Introduction to GT

Directed Graph

- A **directed graph (digraph)** is a tuple $G = (V, E)$ where V is a (finite) set of vertices and E is a collection of elements contained in $V \times V$. That is, E is a collection of ordered pairs of vertices.

More GRaphs

- **Empty and Trivial Graphs:** A graph G = (V, E) in which V = ∅ is called the *empty graph*. A graph in which V = {v} and E = ∅ is called the *trivial graph* (*Only one vertex*).

only one vertex & no edges

empty graph

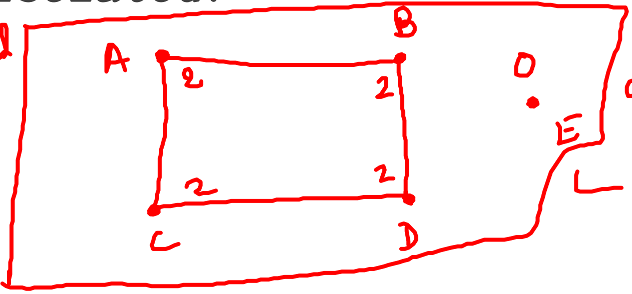
⇒ A graph without edges is called Null graphs

N_n

N_4

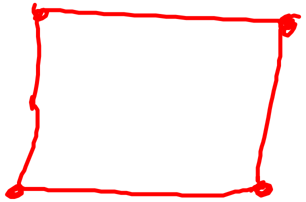
- **Isolated vertex:** Let G = (V, E) be a graph and let v ∈ V . If **deg(v) = 0** then v is said to be *isolated*.

Disconnected graph



deg(E) = 0

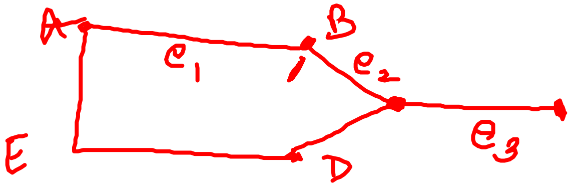
Isolated vertex



Connected graph

- Pendent Vertext: A vertex with degree one is called *pendent vertex*.

Note: Two adjacent edges are said to be in series if their commaon vertex is of $\boxed{\text{degree 2.}}$



C is pendent vertex

e_2 & e_3 is not in series because the common vertex degree is 3

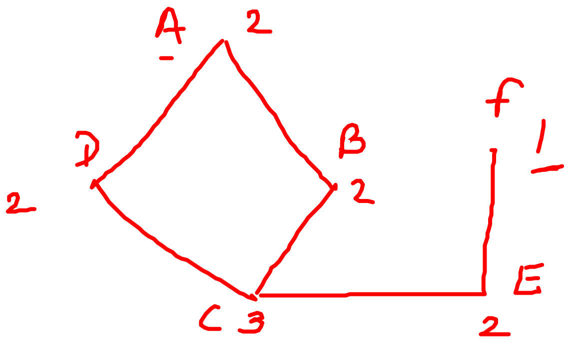
Degree and Degree Sequence

- **Degree Sequence:** Let G = (V, E) be a graph with |V| = n. The degree sequence of G is a tuple d ∈ Z^n composed of the degrees of the vertices in V arranged in decreasing order.

1 2 2 2 2 3

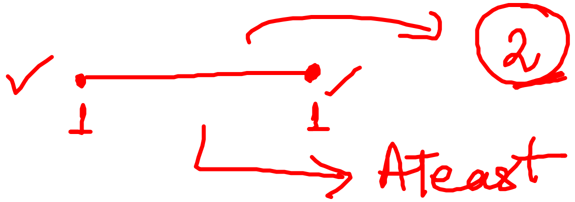
✓ F A B D E C

$\boxed{F \quad B \quad A \quad E \quad D \quad C}$



Imp

- **Theorem1:** Let G = (V, E) be a non-empty, non-trivial graph. Then G has at least one pair of vertices with equal degree.



