Radiation

Radiation heat transfer between two surfaces

- Till now, all discussions were about radiation from a single body
- It is now necessary to look at radiative heat transfer between two surfaces

(a) Radiation heat exchange between two black bodies

- Two black bodies 1 and 2 have surface area A_1 and A_2 and temperatures T_1 and T_2
- The total emissive powers were E_{b1} and E_{b2}, respectively
- Rate at which radiation is emitted by body 1 and absorbed by body 2 $= Q_{12} = A_1 E_{b1} F_{12}$
- Rate at which radiation is emitted by body 2 and absorbed by body 1 $= Q_{21} = A_2 E_{b2} F_{21}$
- Therefore, net heat exchange,

$$Q = A_1 E_{b1} F_{12} - A_2 E_{b2} F_{21}$$

- Putting, $E_{b1} = \sigma T_1^4$ and $E_{b2} = \sigma T_2^4$ and using, $A_1 F_{12} = A_2 F_{21}$

$$Q = A_1 F_{12}(E_{b1} - E_{b2}) = A_1 F_{12}(\sigma T_1^4 - \sigma T_2^4)$$

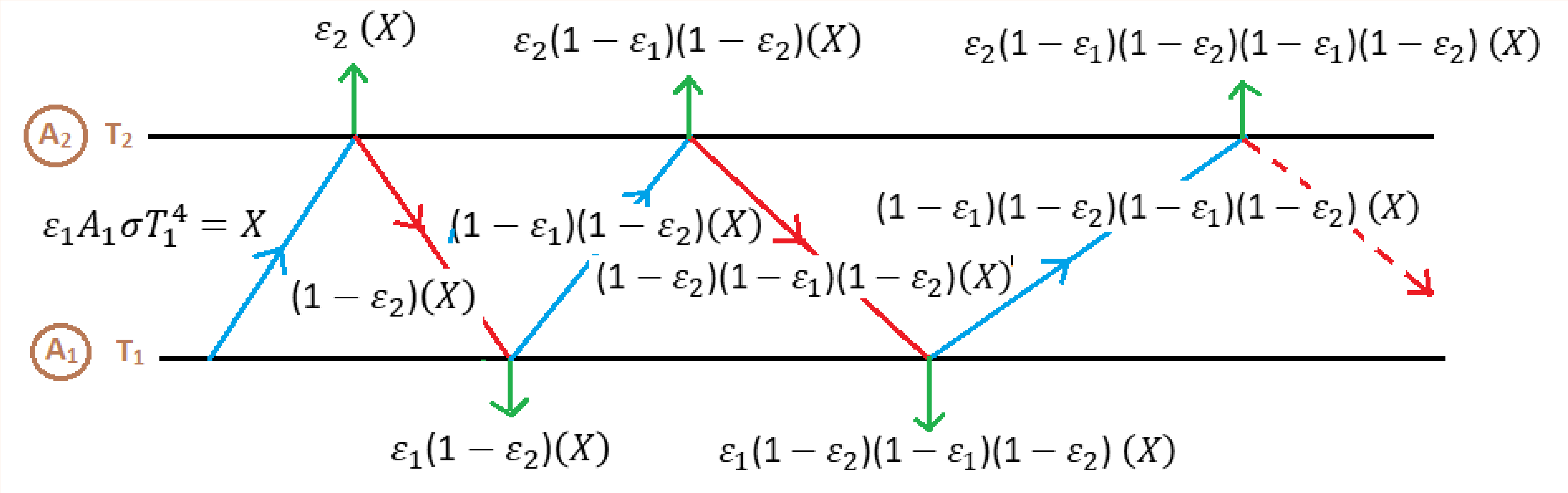
$$\boxed{Q = A_1 F_{12}\, \sigma (T_1^4 - T_2^4)}$$

