Radiation

Heat exchange between non-black bodies

- The estimation of radiation heat transfer between black surfaces is relatively easy as all the radiant energy which strikes a surface is absorbed
- The main problem is the estimation of the view factor $[Q = A_1 F_{12} \sigma (T_1^4 - T_2^4)]$

- When non-black bodies are involved, the situation is more complex
- For all the energy striking a surface, part is absorbed and part reflected (part may also be transmitted if transmittance is not zero)
- Problem becomes more complicated if we consider the reflections occurring back and forth several times (as was done earlier for gray bodies)

For the development of a general expression, all surfaces are considered (i) diffuse, (ii) uniform in temperature and (iii) the reflective and emissive properties are constant over the surface

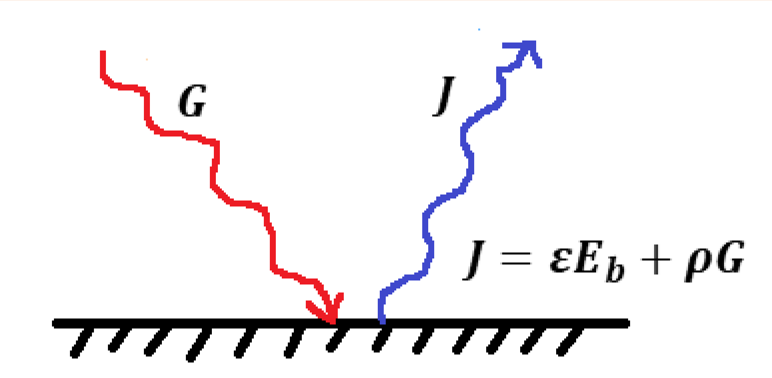
Two new terms are defined –

- **Irradiation** (G) - total radiation incident upon a surface per unit time per unit area
- **Radiosity** (J) - total radiation leaving a surface per unit time per unit area (includes both emission and reflection)

$$J = \varepsilon E_b + \rho G$$

transmittance = 0

energy emitted (εE_b) energy reflected (ρG)



Since $\alpha + \rho + \tau = 1$

If $\tau = 0$, $\rho = 1 - \alpha = 1 - \varepsilon$ (using Kirchoff's law)

$$\therefore \quad J = \varepsilon E_b + (1 - \varepsilon)G$$

$$G = \frac{J - \varepsilon E_b}{(1 - \varepsilon)}$$

Net energy leaving the surface is the difference of radiosity and irradiation

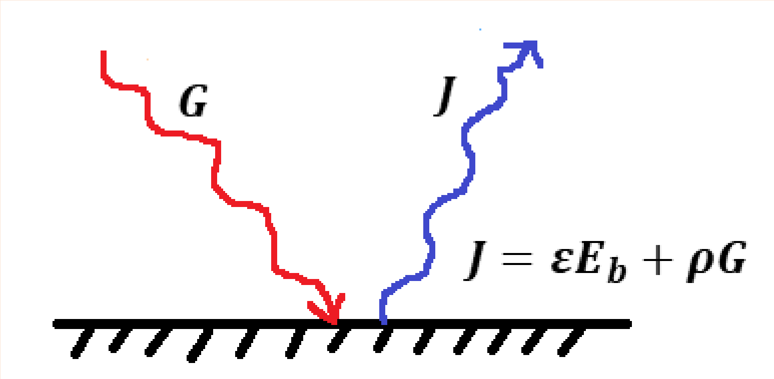
$$\frac{Q}{A} = J - G = \varepsilon E_b + (1 - \varepsilon)G - G$$

$$\therefore \quad \frac{Q}{A} = \varepsilon(E_b - G)$$

$$\frac{Q}{A} = \varepsilon\left(E_b - \frac{J - \varepsilon E_b}{(1-\varepsilon)}\right) = \varepsilon\left(\frac{E_b - \varepsilon E_b - J + \varepsilon E_b}{(1-\varepsilon)}\right)$$

