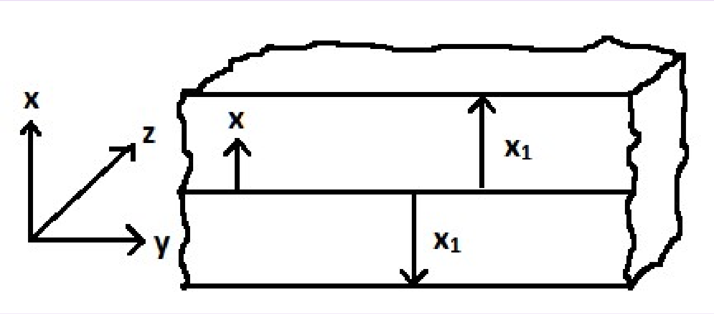
Introduction to unsteady state heat transfer

Unsteady state heat conduction in a large flat plate



- A geometry that often occurs in heat-conduction problems is a flat plate
- The flat plate considered here has a of thickness $2x_1$ in the $x-direction$ and large or infinite dimensions in the y and $z-directions$
- Heat is conducted only from the two flat and parallel surfaces in the $x-direction$
- The initial temperature at $t = 0$ in the plate is T_o
- The solid is exposed to an environment at temperature T_1 and unsteady state heat conduction occurs
- A surface resistance is present

- The numerical solutions for this case is plotted graphically
- Using the graphs or charts (shown in the next slide) , the temperature at any position in the plate x and at any time t can be determined

The dimensionless parameters used in these graphs and subsequent unsteady-state charts are the following:

$$m = \frac{k}{hx_1} = \frac{1}{Bi}$$

$$n = \frac{x}{x_1}$$

$$X = \frac{\alpha t}{x_1^2} = Fo\ (Fourier\ Number)$$

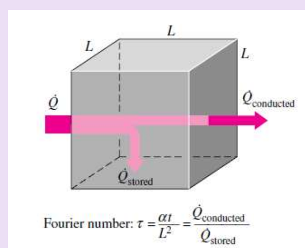
Here, x is the distance from the centre of the flat plate, cylinder, or sphere

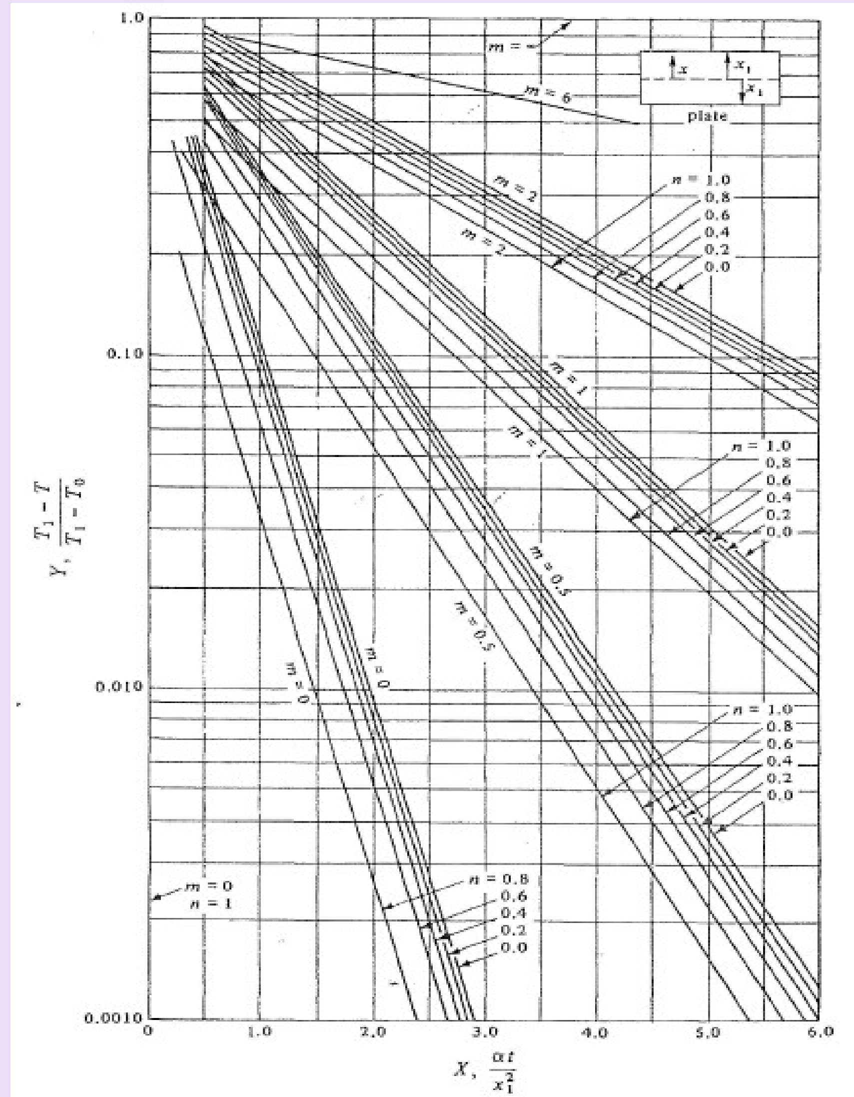
x_1 is one half the thickness of the flat plate (or radius of cylinder, or radius of sphere)