- *If the bodies are infinite parallel plates,* $F_{12} = F_{21} = 1$

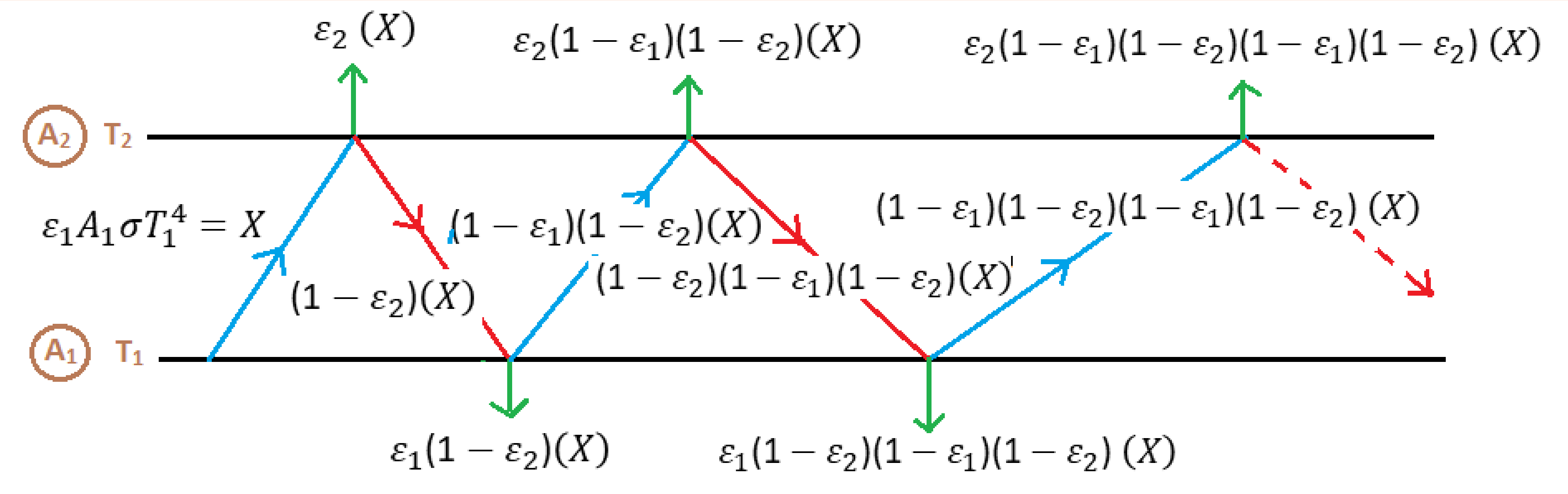
$$\boxed{Q = A_1\, \sigma (T_1^4 - T_2^4)}$$

(b) Radiation heat exchange between two parallel gray (non-black) planes

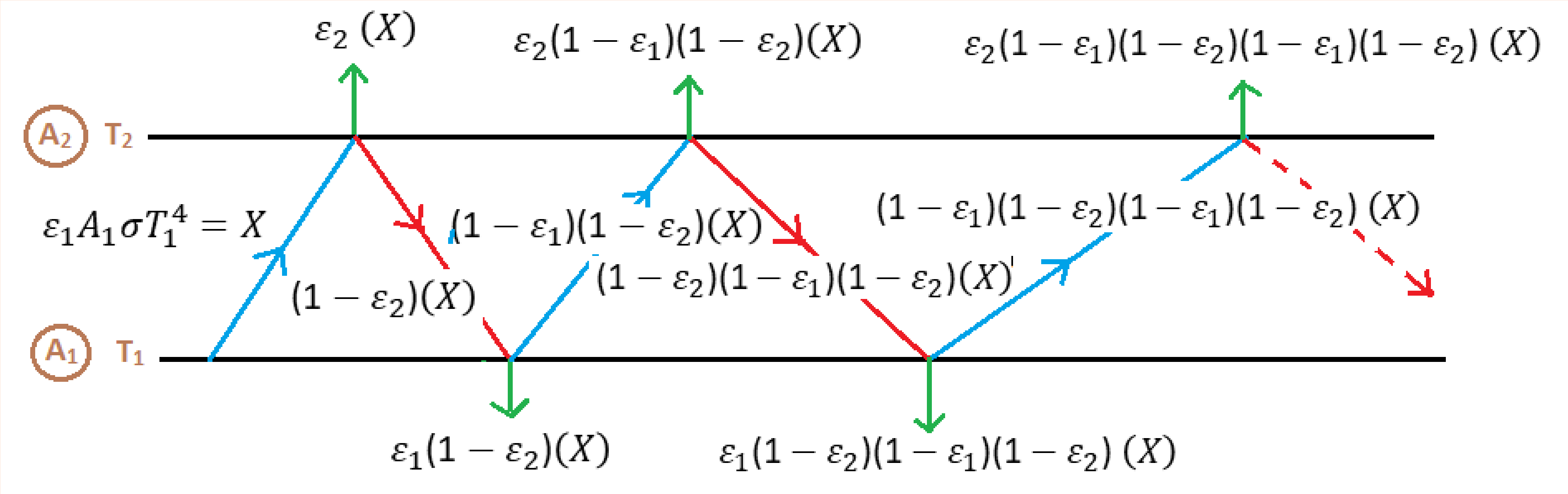
- If the two bodies are not black bodies , they have different emissivities and the net energy exchange is different
- Some of the energy emitted from the first plane will be absorbed, and the remainder radiated back to the source
- The emissivities and absorptivities of plane 1 and 2 are $\varepsilon_1 = \alpha_1$ and $\varepsilon_2 = \alpha_2$, respectively, areas are A_1 and A_2 and view factors are $F_{12} = F_{21} = 1$ *(parallel plates)*