$$\frac{Q}{A} = \frac{\varepsilon(E_b - J)}{(1-\varepsilon)}$$

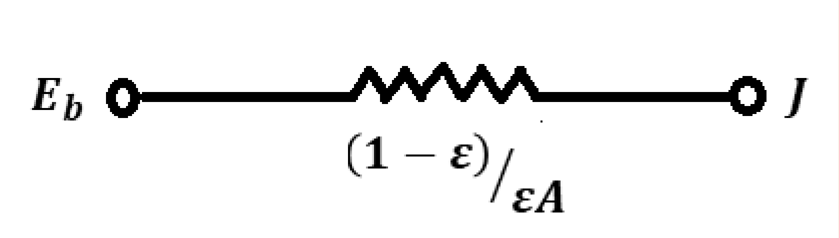
$$\boxed{Q = \frac{(E_b - J)}{\frac{(1-\varepsilon)}{\varepsilon A}}}$$



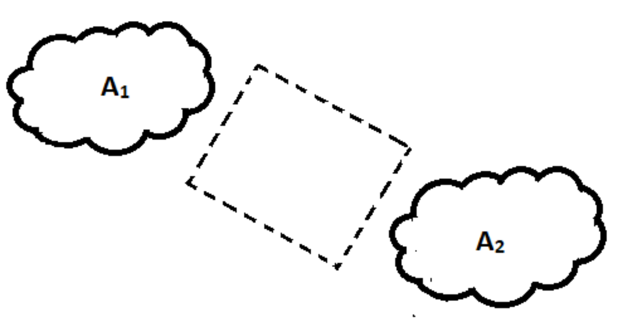
Numerator is the ***potential difference***

Denominator is the ***surface resistance to heat transfer by radiation***

This is often represented as an electrical network



- Let us now consider exchange of energy (radiant energy) by two surfaces A_1 and A_2
- Of the total radiation that leaves surface 1, the amount that reaches surface 2 is $= J_1 A_1 F_{12}$
- Of the total radiation that leaves surface 2, the amount that reaching surface 1 is $= J_2 A_2 F_{21}$
- Net interchange of energy between two surfaces, $Q_{12} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$
- Since, $A_1 F_{12} = A_2 F_{21}$



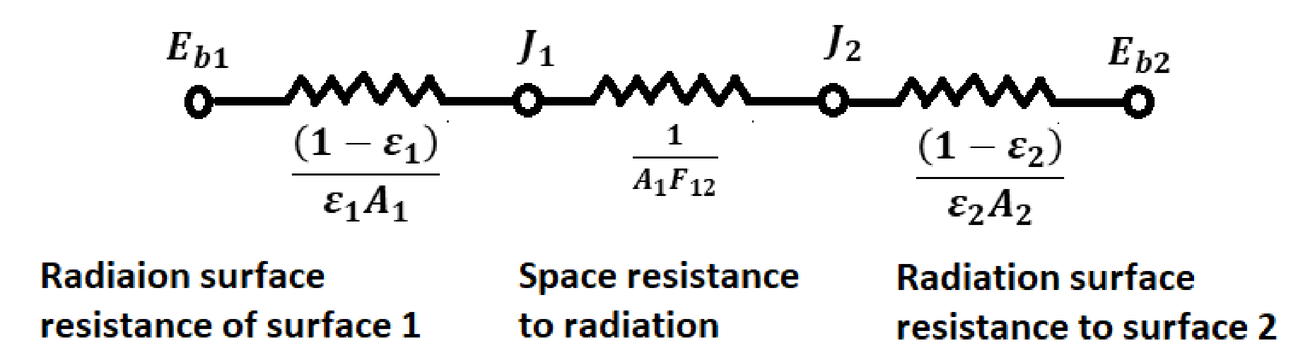
$$Q_{12} = (J_1 - J_2) A_1 F_{12}$$

$$\boldsymbol{Q_{12} = \frac{(J_1 - J_2)}{\frac{1}{A_1 F_{12}}}}$$

- Denominator is the ***space resistance between the radiosity potentials***

Therefore, two surfaces that exchange heat with each other and nothing else is represented by,

$$\boldsymbol{Q_{12} = \frac{(E_{b1} - E_{b2})}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}}}$$



