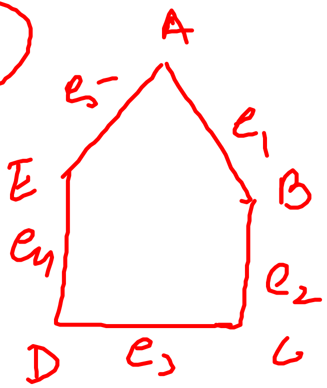
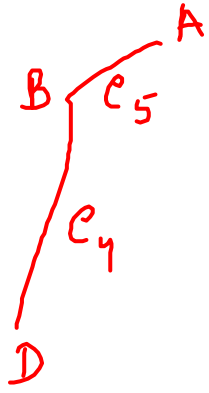
Subgraph and Complement

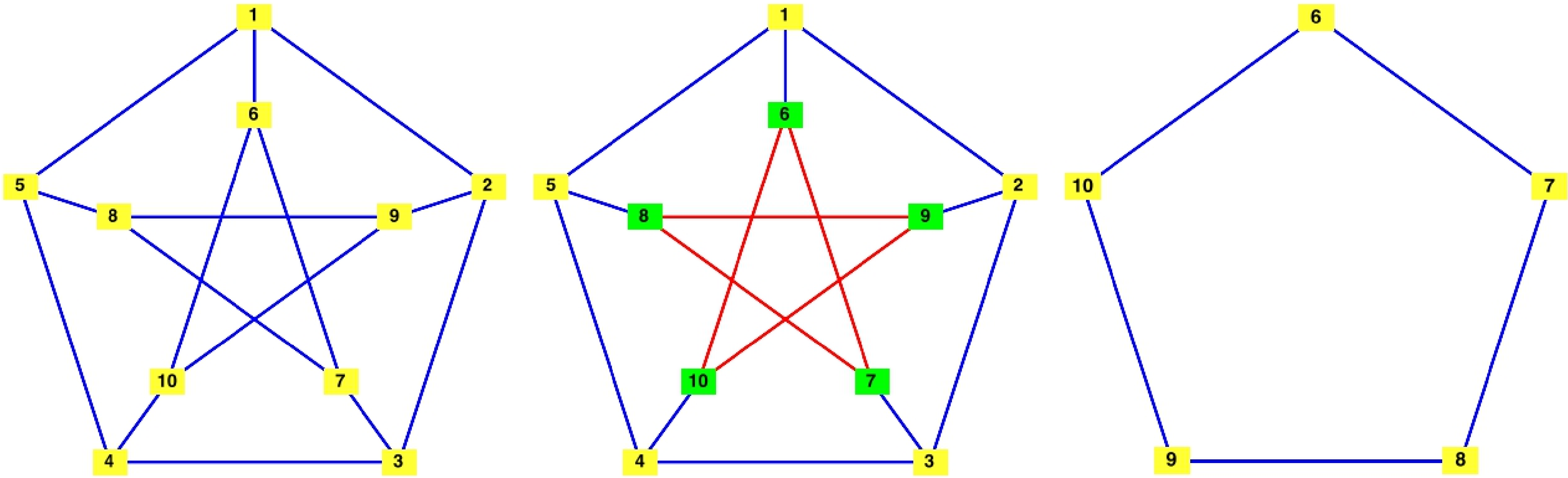
Subgraph

$V = \{A, B, C, D, E\}$

- Let $G = (V, E)$. A graph $H = (V`, E`)$ is a subgraph of G if $V` \subseteq V$ and $E` \subseteq E$.

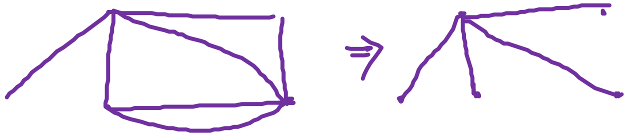






(a) Petersen Graph (b) Highlighted Subgraph (c) Extracted Subgraph

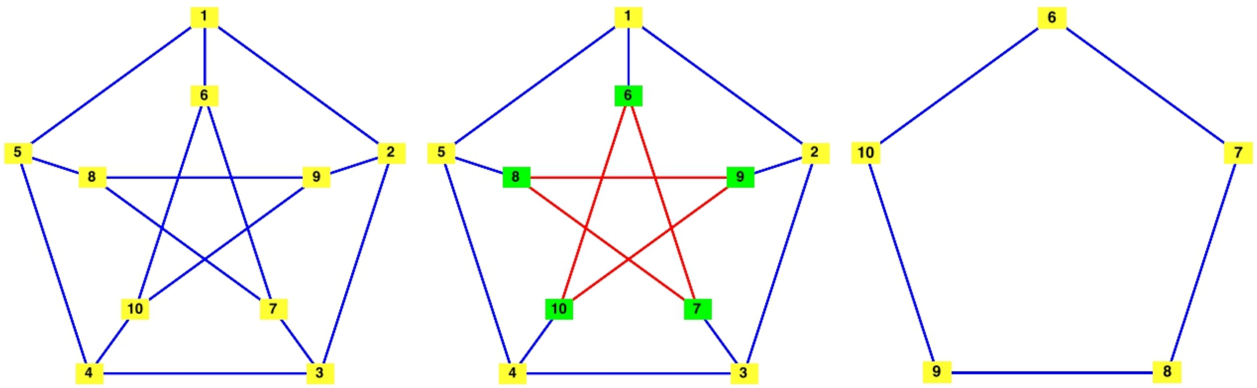
1) Every graph is its own subgraph.



Types of Subgraph

- Spanning Subgraph: Let G = (V, E). A graph H = (V`, E`) is a subgraph of G. The subgraph H is spanning subgraph of G if **V` = V**.