Assignment 1

Theorem : Let G = (V, E) be a (general) graph then: $\boxed{2E = \sum_{v\in V} deg(v).}$ ~~sum of deg~~

sum of degree is twice of no. of edges

Imp

Corollary: Let G = (V, E). Then there are an even number of vertices in V with odd degree.

connected graph

1, 1, 2, 2, 3, 3

3, 3, 2, 2, 1, 1

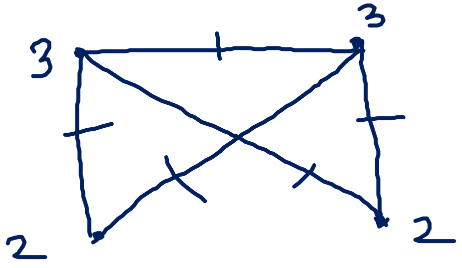
$\boxed{1, 1, 2, 3}$

graph?

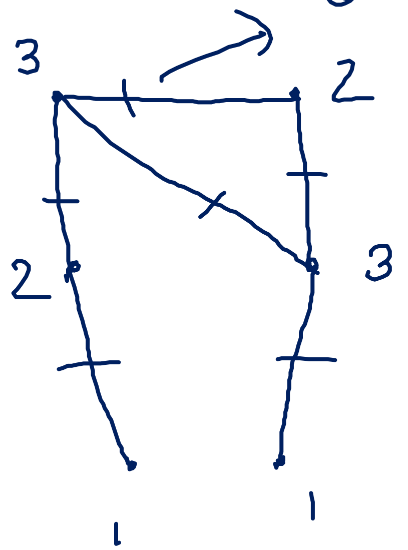
$n = 6, e = 15$

$$\boxed{e = {}^{n}C_{2} = \frac{n(n-1)}{2}}$$

→ max possible edges



Disconnected graph



Theorem 1: Let G = (V, E) be a non-empty, non-trivial graph. Then G has at least one pair of vertices with equal degree.

Theorem 2 : Let G = (V, E) be a (general) graph then: $2E = \sum_{v \in V} deg(v)$.

Corollary: Let G = (V, E). Then there are an even number of vertices in V with odd degree.

- Order of the graph = number of vertices in graph $= |V|$
- Size of the graph = number of edges in a graph $= |E|$
- Max number of edge in a graph: $\underline{Max\ E} = \frac{n(n-1)}{2} = {}^{n}C_{2} = \binom{n}{2}$

EXAMPLE → $e = 21$ edges, 3 vertices of degree 4 & other vertices are degree 3. find no. of vertices.

$$\Rightarrow e = \frac{n(n-1)}{2} \Rightarrow 21 \times 2 = n(n-1) \Rightarrow 7 =$$

Let no. of vertices is n

$$\sum deg(v) = 2e$$

$$deg(v_1) + deg(v_2) + \ldots + = 2e$$

$$3 \times 4 + (n-3) \times 3 = 2e$$

$$12 + 3n - 9 = 42$$

$$\boxed{n = 13}$$

Ex2 → $e = 8$, $n = ?$ – even
degree of each vertex is 3 → 4

$$n \times 3 = 2 \times 8$$

$$n = \frac{2 \times 8}{3} \Rightarrow 5$$

3, 3, 3, 3, 3

$$n \times 4 = 2 \times 8$$

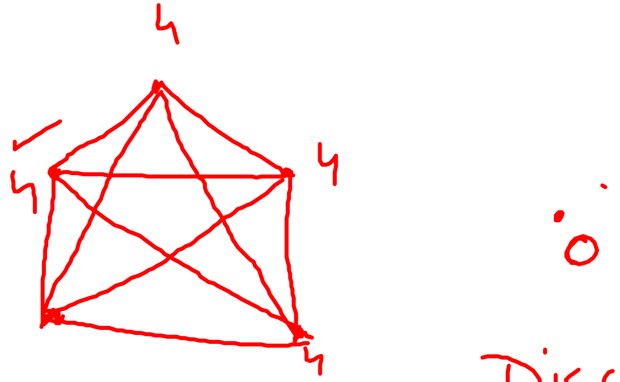
$$\boxed{n = 4}$$

1. (2, 2, 2, 2, 2) → Yes

   2 2 2

   ~~3~~ 3 3 3

2. (~~4~~, 4, 4, 4, 4, 0) → Yes



Disconnected

3. (3, 3, 2, 2, 2) → Yes

4. (5, 3, 3, 3, 2, 2) → Yes