Types of Graph

① $\sum \deg(v) = 2e$

② max $e = {}^nC_2 = \frac{n(n-1)}{2}$

③ $n \geq \Delta(G) \Rightarrow$ graph is exist

④ degree sequence

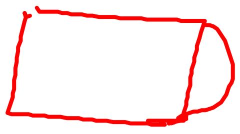
$\vdash$ odd degree of vertice is always even trivial graph

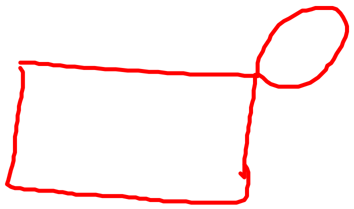
$o(G)=6$
size $=0$
Null graph N_n

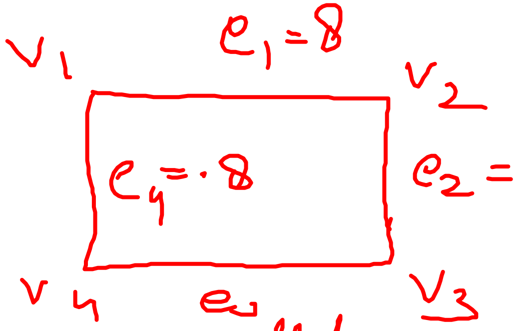
Types of Graph

- **Simple Graph:** A graph without self-loop and parallel edge is called simple graph.
- **Finite and Infinite Graph:** A graph having finite number of verties and finite number of edges is called finite graph, otherwise infinte graph.

  finite — edges finite
  infinite — edges infinite
- **Trivial:** A finite graph with one vertex and no edges is called a trivial graph.
- **Null Graph:** A graph of order n and size zero is called null graph.
- **Multi-Graph:** A graph having some parallel edges but no self-loop, called multi-graph.
- **Pseudo Graph:** A graph having self-loop but no parallel edges, is called pseudo graph.
- **Labled Graph:** If the vertices and edges of a graph G are labelled with name or data then the graph is labelled graph.
- **Weighted Graph:** When in graph some additional information is given by assigning positive number called weight, graph is called weighted graph.