Fourier number (Fo) is a measure of *heat conducted* through a body relative to *heat stored*

Thus, a large value of the Fourier number indicates faster propagation of heat through a body

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2\,(1/L)}{\rho c_p L^3/t}\frac{\Delta T}{\Delta T} = \frac{\text{The rate at which heat is } conducted \text{ across } L \text{ of a body of volume } L^3}{\text{The rate at which heat is } stored \text{ in a body of volume } L^3}$$



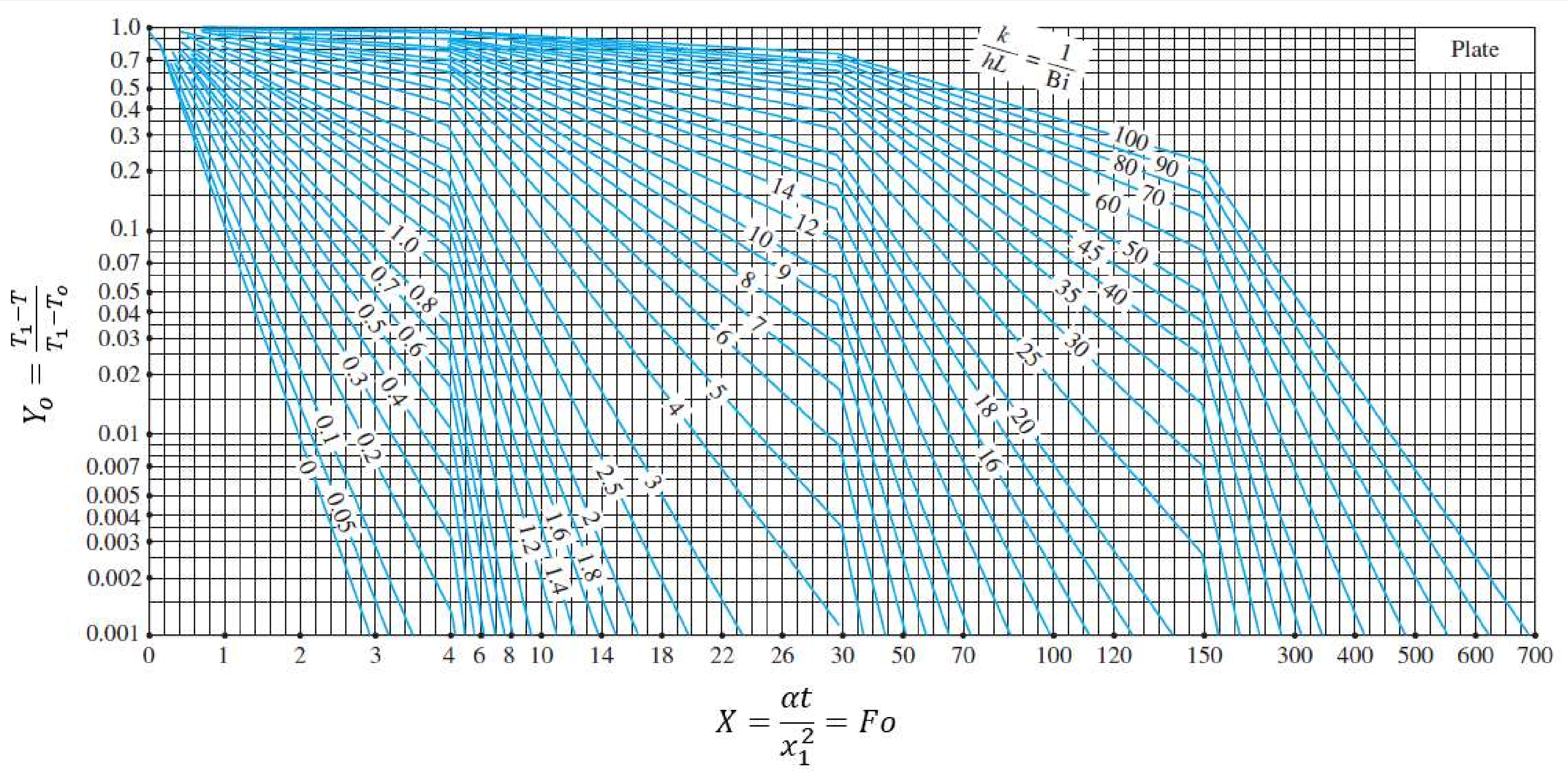


- Often the temperature history at the centre of the plate (when *n* = 0) is quite important
- A more accurate chart for determining only the centre temperature is given in the ***Heisler Charts***
- Heisler charts are transient temperature nomographs
- Apart from the centre temperature graphs, Heisler has also prepared multiple charts for determining the temperatures at other positions

