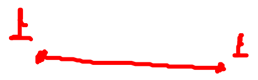
Introduction to GT

Degree Sequence

Degree and Degree Sequence

- **Empty and Trivial Graphs:** A graph G = (V, E) in which V = ∅ is called the ***empty graph (or null graph)***. A graph in which V = {v} and E = ∅ is called the ***trivial graph***.
- **Isolated vertex:** Let G = (V, E) be a graph and let v ∈ V . If **deg(v) = 0** then v is said to be ***isolated***.

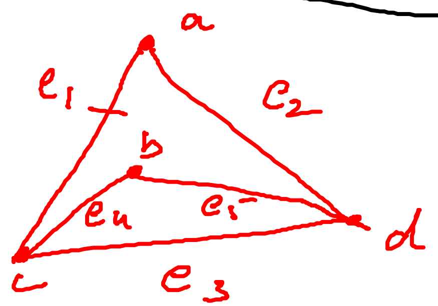


Theorem 1: Let G = (V, E) be a non-empty, non-trivial graph. Then G has at least one pair of vertices with equal degree.

Theorem 2 : Let G = (V, E) be a (general) graph then: $2E = \sum_{v \in V} deg(v)$.

Corollary: Let G = (V, E). Then there are an even number of vertices in V with odd degree.

- **Order of the graph = number of vertices in graph** n = 4
- **Size of the graph = number of edges in a graph** e = 5
- **Max number of edge in a graph:** $Max\ E = \frac{n(n-1)}{2}$ $^{n}C_2$



Degree and Degree Sequence

- **Degree Sequence:** Let G = (V, E) be a graph with |V| = n. The degree sequence of G is a tuple d ∈ Z^n composed of the degrees of the vertices in V arranged in decreasing order.

Ex → [5 3 4 1 2] ⇒ [5 4 3 2 1] → graph?

$n = 5$, $e = \frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$

$2e = \sum deg(v) = 5 + 4 + 3 + 2 + 1$

$2 \times 10 = 5 + 4 + 3 + 2 + 1$

graph is not possible.

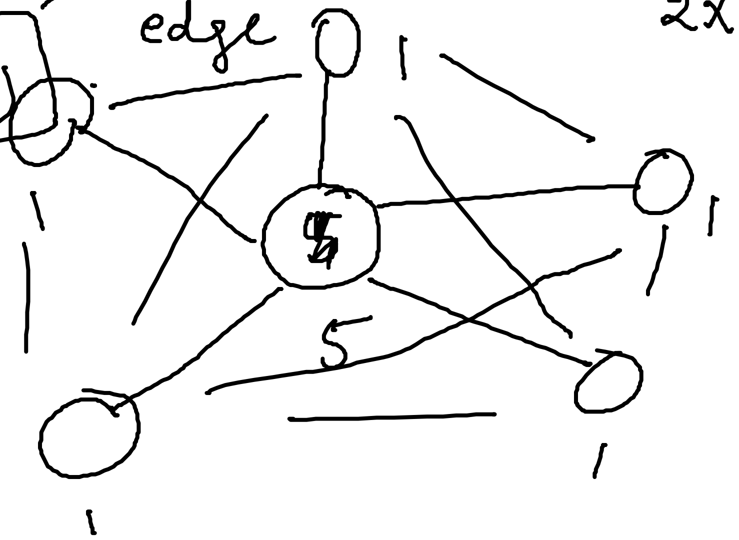
Ex → [5 1 1 1 1 1]

$n = 6$, $e = \frac{6 \times 5}{2} = 15$ → max possible edge

$\sum deg(v) = 10$

$2e = \sum deg(v)$

$2 \times 15 = 10$



$2e \geq \sum deg(v)$

↓

max possible edge

Havel–Hakimi Algorithm

Ex → 4 3 3 2 2

(1) Arrange in degree sequence

[4 3 3 2 2]
 X -1 -1 -1 -1

(2) Remove first element k=4 & subtract ~~nex~~ 1 from next 4 = k (position)