Multigraph


Pseudo graph



Labelled

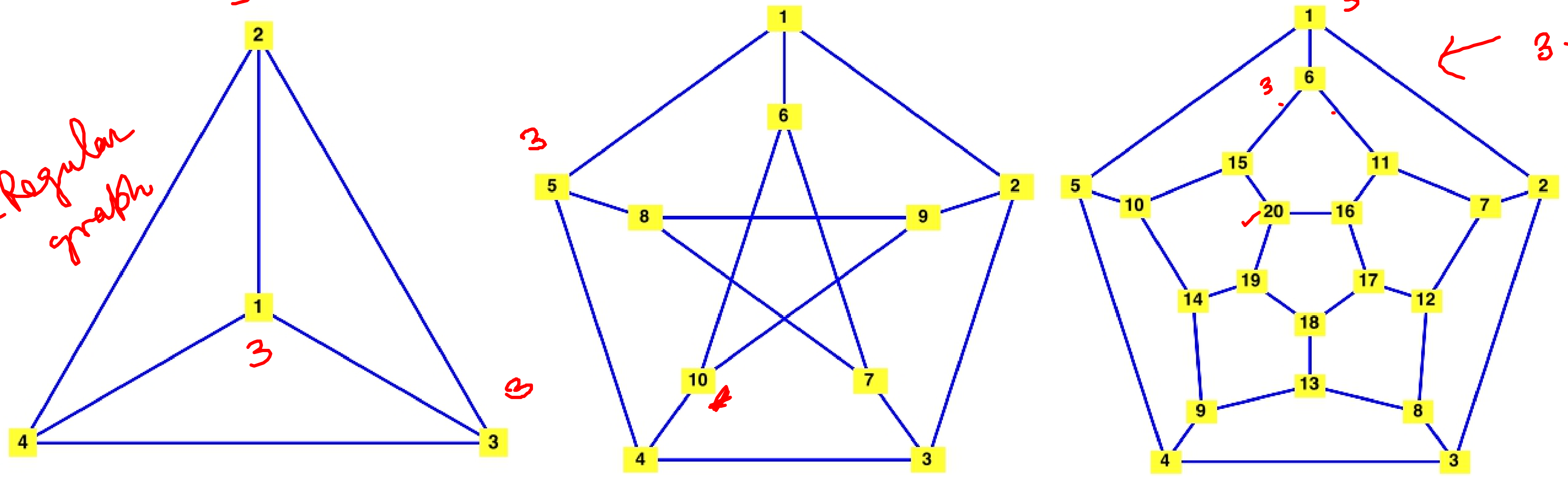
Regular Graph

- A graph in which all the vertices have same degree is called a **regular graph.**
- A regular graph where degree of each vertex is **k** is called as **k-regular.**

↳ degree

3-Regular graph

3-regular graph order 20



(a) K_4

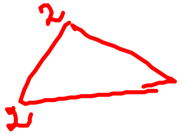
(b) Petersen Graph

(c) Dodecahedron

3 Regular graph with order 4

n = 10
degree = 3
3-Regular graph

n = 20
degree = 3
3 Regular graph



$n = 3$

$d_g = (n-1) = 2$

Complete Graph or Full Graph

K_n

- A graph in which each vertex is connected to every other vertex is called a **complete graph.**
- Note that degree of each vertex will be n–1, where n is the order of graph.
- So we can say that a complete graph of order n is nothing but a (n–1)–regular graph of order n.

K

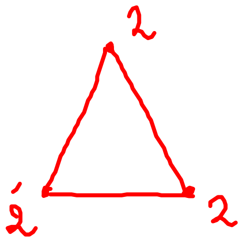
- A complete graph of order n is denoted by K_n.

K_1

N_1

K_2

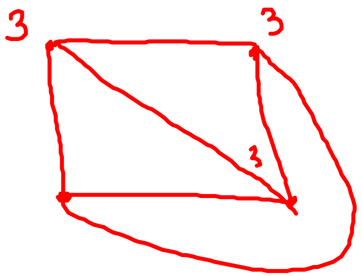
K_3



2 - regular graph of order 3

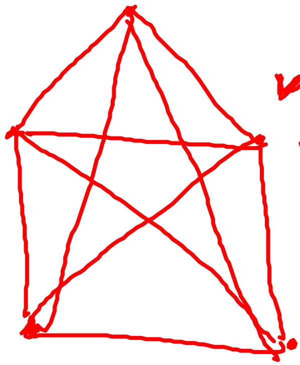
K_4

3-Regular graph of order 4

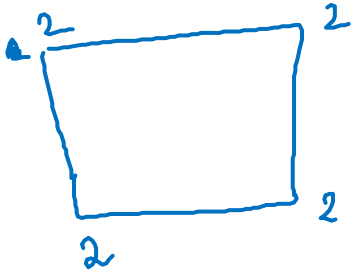


K_5

4



4-regular graph of order n

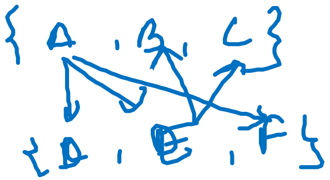


2 - Regular of order 4

⇒ Each complete graph is Regular graph but vice-versa is not true.

Bipartite Graph

$\boxed{K_{m,n}}$



- A graph is said to be bipartite if we can divide the set of vertices in two disjoint sets such that there is no edge between vertices belonging to same set.
- Each vertex has only one label. So the two sets are disjoint i.e. the two sets don't have any vertex in common.
- And there should not be any edge running within the same set. This means that every edge runs between two vertices belonging to different sets — one labelled as A and other as B.

