1}{A_2} F_{12} = F_{12} \times \frac{\sqrt{2}L \times B}{L \times B} = F_{12} \times \frac{\sqrt{2}L}{L}$$

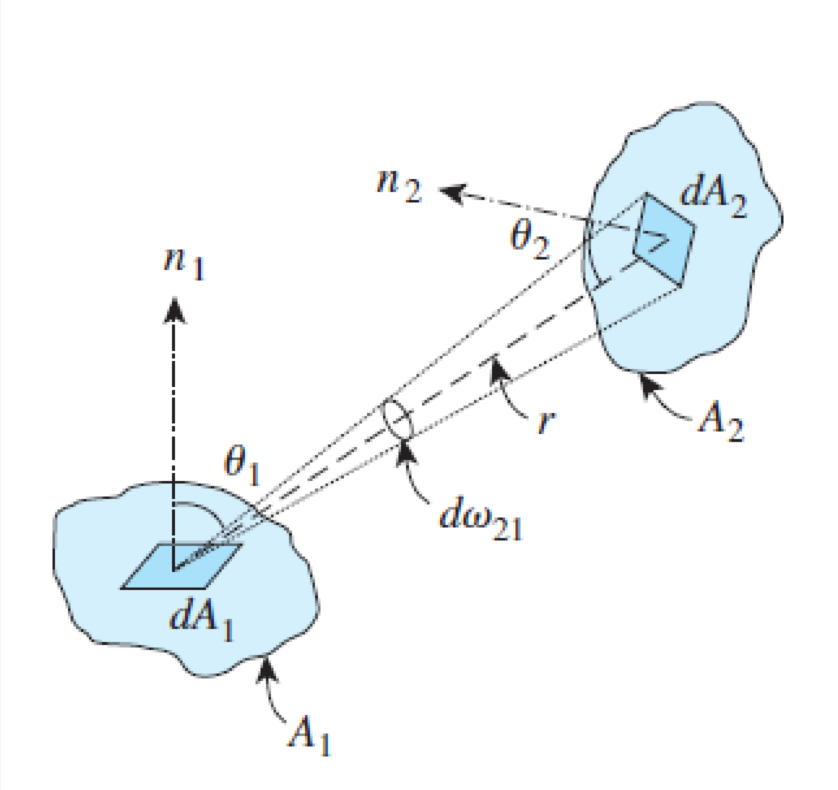
$$F_{21} = F_{12} \times \sqrt{2} = 0.5 \times \sqrt{2} = 0.71$$