- There are *three* charts associated with each geometry:
  - the first chart is to determine the temperature T_0 at the *center* of the geometry at a given time *t*
  - *t*he second chart is to determine the temperature at *other locations* at the same time in terms of T_0
  - the third chart is to determine the total amount of *heat transfer* up to the time *t*

- The next six slides show the Heisler charts for three different geometries – flat plate, cylinder, and sphere

Transient temperature and heat transfer charts for a **plane wall of thickness $2x_1$** initially at a uniform temperature T_o subjected to convection from all sides to an environment at temperature T_1 with a convection coefficient of h



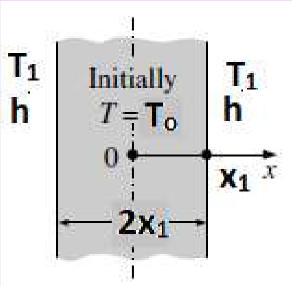


Chart for determining temperature at the centre of a large flat plate for unsteady state heat conduction

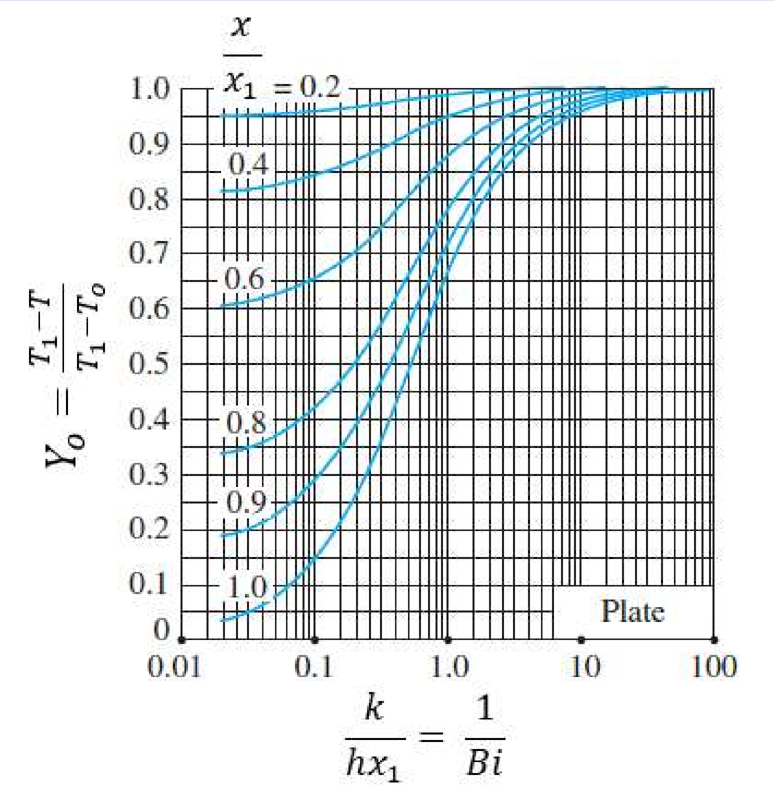


Chart for determining temperature distribution at different locations in a large flat plate for unsteady state heat conduction