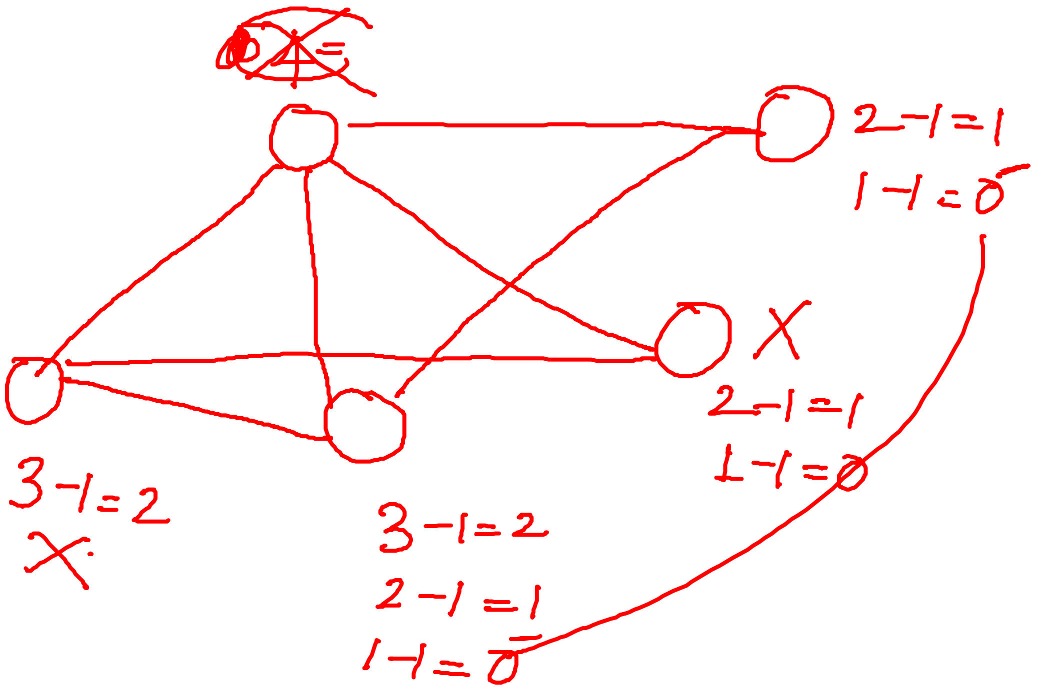
[2 2 1 1]
 X -1 -1

(3) Repeat, remove 2 & subtract 1 from next two position

[1 0 1] ⇒ [1 1 0]
 X -1

(4) Remove 1 & subtract 1 from next 1 position

[0 0] ⇒ If o/p contain all zeros then the the graphical order is correct the we make a g. graph.