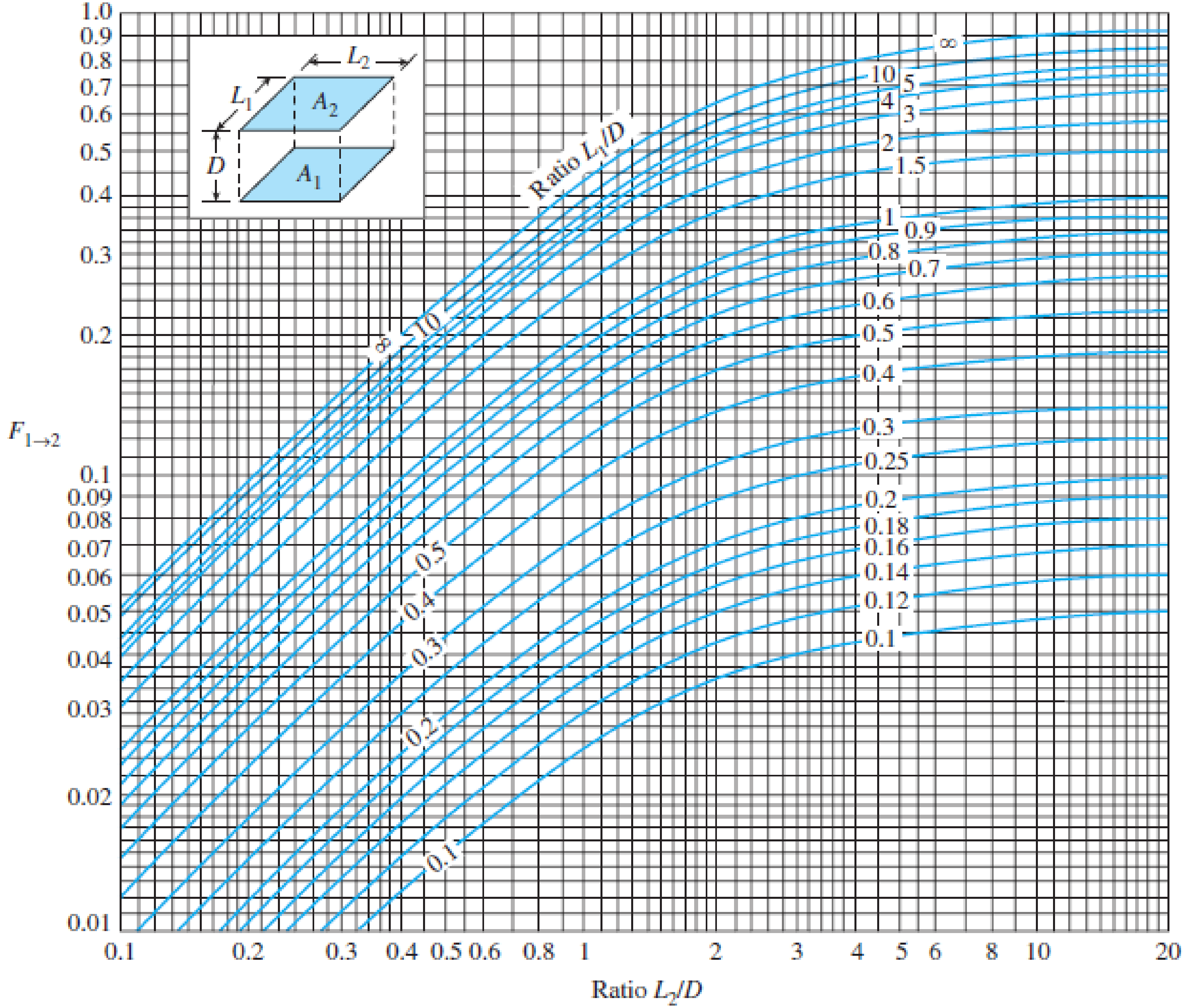
Therefore, $\boldsymbol{F_{12} = 0.5}$, , $\boldsymbol{F_{21} = 0.71}$

- The examples given before helps us estimate view factors for simple geometries
- However, if the shapes and orientations are more complicated the view factors need to be estimated
- For such cases, a general relationship has been developed