Radiation heat transfer between two surfaces

(i) For two parallel planes (also discussed earlier)

As the surfaces are infinite in length, $A_1 = A_2 = A$ and $F_{12} = 1.0$

$$Q_{12} = \frac{(E_{b1} - E_{b2})}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}}$$

$$Q_{12} = \frac{(\sigma T_1^4 - \sigma T_2^4)}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A} + \frac{1}{A} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A}}$$

$$Q_{12} = \frac{\sigma (T_1^4 - T_2^4) A}{\frac{1}{\varepsilon_1} - 1 + 1 + \frac{1}{\varepsilon_2} - 1}$$

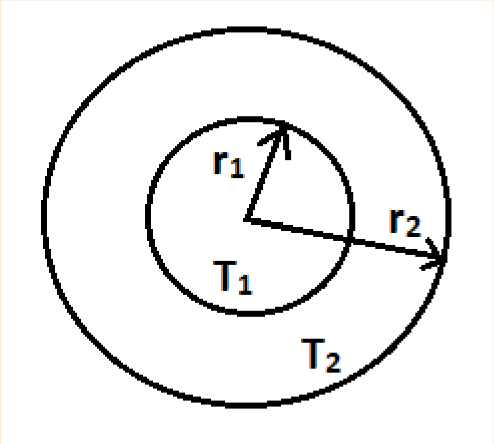
$$\boxed{Q_{12} = \frac{\sigma A (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}}$$

The same equation was derived earlier

For black surfaces, $\varepsilon_1 = \varepsilon_2 = 1$

$$Q_{12} = \sigma A (T_1^4 - T_2^4)$$

(ii) For two concentric infinitely long cylinders



As the cylinders are concentric, $F_{11} = 0$ and $F_{11} + F_{12} = 1.0$,

Since $F_{11} + F_{12} = 1.0$, $F_{12} = 1.0$

$$Q_{12} = \frac{(E_{b1} - E_{b2})}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}}$$