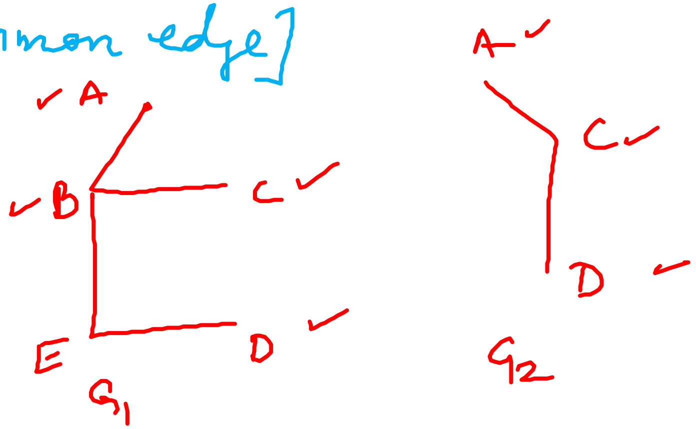
①



(a) Petersen Graph (b) Highlighted Subgraph (c) Extracted Subgraph

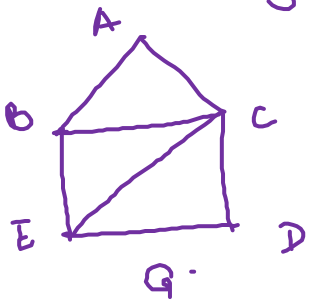
③ Edge Disjoint Subgraph –

$E_1(G_1) \cap E(G_2) = \phi$ [i.e., G_1 & G_2 don't have common edge]



Note → edge disjoint subgraph may have vertices in common but vertex disjoint subgraph can't have common edge

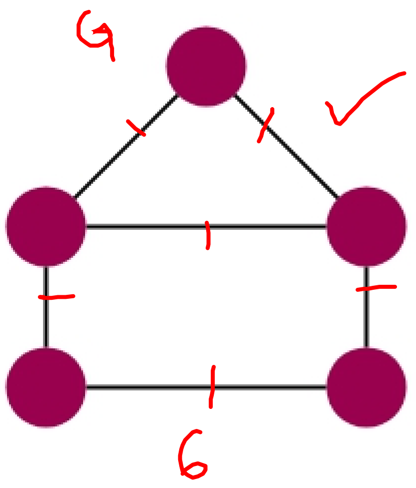
② Vertex Disjoint ⇒ Any two subgraph $G_1 = (V_1, E_1)$ & $G_2(V_2, E_2)$ are said to be vertex disjoint if $V_1(G_1) \cap V_2(G_2) = \phi$ [no common vertex b/w G_1 & G_2]



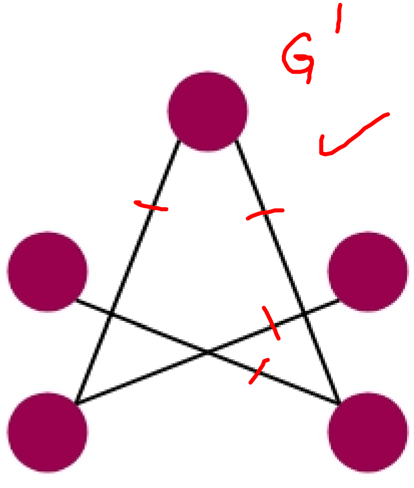
Vertex Disjoint

Complement of graph

- Complement of a simple graph G is a simple graph G' having–
  - All the vertices of G.
  - An edge between two vertices v and w iff there exists no edge between v and w in the original graph G

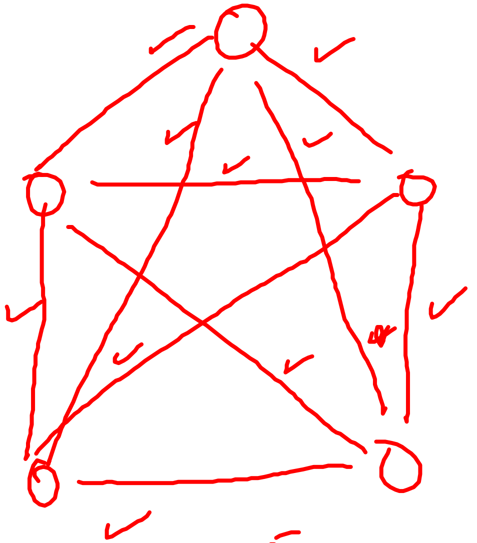


Graph G



Complement Graph $\overline{G}$

$\Rightarrow$



$$\boxed{E(G) + E(\overline{G'}) = \frac{n(n-1)}{2}}$$

$n = 5$

$$e = \frac{n(n-1)}{2}$$

$= 10$

Complement of graph

1. Ex1: A simple graph G has 10 vertices and 21 edges. Find total number of edges in its complement graph G'.

n e

$$E(G) + E(\bar{G}) = \frac{n(n-1)}{2}$$

$$\frac{10 \times 9}{2}$$

$$21 + E(\bar{G}) = 45$$

$$E(\bar{G}) = 24$$

2. Let G be a simple graph of order n. If the size of G is 56 and the size of G' is 80. What is n?

$n = ?$ $e(G)$ $e(\bar{G})$

$$E(G) + E(\bar{G}) = \frac{n(n-1)}{2}$$

$$56 + 80 = \frac{n(n-1)}{2}$$

$$136 \times 2 = n(n-1) \Rightarrow$$

$$n = 17$$

3. A simple graph G has 30 edges and its complement graph G' has 36 edges. Find number of vertices in G.

$E(G)$ $E(\bar{G})$ $n = ?$