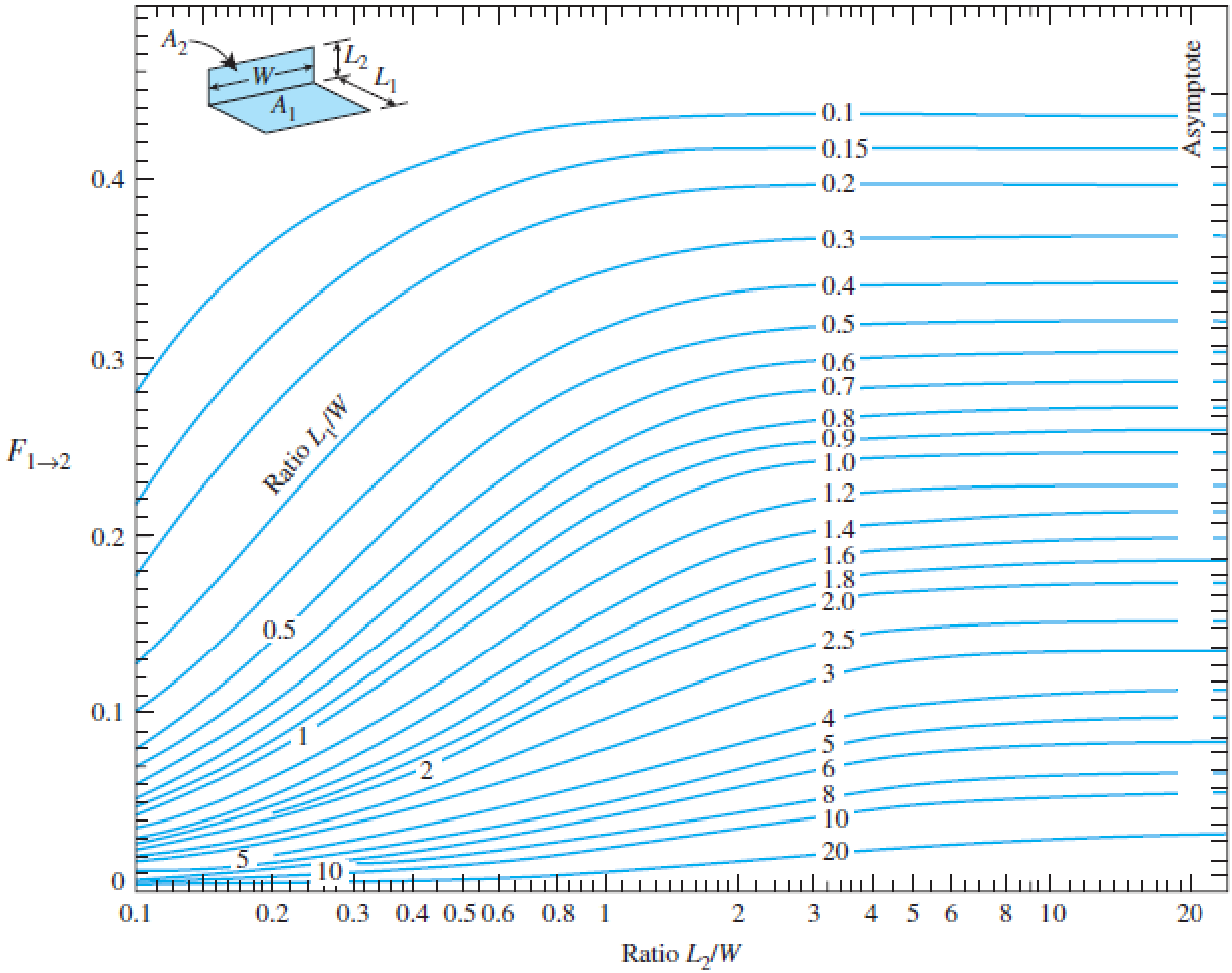
$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{cos\phi_1 cos\phi_2}{\pi r^2} dA_1 dA_2$$

- The values of F_{12} can be determined depending on the geometry of the surfaces
- Hottel integrated this relationship for some common geometries and the values are available in the form of charts
- The charts (a few samples are given in the next three slides) along with the relationships for view factors are used to determine the values for different F_{ij}s
- There are several other Hottel charts apart from the ones shown here