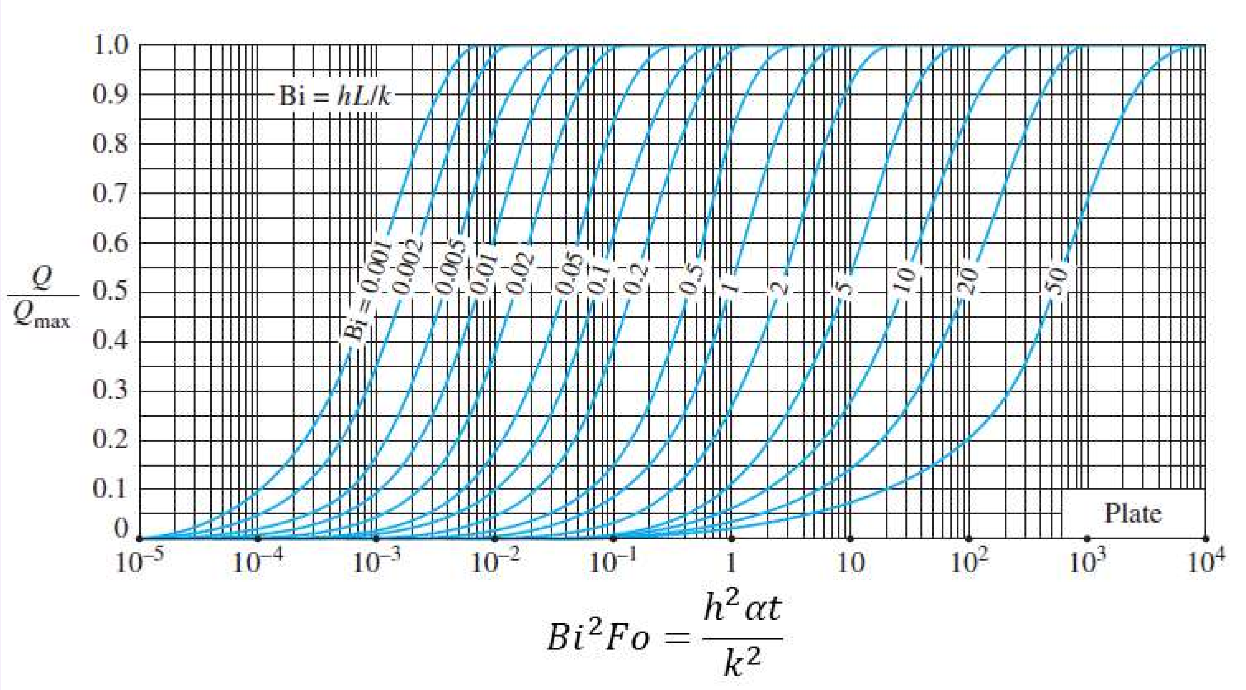
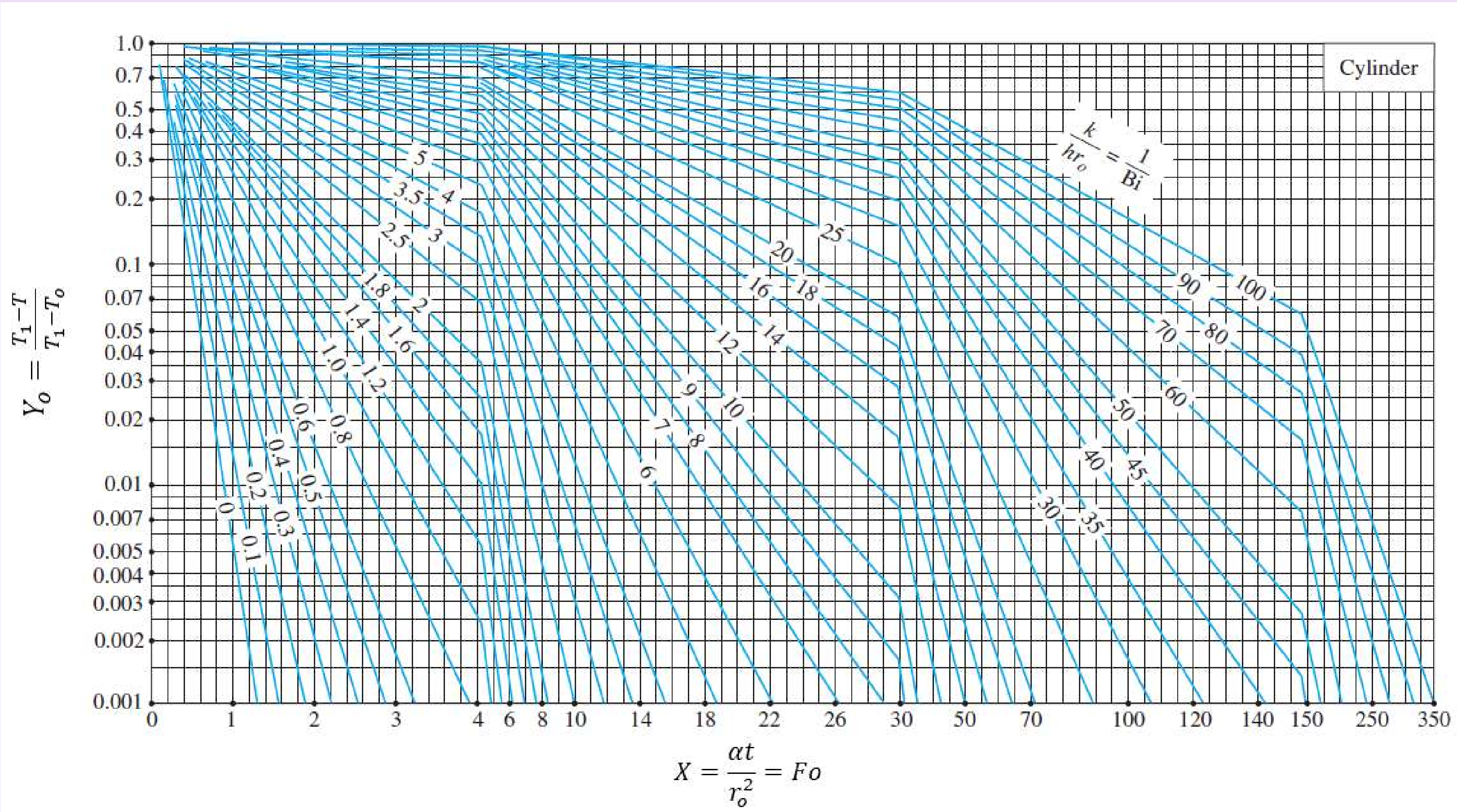
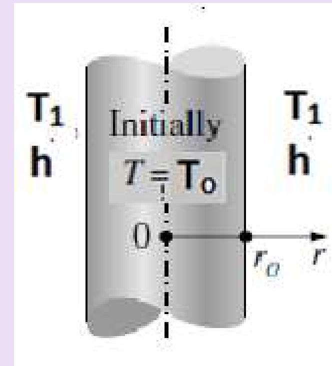