- In unit time, surface A_1 emits radiation = $\varepsilon_1 A_1 \sigma T_1^4$ to surface A_2 and a fraction of this energy, ε_2 ($\varepsilon_2 = \alpha_2$) is absorbed by A_2
- $Energy\ \boldsymbol{radiated}\ from\ A_1 = \boldsymbol{\varepsilon_1 A_1 \sigma T_1^4 = X}$
- $Energy\ \boldsymbol{absorbed}\ by\ A_2 = \varepsilon_2\ (\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{\varepsilon_2\ (X)}$
- $Energy\ \boldsymbol{reflected\ back\ by\ A_2\ to\ A_1} = (1-\varepsilon_2)(\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{(1-\varepsilon_2)(X)}$



- *Of this amount, energy* ***reabsorbed*** *by* $A_1 = \varepsilon_1(1-\varepsilon_2)(\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{\varepsilon_1(1-\varepsilon_2)(X)}$
- *and energy* ***reflected by*** $\boldsymbol{A_1} = (1-\varepsilon_1)(1-\varepsilon_2)(\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{(1-\varepsilon_1)(1-\varepsilon_2)(X)}$
- *Of this reflected amount, energy* ***absorbed*** *by* $A_2 = \varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{\varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(X)}$
- *Energy* ***reflected back by*** $\boldsymbol{A_2}$ ***to*** $\boldsymbol{A_1} = (1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)(\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)(X)}$
- *Again of this amount, energy* ***reabsorbed*** *by* $A_1 = \varepsilon_1(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)(\varepsilon_1 A_1 \sigma T_1^4) = \boldsymbol{\varepsilon_1(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)\,(X)}$
- *and energy* ***reflected by*** $\boldsymbol{A_1 = (1-\varepsilon_1)(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)\,(X)}$
- *Of this reflected amount, energy* ***absorbed*** *by* $A_2 = \boldsymbol{\varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)\,(X)}$



- *This continues and the total energy sent from A_1 and absorbed by A_2*

$$Q_{12} = \varepsilon_2\ (X) + \varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(X) + \varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)\ (X) + \cdots$$

$$Q_{12} = \varepsilon_2\left(\varepsilon_1 A_1 \sigma T_1^4\right) + \varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)\left(\varepsilon_1 A_1 \sigma T_1^4\right) + \varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2)\left(\varepsilon_1 A_1 \sigma T_1^4\right) + \cdots$$

$$Q_{12} = A_1 \sigma T_1^4[\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2) + \varepsilon_1\varepsilon_2(1-\varepsilon_1)(1-\varepsilon_2)(1-\varepsilon_1)(1-\varepsilon_2) + \cdots]$$

$$Q_{12} = A_1 \sigma T_1^4 \frac{\varepsilon_1\varepsilon_2}{1-(1-\varepsilon_1)(1-\varepsilon_2)}$$

$$Q_{12} = A_1 \sigma T_1^4 \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

- Similarly, the amount absorbed at A_1 coming from A_2

$$Q_{21} = A_1 \sigma T_2^4 \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

- The net transfer of energy from 1 to 2 is

$$Q_{12} = A_1 \sigma \left(T_1^4 - T_2^4\right) \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

$\Rightarrow$ for ***parallel plates***

- In case of transfer of energy between ***concentric spheres or cylinders or other geometries***, then the equation becomes,

$$Q_{12} = A_1 \sigma \left(T_1^4 - T_2^4\right) \frac{1}{1/\varepsilon_1 + \frac{A_1}{A_2}\left(1/\varepsilon_2 - 1\right)}$$

Problem

Liquid nitrogen boiling at 77K (-196° C) is stored in a 15 litre spherical container of diameter 32 cm. The container is surrounded by a concentric spherical shell of diameter 36 cm at a temperature of 303 K (30°C) and the space between the two spheres is evacuated. The surfaces of the spheres facing each other are silvered and have an emissivity of 0.03. Taking the latent of vaporization for liquid nitrogen to be 201 kJ/kg, find the rate of which nitrogen evaporates. Also, find the rate of evaporation, if often the surfaces were black.