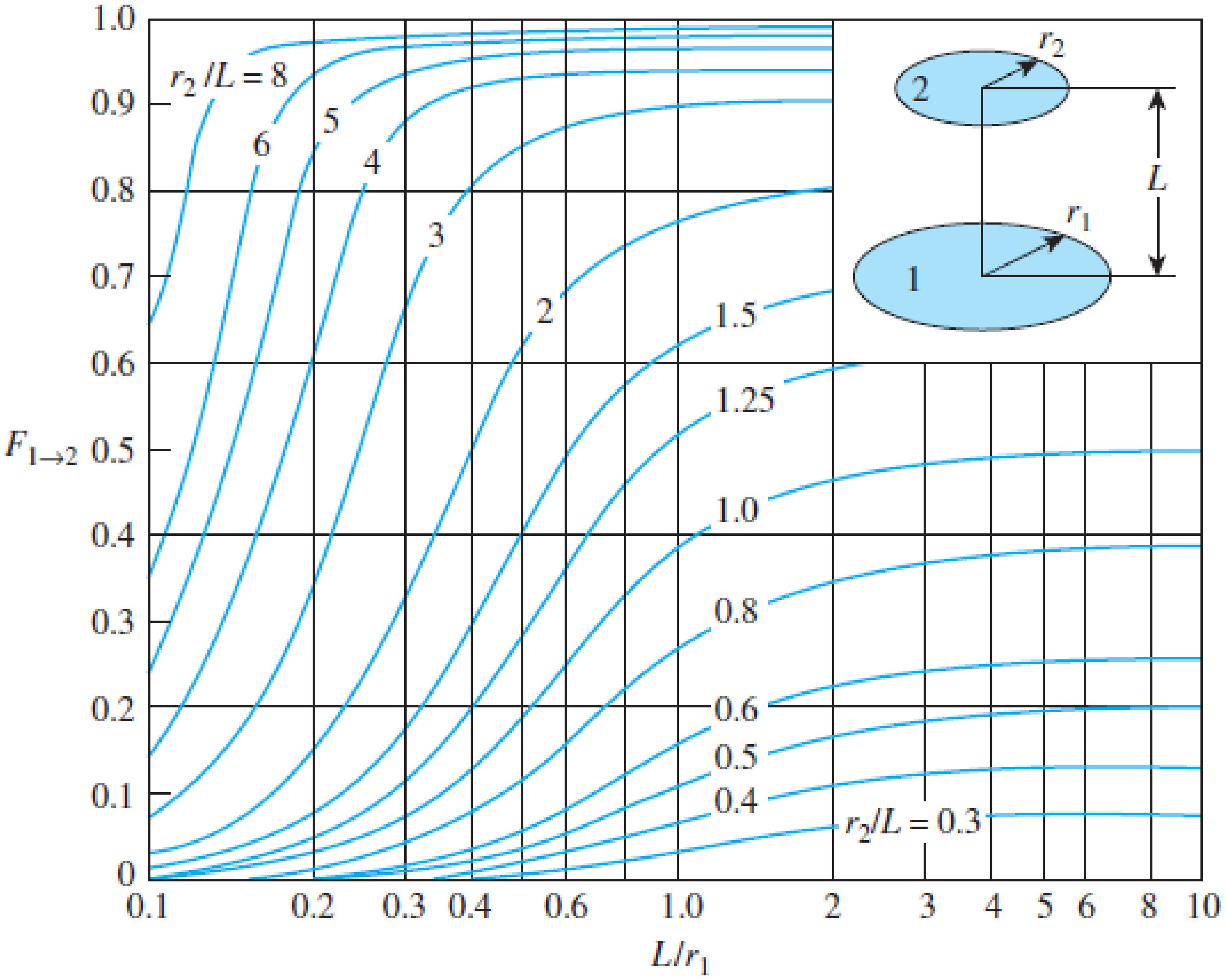




(a) View factor between two aligned parallel rectangles of equal size



(b) View factor between two perpendicular rectangles with a common edge.



(c) View factor between two coaxial parallel disks.

Superposition rule

- The view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to parts of surface j

- This is called the ***superposition rule***

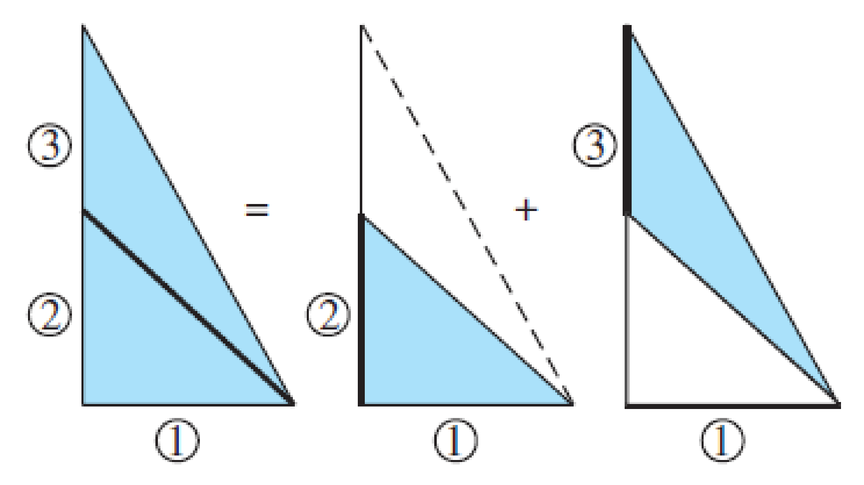
$$F_{1(23)} = F_{12} + F_{13}$$

- $F_{1(23)}$ and F_{12} can be determined from the charts (Chart b) and then F_{13} can be estimated from the above relation