Chart for determining heat transfer in a large flat plate for unsteady state heat conduction

Transient temperature and heat transfer charts for a ***long cylinder*** of radius r_o initially at a uniform temperature T_o subjected to convection from all sides to an environment at temperature T_1 with a convection coefficient of h





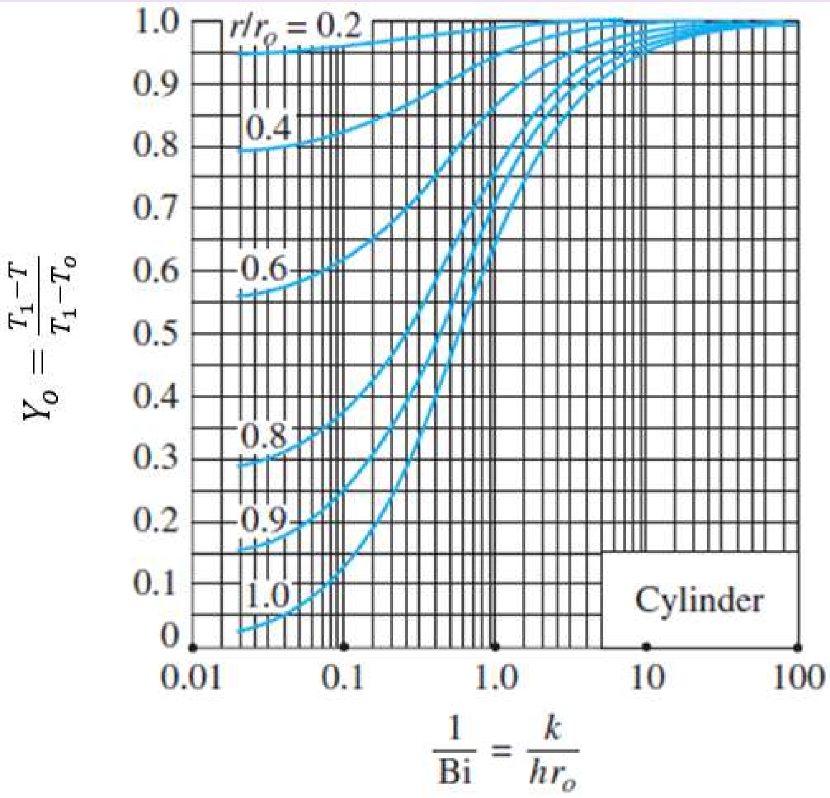


Chart for determining temperature distribution at different locations in a cylinder for unsteady state heat conduction