Here, $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$, $T_1 = 77$ K, $T_2 = 303$ K, $D_1 = 0.32$ m, $D_2 = 0.36$ m, $\varepsilon_1 = \varepsilon_2 = 0.03$

$A_1 = \pi D_1^2 = 0.3217 m^2$ $A_2 = \pi D_2^2 = 0.407\ m^2$

$$Q_{12} = A_1 \sigma (T_1^4 - T_2^4) \frac{1}{{}^1/_{\varepsilon_1} + \frac{A_1}{A_2}\left({}^1/_{\varepsilon_2} - 1\right)}$$

$$Q_{12} = 0.3217 \times 5.67 \times 10^{-8} (77^4 - 303^4) \frac{1}{{}^1/_{0.03} + \frac{0.3217}{0.407}\left({}^1/_{0.03} - 1\right)} = -2.5998\, W$$

Now, $\lambda = 201\ kJ/kg$

$$\text{Rate of evaporation} = \frac{Q}{\lambda}$$

$$= \frac{2.5998}{201 \times 1000} = 0.047\ kg/h$$

If both the bodies were black,

$$\varepsilon_1 = \varepsilon_2 = 1$$

and

$$Q_{12} = A_1 \sigma (T_1^4 - T_2^4) \frac{1}{{}^{1}/_{\varepsilon_1} + \frac{A_1}{A_2}\left({}^{1}/_{\varepsilon_2} - 1\right)}$$

$$Q_{12} = A_1 \sigma (T_1^4 - T_2^4)$$

$$Q_{12} = 0.3217 \times 5.67 \times 10^{-8} (77^4 - 303^4)$$

$$= -153.105\ W$$

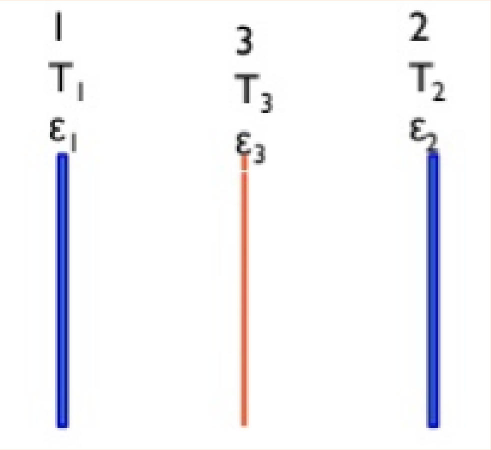
Now, $\lambda = 201\ kJ/kg$

Rate of evaporation $= \frac{Q}{\lambda}$

$$= \frac{153.105}{201 \times 1000} = 2.742\ kg/h$$

Radiation shield

- A radiation shield is a barrier wall of low emissivity placed between two walls in order to reduce the exchange of radiation between them
- The shield puts an additional resistance
- The net exchange between two initial planes is $Q = A_1\sigma\left(T_1^4 - T_2^4\right)\frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$
- If $\varepsilon_1 = \varepsilon_2$ but $\varepsilon_1 \neq \varepsilon_3$, the net exchange from 1 to 2 is given by,



$$Q_1 = A_1\sigma\left(T_1^4 - T_3^4\right)\frac{1}{1/\varepsilon_1 + 1/\varepsilon_3 - 1} = A_1\sigma\left(T_3^4 - T_2^4\right)\frac{1}{1/\varepsilon_3 + 1/\varepsilon_2 - 1}$$

or,

$$\left(T_1^4 - T_3^4\right) = \left(T_3^4 - T_2^4\right) \qquad \Rightarrow \qquad T_3^4 = \frac{T_1^4 + T_2^4}{2}$$

- Putting this value in the first equation we get,

$$Q_1 = A_1\sigma\left(T_1^4 - \frac{T_1^4 + T_2^4}{2}\right)\frac{1}{1/\varepsilon_1 + 1/\varepsilon_3 - 1} = A_1\sigma\frac{1}{2}\left(T_1^4 - T_2^4\right)\frac{1}{1/\varepsilon_1 + 1/\varepsilon_3 - 1}$$

- If the $\varepsilon_1 = \varepsilon_3$, $\quad Q_1 = \frac{Q}{2}$
- Or, $\quad Q_n = \frac{Q}{n+1}$

If the emissivity of the shield is the same as the initial plane, the heat exchange reduces by $\left(\frac{1}{n+1}\right)$

This kind of shield is often used in the cryogenic industry