Graph Theory

MTH-S503

Syllabus

- UNIT-1: Graphs, Sub graphs, Some basic properties, Different types of graphs (Regular, Bipartite, Induced, Quotient etc.) walks, paths & circuits, connected graphs, disconnected graphs and its components, Euler graphs and its properties, Fluery' s algorithms and Chinese postman problem Operation on graphs, Hamiltonian graphs and its properties, Hamiltonian paths and circuits, the traveling sales man problem. Shortest distance algorithms (Dijkstra' s)
- UNIT-2: Cuts sets and cut vertices, some properties, all cut sets in a graph, fundamental circuits and cut sets, connectivity and separability , Rank, Nullity of a graph. Digraphs : Definition, Types of Digraphs, Digraphs and Binary relations, Directed path and connectedness,Euler Digraphs. Chap 3
- UNIT-3: Trees and its characterization, Distance, Height, Diameters, Radius of a tree, Weighted Tree, Rooted and Binary trees, Spanning trees , Weighted spanning tree , Minimum weight spanning tree algorithms prim' s and Kruskal' s. Chords, Branches, Fundamental circuits.
DS
- UNIT-4: Planarity: Planer graphs, Regions, Euler formula, Kuratowski two graphs, Characterization of planarity, detection of planarity, Thickness and Crossings number of a graph. Colouring of graphs: Vertex colouring , Edge colouring, Five colour Theorem, Chromatic number, chromatic polynomials, Methods of finding the chromatic polynomial, Chromatic partitioning, Independence number and Covering number. Matchings , Maximal matching, Augmenting path, Hall' s marriage problem.
- UNIT5: Matrix representation of graphs : Incidence, Adjacency, Circuit, Cut-set and Path matrices and their properties. Matrix representation of Digraphs (Adjacency matrix).

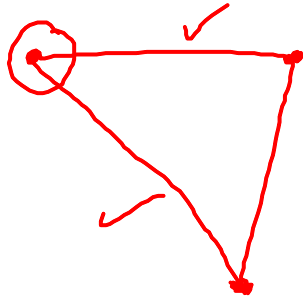
Overview

$G = (V, E)$

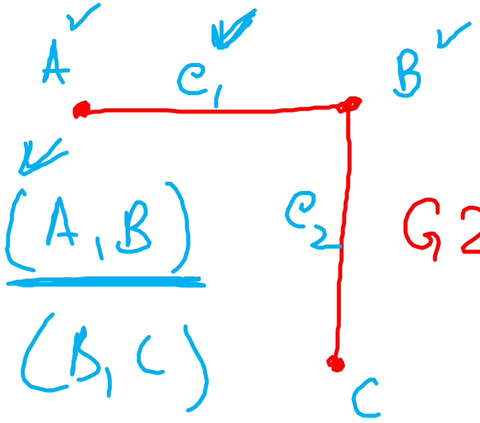
V — finite or infinite

E —

- Introduction to Graph Theory
  - Graphs, Multigraphs, Simple Graphs — undirected
  - Directed Graphs
  - Degree and Degree Sequence
  - Subgraphs
  - Graph Complement, Cliques and Inependent Sets