$A = \{1, 2, 3\}$

$B = \{4, 5, 6, 7\}$

$K_{m,n}$

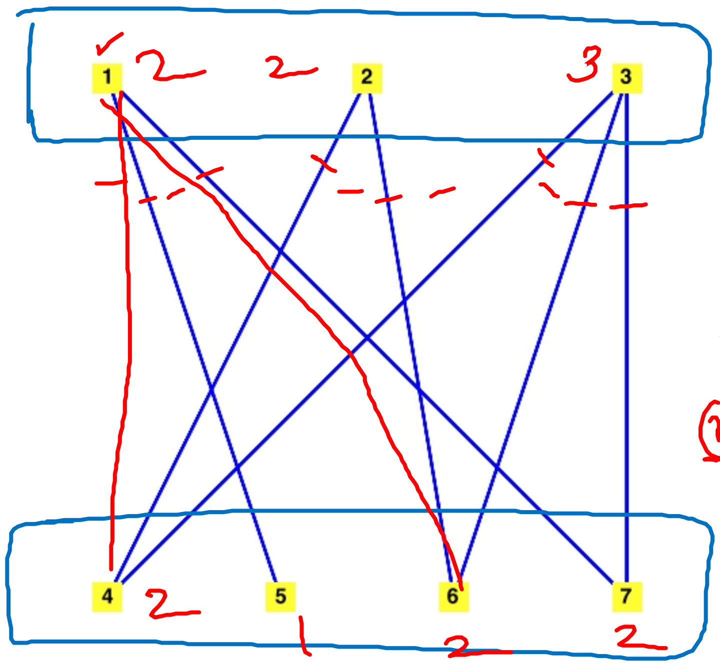
$n \rightarrow$ No. of vertice in set B

$m \rightarrow$ No. of vertice in set A

$V = \{1, 2, 3, 4, 5, 6, 7\}$

no. of vertices = order = 7 = $m+n$

max No. of edges (size) = $\dfrac{(m+n)(m+n-1)}{2}$



12

7

No. of vertice

$\dfrac{n(n-1)}{2}$

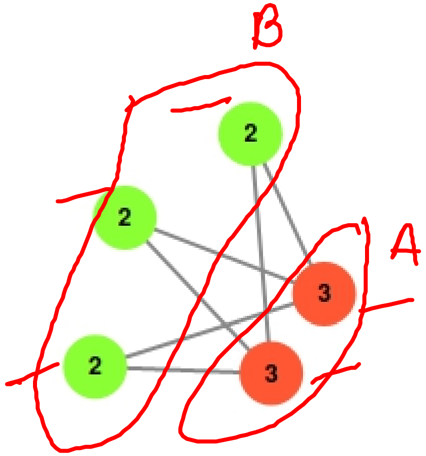
$\dfrac{(m+n)(m+n-1)}{2}$

$\Rightarrow \dfrac{7(7-1)}{2}$

$\Rightarrow 7 \times 3 = \boxed{21}$

Complete Bipartite Graph

- Complete bipartite graph is a special type of bipartite graph where every vertex of one set is connected to every vertex of other set
- The figure shows a bipartite graph where set A (orange-colored) consists of 2 vertices and set B (green-colored) consists of 3 vertices
- If the two sets have p and q number of vertices, then we denote the complete bipartite graph by $K_{p,q}$.



Theorem

- Maximum number of edges in a simple graph with n verties is $\frac{n(n-1)}{2}$

max degree of each vertex in a simple graph of n vertices $= n\cdot(n-1)$

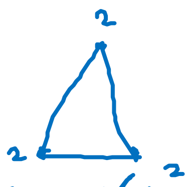
$$\sum \deg(v) \ngtr n(n-1)$$

$$2e \ngtr n(n-1)$$

$$\boxed{e \ngtr \frac{n(n-1)}{2}}$$

$n=1$, $e=0$

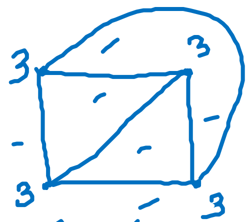
$n=2$, $e=1$, $\sum \deg(v)=2$



$n=3$

$e=3$

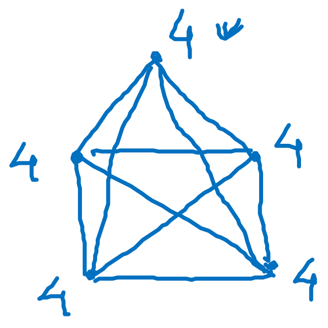
$\deg(v) = 2+2+2 = 6 = 3\times 2 = n(n-1)$