5 5 5 5 5 5

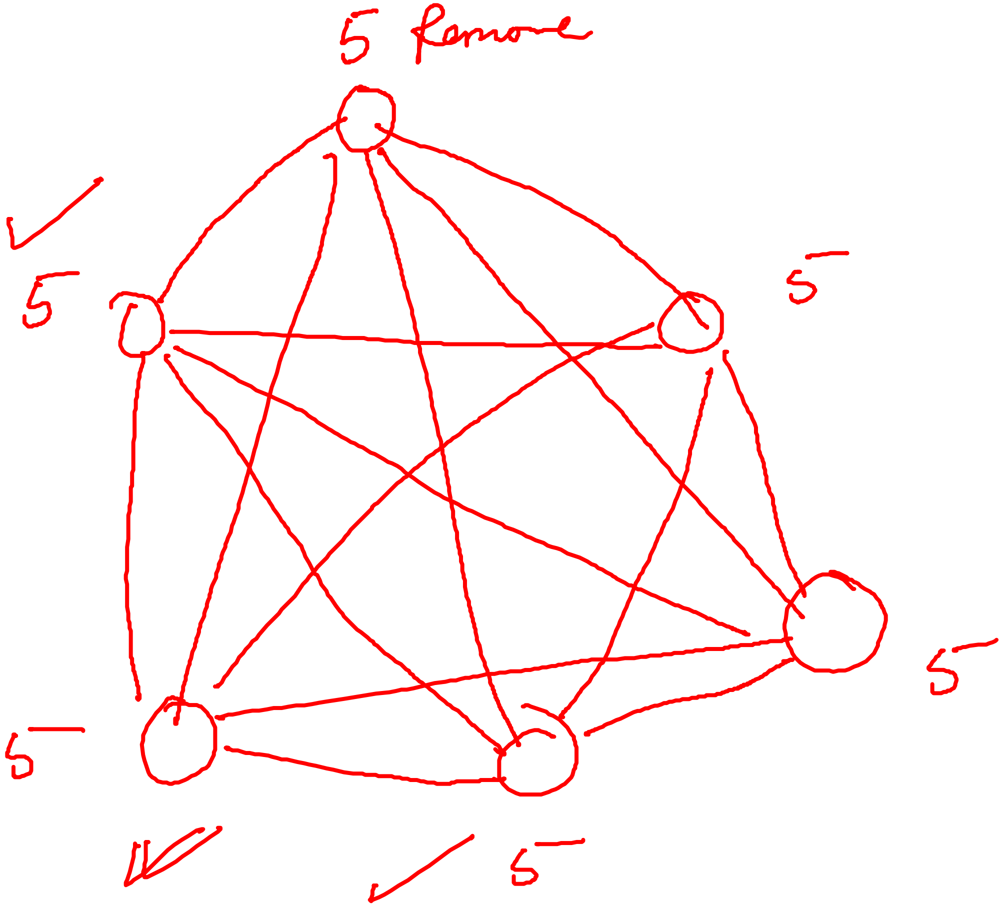
Ex → [4 5 3 4 3 3 2] ⇒ even

⇒ { 0 4 4 4 4 4
4 4 4 4 4 0 }

⇒ { 0 3 3 3 3 0
3 3 3 3 0 0 }

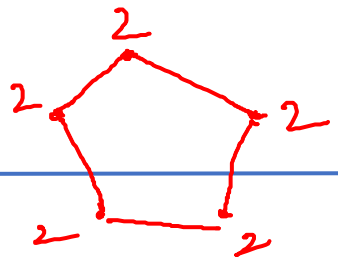
⇒ { 0 2 2 2 0 0
2 2 2 0 0 0 }

⇒ { 0 1 1 0 0 0
1 1 0 0 0 0
0 0 0 0 0 0 } ⇒



All Function

① (2 2 2 2 2) → yes

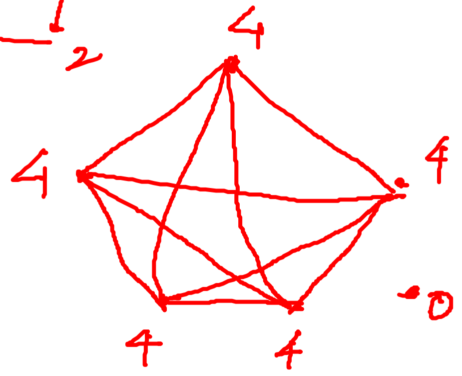


② (4, 4, 4, 4, 4, 0) → yes n = 6, e = 15

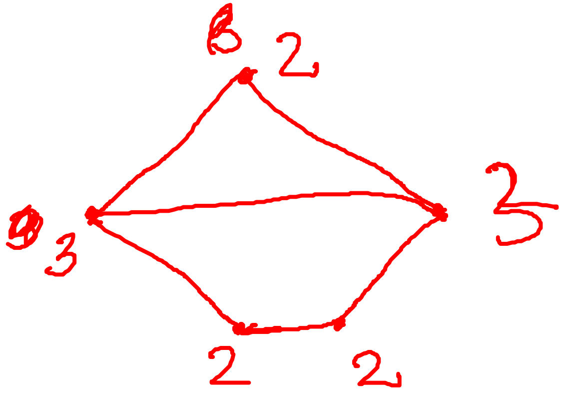
Σdeg(v) = 20

2e ≥ Σdeg(v)

30 ≥ 20



③ (3 3 2 2 2) → yes



④ (5 3 3 3 2 2) → yes

4 = even

1 3 5 4 3 2

[5 4 4 3 3 3 2]

4   −1

Remove

[3 3 2 2 2 2]

−1

[2 1 1 2 2]

[2 2 2 1 1]

[1 1 1 1]

[0 1 1]

[1 1 0]

[0 0] ⇒

Remove 5

[5 4 4 2 1]

3 3 1 0

−1   −1

[2 0 −1]

5

max deg

5 < m