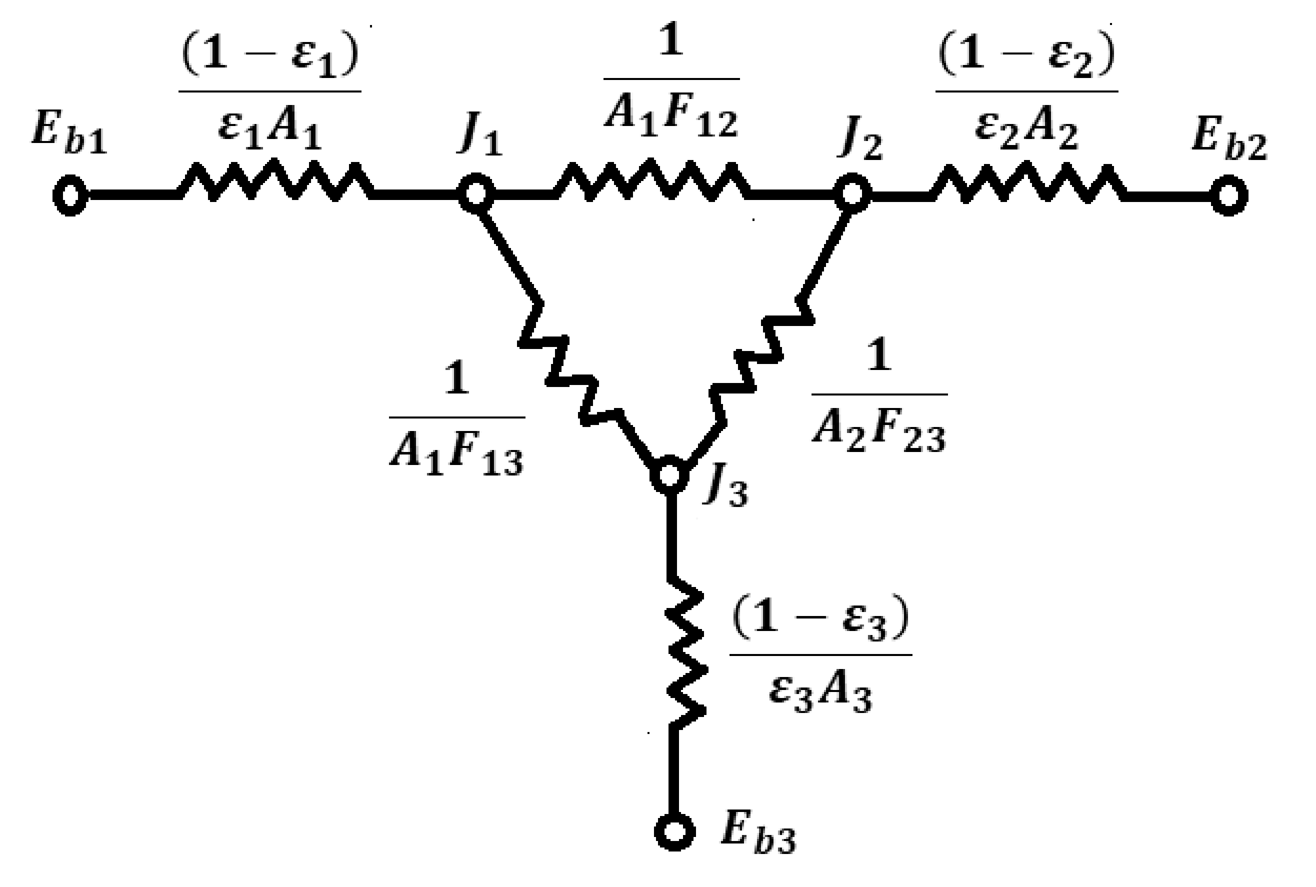
$$Q_{12} = \frac{(\sigma T_1^4 - \sigma T_2^4)}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} + \frac{1}{A_1} + \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}}$$

$$Q_{12} = \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\varepsilon_1} - 1 + 1 + \frac{(1-\varepsilon_2)}{\varepsilon_2}\frac{A_1}{A_2}}$$

$$\boxed{Q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2}}}$$

The same equation was also shown earlier

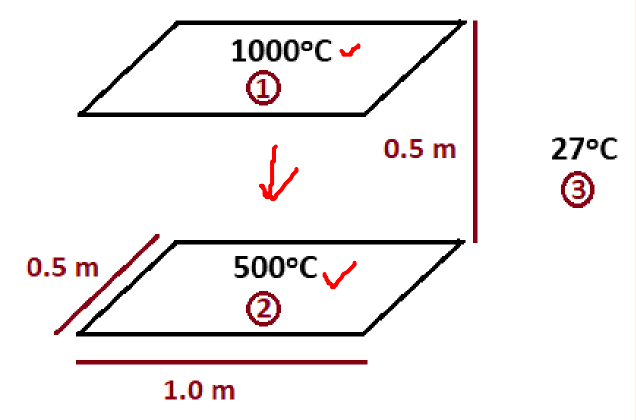
- For a three-body problem – two surfaces exchanging heat with each other and with the surroundings we have,