$n=4$

$e=6$

$\sum \deg(v) = 12$



$n=5$

$\sum = 20 = 5\times 4 = n(n-1)$

Theorem

- number of edges is a k-regular graph is $\frac{n.k}{2}$

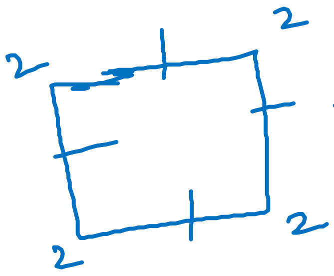
$$\sum \deg(v) = n \times k$$

$$2e = n \cdot k$$

$$\boxed{e = \frac{n \cdot k}{2}}$$

$e = 1$

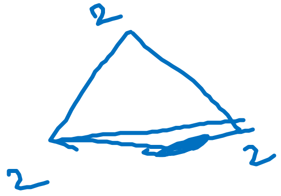
$K=1$, $n=2$, $\Sigma d = \frac{2}{2}$



$=$

$K=2$, $n=4$, $\Sigma \deg = \frac{8}{2}$

$e=4$



$=$

$n=3$

$K=2$, $\Sigma dg = \frac{6}{2}$

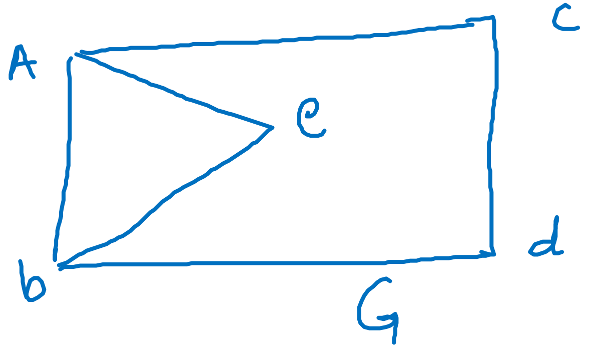
$e=3$

⇒ **COMPLEMENT OF SIMPLE GRAPH** ⇒ $G = (V, E)$ is $\bar{G} = (\bar{V}, \bar{E})$

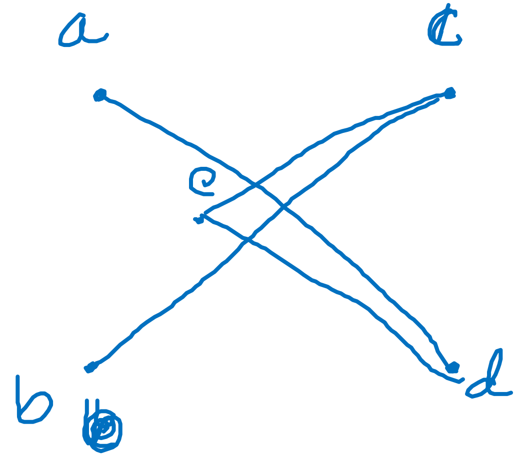
$\bar{V}$ → contains all the vertices & $\bar{E}$ → set of edges $= \{ (v_i, v_j) / (v_{i,j}) \notin E \}$

i.e., if two vertices are not adjacent in G (

Ex →

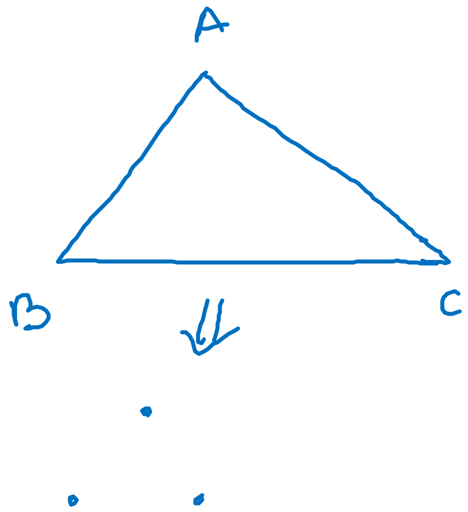


$\xRightarrow{c}$



$\bar{G}$

Ex → K_3



$\bar{V} = \{a, b, c, d, e\}$

$\bar{E} = \{ \cancel{(a,d), (b,c), (b,e), (d,e)},$

$\{(a,d), (b,c), (c,e), (d,e)\}$