- We can also estimate $F_{(23)1}$ by

$$(A_2 + A_3)F_{(23)1} = A_2\, F_{21} + A_3\, F_{31}$$

$$F_{(23)1} = \frac{A_2\, F_{21} + A_3\, F_{31}}{(A_2 + A_3)}$$



Symmetry rule

- Two or more surfaces that possess symmetry about a third surface will have identical view factors from the surface
- For e.g., the view factor from the face of a pyramid to each of the four sides - the box is a square and the sides are isosceles triangles

$$F_{12} = F_{13} = F_{14} = F_{15}$$

- According to summation rule,

$$F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

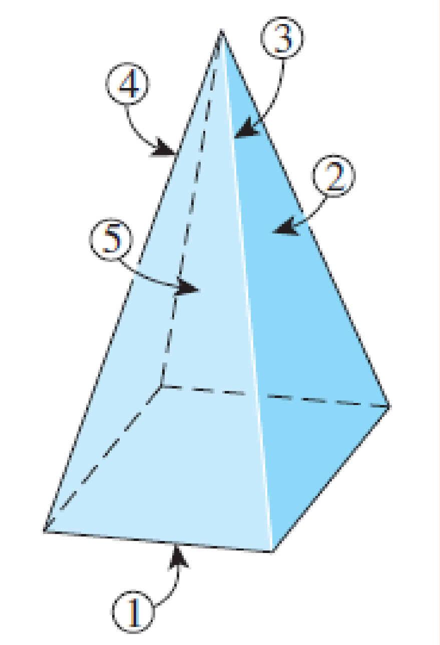
- Since $F_{11} = 0$,

$$F_{12} + F_{13} + F_{14} + F_{15} = 1$$

$$4F_{12} = 1$$

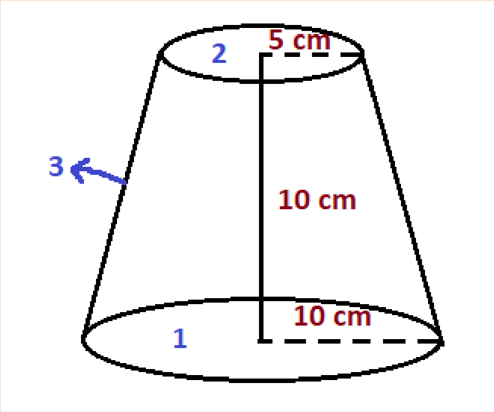
$$F_{12} = 0.25$$

$$F_{12} = F_{13} = F_{14} = F_{15} = 0.25$$



Problem

A truncated cone has top and bottom diameters of 10 cm and 20 cm. Calculate the shape factor between the top surface and the side and also the shape factor between the side and itself.



Top surface – 2, Bottom surface – 1, Side surface – 3

The problem is solved using the Hotell chart

We need to **find F_{23} and F_{33}**

Now, $\frac{L}{r_1} = \frac{10}{10} = 1$ and $\frac{r_2}{L} = \frac{5}{10} = 0.5$