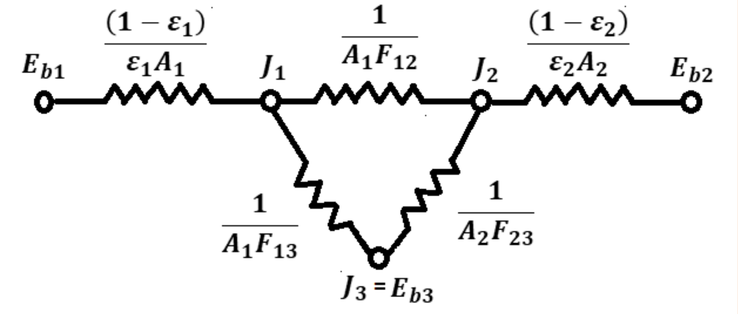


- If one surface (say A_3) is very large, $J_3 = E_{b3}$ as $\frac{(1-\varepsilon_3)}{\varepsilon_3 A_3} \to 0$

Problem

Two parallel plates 0.5 m by 1.0 m are spaced 0.5 m apart. One plate is maintained at 1000°C and the other at 500°C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27°C. The plates exchange energy with each other and the room but the plate surface facing each other should be considered in the analysis. Find the net heat transfer to each plate and to the room.





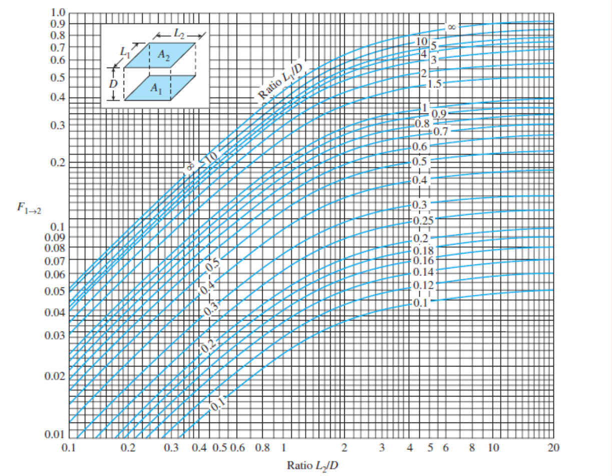
Here, $T_1 = 1273$ K, $T_2 = 773$ K, $T_2 = 300$ K $D = 0.5$ m,

σ = 5.67 x 10^{-8} W/m^{2}K^4 , $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.5$, $A_1 = A_2 = 0.5\ m^2$

As surface A_3 (surrounding) is very large, $\frac{(1-\varepsilon_3)}{\varepsilon_3 A_3} \rightarrow 0$ and $J_3 = E_{b3}$

Shape factor F_{12} is obtained from the Hottel charts

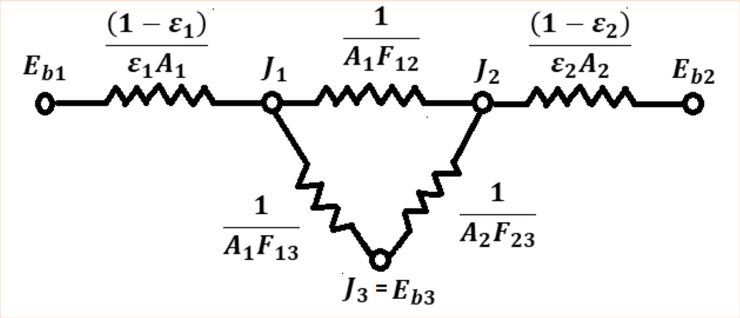
$$F_{12} = 0.285$$



(a) View factor between two aligned parallel rectangles of equal size

As $A_1F_{12} = A_2F_{21}$,

$$F_{21} = 0.285$$



- Again, $F_{11} + F_{12} + F_{13} = 1$ and $F_{11} = 0$

  Therefore, $F_{13} = 1 - F_{12} = 1 - 0.285 = 0.715$
- Now, $F_{21} + F_{22} + F_{23} = 1$ and $F_{22} = 0$