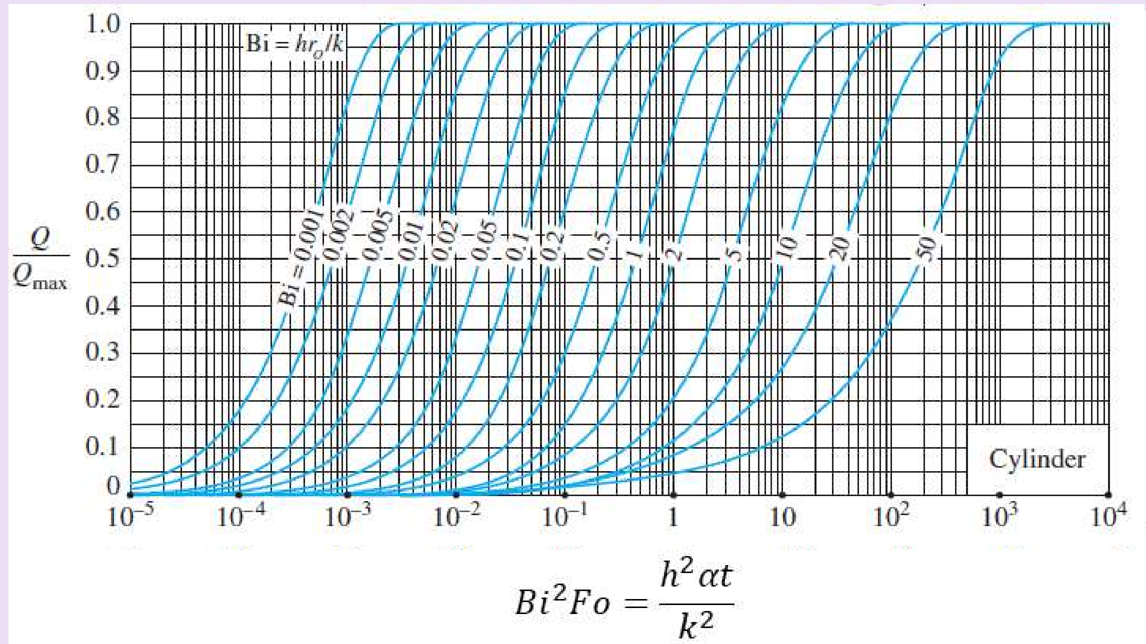
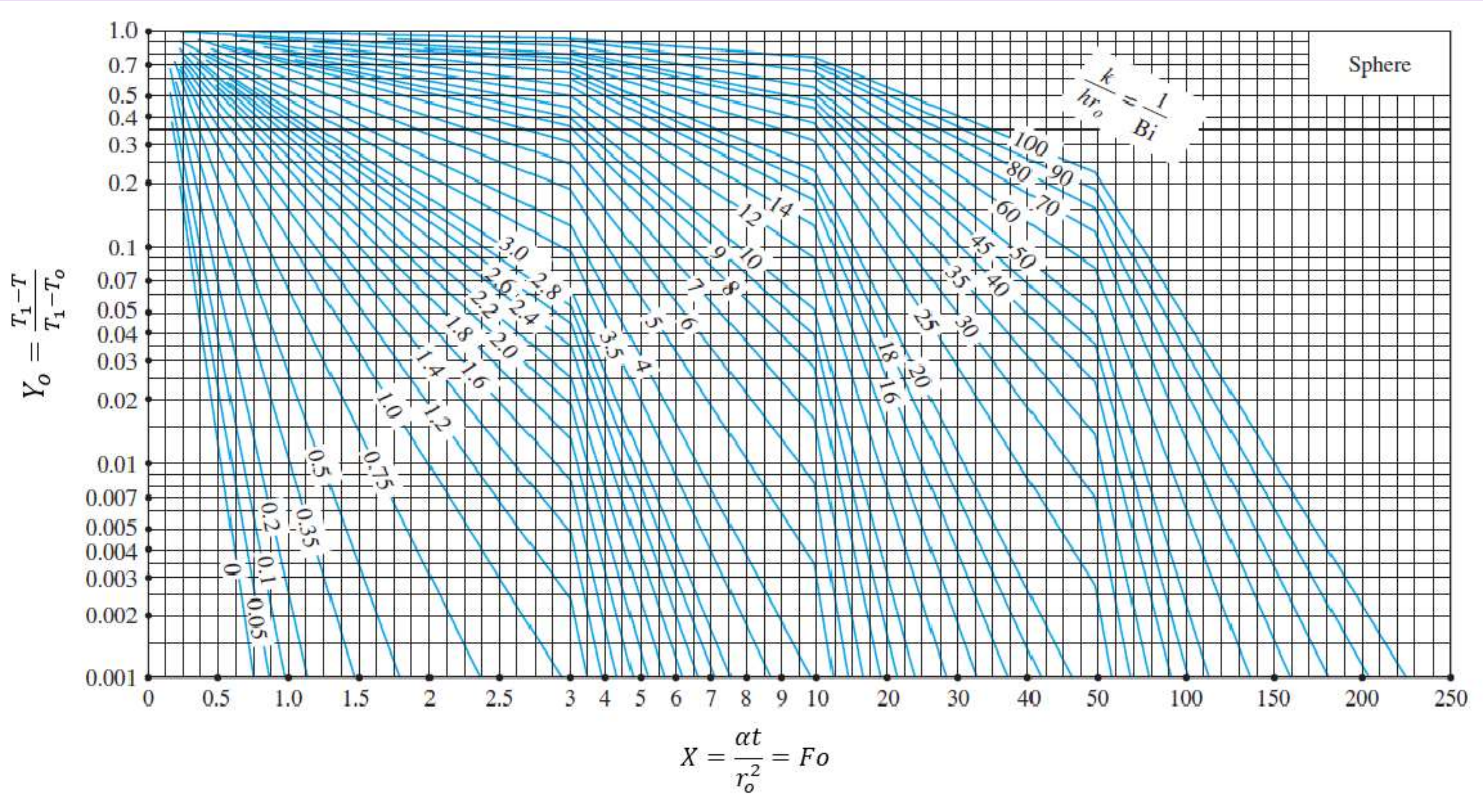
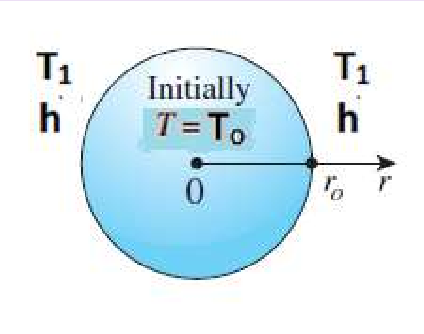


Chart for determining heat transfer in a cylinder for unsteady state heat conduction

Transient temperature and heat transfer charts for a ***sphere*** *of radius* r_o initially at a uniform temperature T_o subjected to convection from all sides to an environment at temperature T_1 with a convection coefficient of h