From the figure, $F_{12} = 0.12$

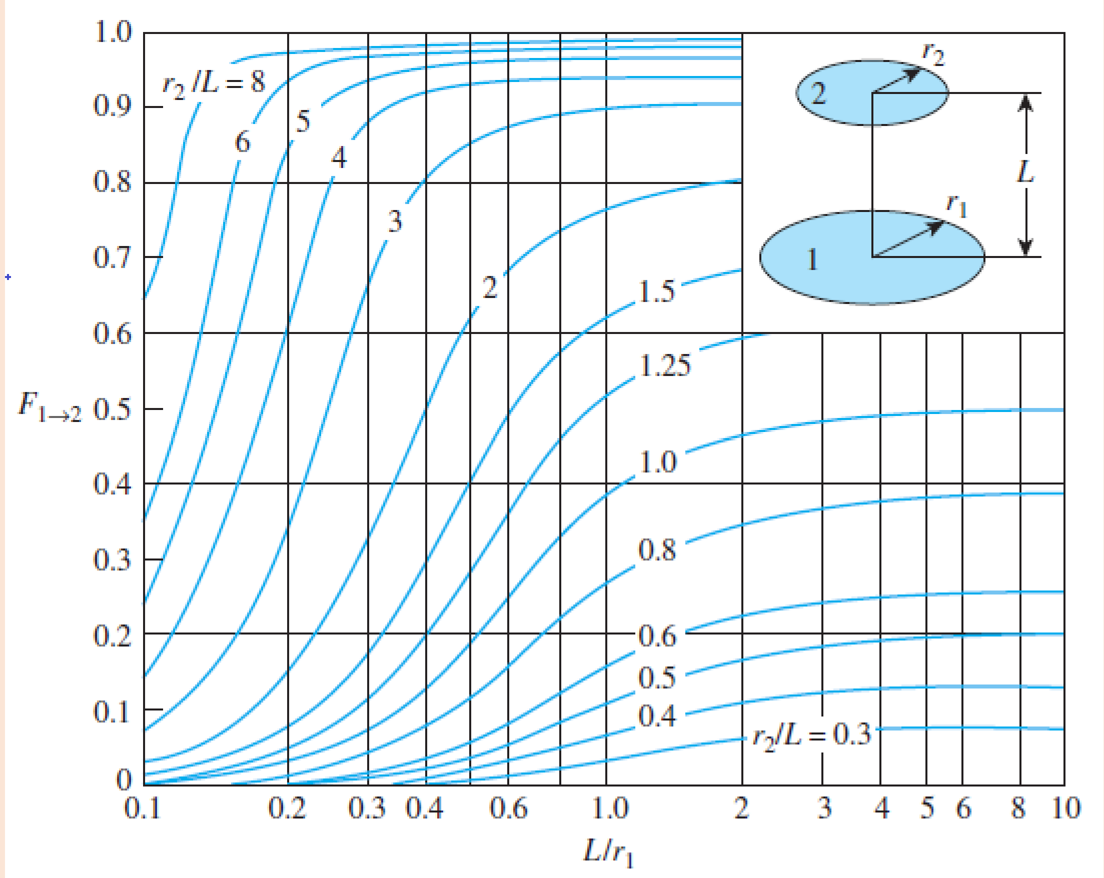
Now, , $A_1 F_{12} = A_2 F_{21}$

$$\therefore \quad F_{21} = \frac{A_1}{A_2} F_{12} = F_{12} \times \frac{\pi r_1^2}{\pi r_2^2} = 0.12 \times \frac{\pi \times 10^2}{\pi \times 5^2} = 0.48$$

Since, $F_{22} = 0$ and $F_{21} + F_{22} + F_{23} = 1$

$$F_{23} = 1 - F_{21} + F_{22} = 1 - 0.48 - 0 = 0.52$$

Therefore, $F_{23} = 0.52$



For surface – 1, $F_{11} + F_{12} + F_{13} = 1$

Since, $F_{11} = 0$, $F_{12} + F_{13} = 1$

$$F_{13} = 1 - F_{12} = 1 - 0.12 = 0.88$$

Now, , $A_1 F_{13} = A_3 F_{31}$ $\therefore \quad F_{31} = \frac{A_1}{A_3} F_{13}$

Surface area (lateral) of the side = $A_3 = \pi(r_1 + r_2)\sqrt{(r_1 - r_2)^2 + L^2}$

$$A_3 = \pi(10 + 5)\sqrt{(10 - 5)^2 + 10^2} = 526.886\, m^2$$

$$\therefore \quad F_{31} = 0.88 \times \frac{\pi \times 10^2}{526.86} = 0.525$$

For surface – 2, $F_{23} = 0.52\ (derived\ earlier)$ and $A_2 F_{23} = A_3 F_{32}$

$$\therefore \quad F_{32} = \frac{A_2}{A_3} F_{23} = 0.52 \times \frac{\pi \times 5^2}{526.86} = 0.0775$$

Finally, for surface – 3, $F_{31} + F_{32} + F_{33} = 1$

$$F_{33} = 1 - F_{31} - F_{32} = 1 - 0.525 - 0.0775 = 0.3975$$

Therefore, $\boldsymbol{F_{33} = 0.3975}$