  Therefore, $F_{23} = 1 - F_{21} = 1 - 0.285 = 0.715$
- The resistances are calculated as,

$$\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1} = \frac{(1-0.2)}{0.2 \times 0.5} = 8.0 \qquad \frac{(1-\varepsilon_2)}{\varepsilon_2 A_2} = \frac{(1-0.5)}{0.5 \times 0.5} = 2.0$$

$$\frac{1}{A_1F_{12}} = \frac{1}{0.5 \times 0.285} = 7.018 \qquad \frac{1}{A_1F_{13}} = \frac{1}{0.5 \times 0.715} = 2.797 \qquad \frac{1}{A_2F_{23}} = \frac{1}{0.5 \times 0.715} = 2.797$$

- **The sum of the currents in each node is zero**,
- At node J_1,

$$\frac{(E_{b1}-J_1)}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1}} + \frac{(J_2-J_1)}{\frac{1}{A_1F_{12}}} + \frac{(E_{b3}-J_1)}{\frac{1}{A_1F_{13}}} = \frac{(E_{b1}-J_1)}{8} + \frac{(J_2-J_1)}{7.018} + \frac{(E_{b3}-J_1)}{2.797} = 0$$

- At node J_2,

$$\frac{(J_1-J_2)}{\frac{1}{A_1F_{12}}} + \frac{(E_{b3}-J_2)}{\frac{1}{A_2F_{23}}} + \frac{(E_{b2}-J_2)}{\frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}} = \frac{(J_1-J_2)}{7.018} + \frac{(E_{b3}-J_2)}{2.797} + \frac{(E_{b2}-J_2)}{2.0} = 0$$

- Now, $E_{b1} = \sigma T_1^4 = 148.87 \frac{\text{kW}}{m^2}$, $E_{b2} = \sigma T_2^4 = 20.24 \frac{\text{kW}}{m^2}$ and $E_{b3} = \sigma T_3^4 = 0.4592 \frac{\text{kW}}{m^2}$
- Putting these values in the two equations at the node, $J_1 = 33.469 \frac{\text{kW}}{m^2}$ and $J_2 = 15.054 \frac{\text{kW}}{m^2}$
- Total heat lost by plate 1,

$$Q_1 = \frac{(E_{b1} - J_1)}{\frac{(1-\varepsilon_1)}{\varepsilon_1 A_1}} = \frac{(148.87 - 33.469)}{8}$$

$$Q_1 = 14.425\ kW$$

Heat lost by plate 2,

$$Q_2 = \frac{(E_{b2} - J_2)}{\frac{(1-\varepsilon_2)}{\varepsilon_2 A_2}} = \frac{(20.241 - 15.054)}{2.0}$$

$$Q_1 = 2.594\ kW$$

Heat received by room,

$$Q_3 = \frac{(J_1 - J_3)}{\frac{1}{A_1 F_{13}}} + \frac{(J_2 - J_3)}{\frac{1}{A_2 F_{23}}} = \frac{(33.469 - 0.4592)}{2.797} + \frac{(15.054 - 0.4592)}{2.797}$$

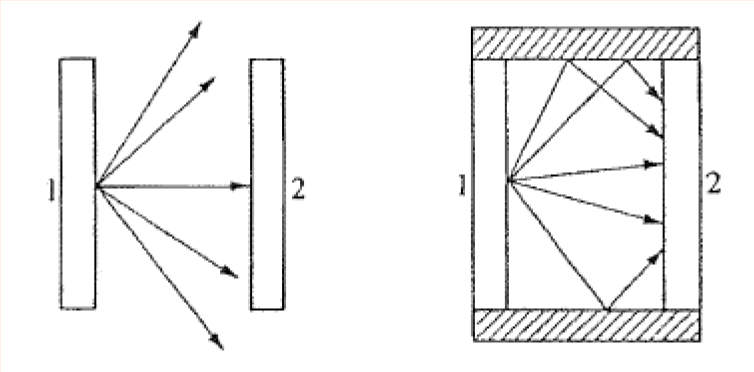
$$Q_1 = 17.020\ kW$$

Also, $Q_1 + Q_2 = Q_3$

Heat transfer when surfaces are connected by re-radiating walls

- *A reradiating surface is one which radiates the entire amount of radiation that it receives from other surfaces*
- If two black bodies having areas A_1 and A_2 are connected by nonconducting (refractory) but reradiating walls, a larger portion of the radiation from surface 1 as intercepted by 2