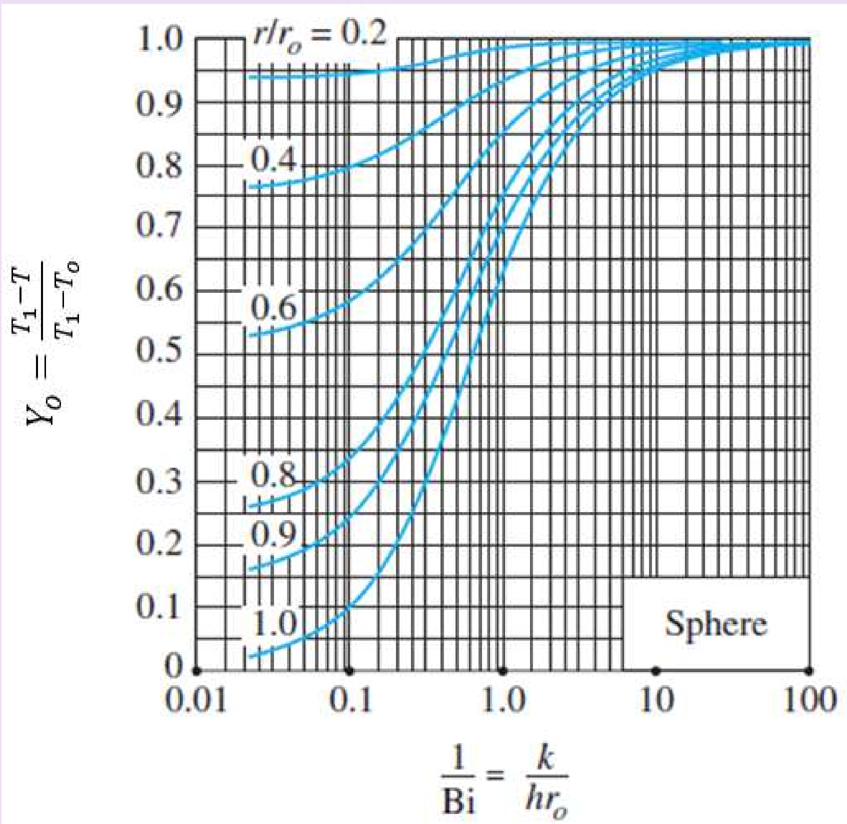


Chart for determining temperature distribution at different locations in a sphere for unsteady state heat conduction

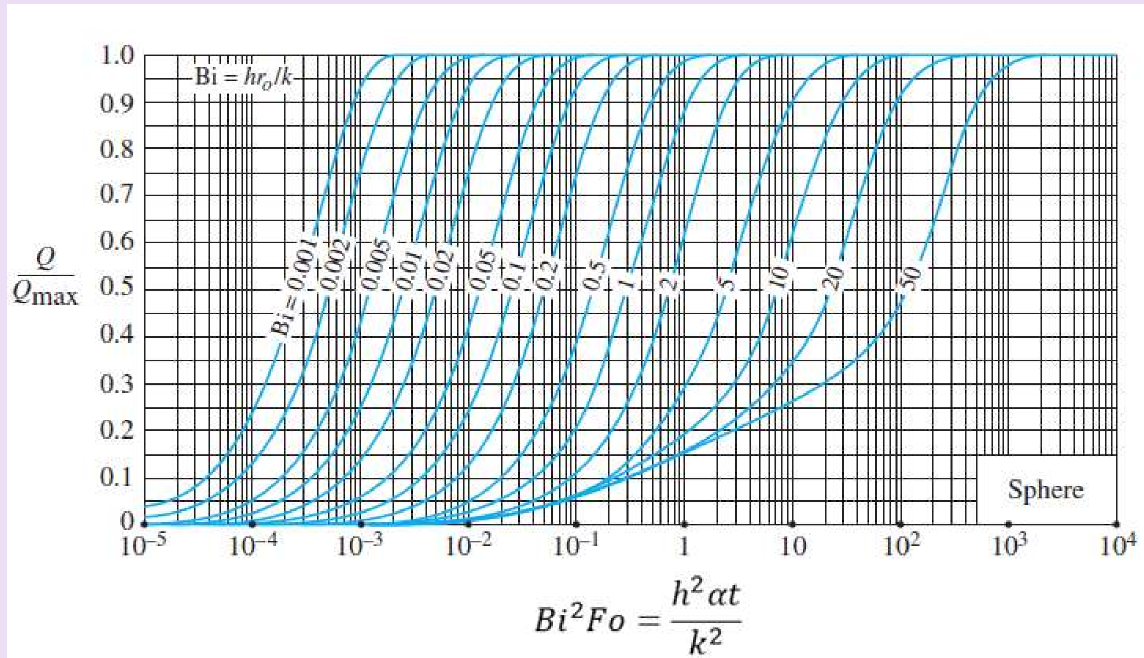


Chart for determining heat transfer in a sphere for unsteady state heat conduction

Problem

A rectangular slab of butter which is 46.2 mm thick at a temperature of 277.6 K in a cooler is removed and placed outside where the temperature is 297.1K. The sides and the bottom of the butter container can be considered to be insulated by the container side walls. The flat top surface of the butter is exposed to the environment. The convective coefficient is constant and is equal to 8.52 W/m^{2}K.