- The new view factor is $\overline{F_{12}}$
- For eg., two surfaces connected by the walls of an enclosure (such as a furnace)
- $\overline{F_{12}}$ calculated assuming uniform refractory temperature and radiant sources A_1 and A_2 that are not concave is given by,



$$\overline{F_{12}} = \frac{A_2 - A_1 F_{12}^2}{A_1 + A_2 - 2A_1 F_{12}} = \frac{1 - \left({}^{A_1}/_{A_2}\right) F_{12}^2}{\left({}^{A_1}/_{A_2}\right) + 1 - 2\left({}^{A_1}/_{A_2}\right) F_{12}}$$

- As before $A_1 \overline{F_{12}} = A_2 \overline{F_{21}}$

$$Q = \overline{F_{12}}\, A_1\, \sigma(T_1^4 - T_2^4)$$

- If surfaces are gray, emissivities have to be taken into consideration

$$Q = \frac{A_1\,\sigma(T_1^4 - T_2^4)}{\frac{1}{\overline{F}_{12}} + \left(\frac{1}{\varepsilon_1} - 1\right) + \left({}^{A_1}/_{A_2}\right)\left(\frac{1}{\varepsilon_2} - 1\right)}$$

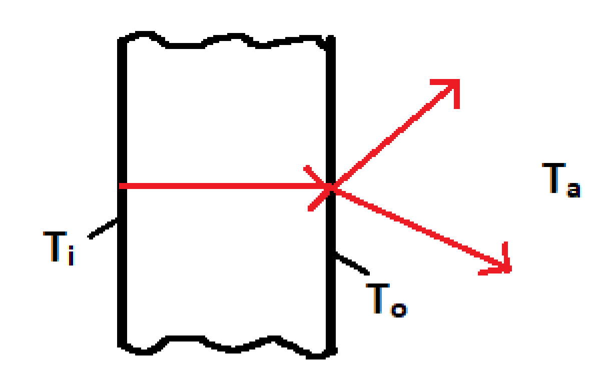
$$Q = \frac{A_1\,\sigma(T_1^4 - T_2^4)}{\frac{A_1 + A_2 - 2A_1F_{12}}{A_2 - A_1F_{12}^2} + \left(\frac{1}{\varepsilon_1} - 1\right) + \left({}^{A_1}/_{A_2}\right)\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Radiation combined with conduction and convection

- Industrial heat transfer by radiation is almost invariably accompanied by conduction and convection
- Heat transfer in a furnace occurs mainly by radiation but a large part of the heat transfer from the flowing combustion gases to the furnace walls or to the tubes or pipes in the furnace occurs by combined conduction, convection and radiation
- Heat flows from inside the furnace to the outer surface by conduction
- This heat is lost from the outer surface to the surroundings by convection and radiation
- If the furnace wall has a thickness l and thermal conductivity k and surface temperatures T_1 and T_o, then

$$\frac{k}{l}(T_i - T_o) = h_c(T_o - T_a) + \varepsilon\sigma(T_o^4 - T_a^4)$$

$$\boxed{\frac{k}{l}(T_i - T_o) = (h_c + h_r)(T_o - T_a)}$$



Where,

$$h_r(T_o - T_a) = \varepsilon\sigma(T_o^4 - T_a^4)$$

$$h_r = \frac{\varepsilon\sigma(T_o^4 - T_a^4)}{(T_o - T_a)}$$