The physical properties of butter are :

$\rho = 998$ kg/m^3, C_p = 2.30 kJ/kg^{o}C, $k = 0.197$ W/mK

Calculate the temperature of the butter (a) at the surface (b) at 25.4 mm below the surface and (c) at 46.2 mm below the surface at the insulated bottom after 5 h exposure

Since the heat is entering the slab of butter only at the top face and the bottom is insulated , the slab can be considered equivalent to a half plate with thickness = $x_1 = 46.2\ mm$

For butter, the thermal diffusivity, $\alpha = \frac{k}{\rho C_p} = \frac{.197}{998 \times 2.3 \times 1000} = 8.58 \times 10^{-8}\ m^2/s$

Here, $x_1 = 0.0462\ m$

$$m = \frac{k}{hx_1} = \frac{0.197}{8.52 \times 0.0462} = 0.50$$

$$X = \frac{\alpha t}{x_1^2} = \frac{8.58 \times 10^{-8} \times 5 \times 3600}{0.0462^2} = 0.72$$

(a) For the top surface, $x = x_1 = 0.0462\, m$

$$n = \frac{x}{x_1} = \frac{0.0462}{0.0462} = 1$$

From the figure, at $X = 0.72$ and $m = 0.5, n = 1$

$$Y = 0.25$$

Therefore, $T = -0.25 \times (297.1 - 277.6) + 297.1 = 292.2\, K$

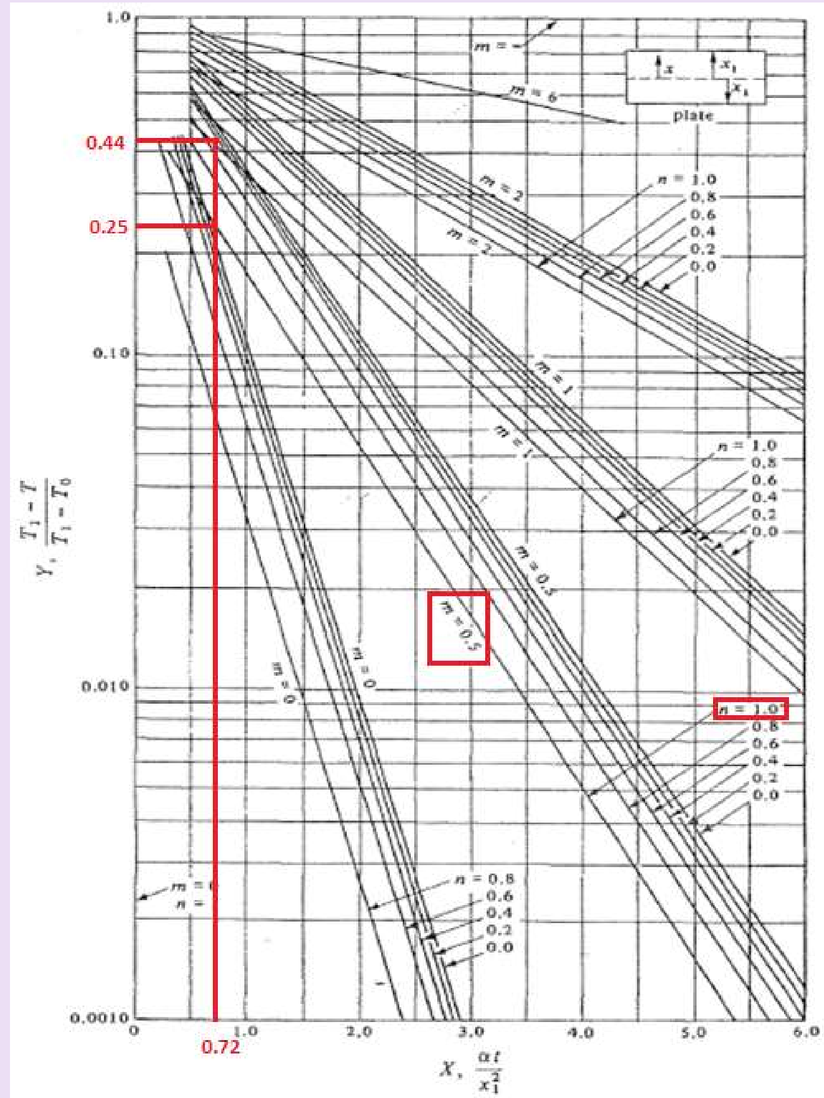
(b) At 25.4 mm from the top, $x = 0.0462 - 0.0254 = 0.0208\, m$

$$n = \frac{x}{x_1} = \frac{0.0208}{0.0462} = 0.45$$

From the figure, at $X = 0.72$ and $m = 0.5$, n =0.45

$$Y = 0.44$$

$$T = -0.44 \times (297.1 - 277.6) + 297.1 = 288.52\, K$$



(c) At 46.2 mm below the surface at the insulated bottom, it is the centre of the slab

For accurate results, the Heisler chart is used

From the figure, at $X = 0.72$ and $m = 0.5$, $Y_o = 0.53$

Therefore, $T = -0.53 \times (297.1 - 277.6) + 297.1 = 286.77\ K$