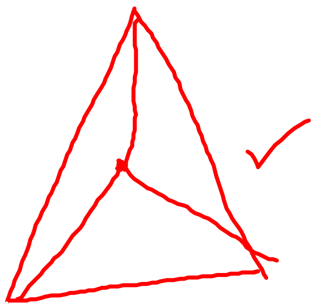
G1

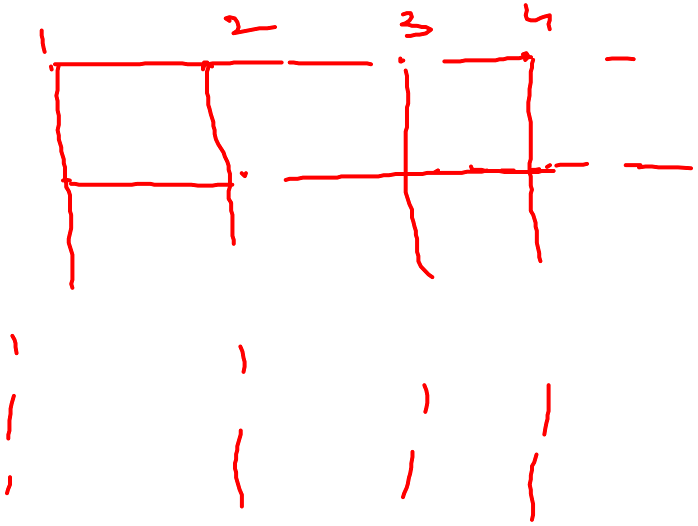


G2

$e_1 = (A, B)$

$e_2 = (B, C)$





$V = \{A, B, C\}$

$E = \{(A, B), (B, C)\}$

or

$E = \{e_1, e_2\}$

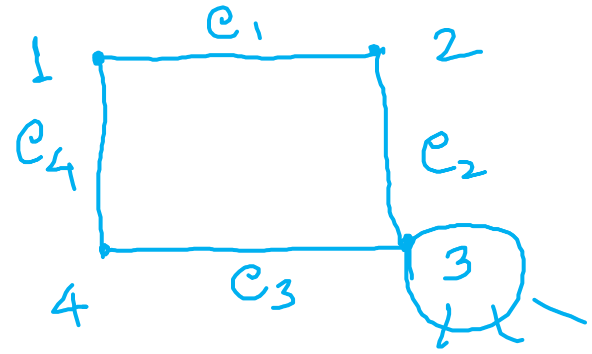
Graphs, Multi-Graphs, Simple Graphs

Graph: A graph is a tuple $G = (V, E)$ where V is a (finite) set of vertices and E is a finite collection of edges. The set E contains elements from the union of the one and two element subsets of V. That is, each edge is either a one or two element subset of V.

Example: V = {1, 2, 3, 4}

E = {{1, 2}, {2, 3}, {3, 4}, {4, 1}}

e_1 e_2 e_3 e_4

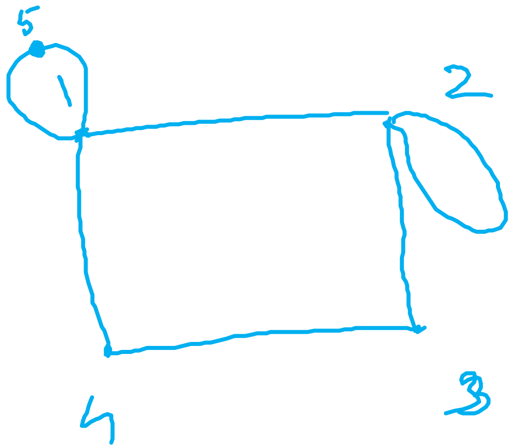


Self-Loop: If G = (V, E) is a graph and **v** ∈ V and **e** = {v}, then edge **e** is called a self-loop. That is, any edge that is a single element subset of V is called a self-loop.

Example: V = {1, 2, 3, 4}

E = {{1, 2}, {2, 3}, {3, 4}, {4, 1}, {1}}

↳ (1,1)



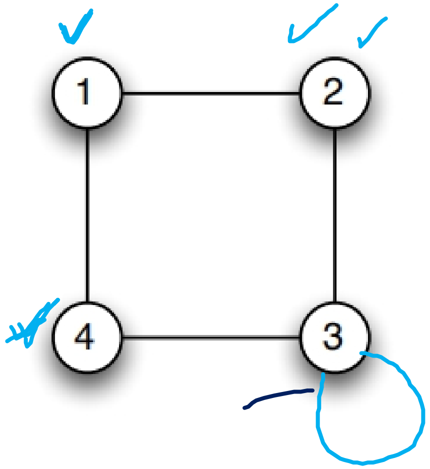
Graphs, Multi-Graphs, Simple Graphs

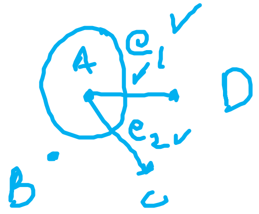
Vertex Adjacency: Let G = (V, E) be a graph. Two vertices v_1 and v_2 are said to be adjacent if there exists an edge **e** ∈ **E** so that **e** = $\{v_1, v_2\}$. A vertex v is self-adjacent if e = {v} is an element of E.

Edge Adjacency: Let G = (V, E) be a graph. Two edges e_1 and e_2 are said to be adjacent if there exists a vertex **v** so that **v** is an element of both e_1 and e_2 (as sets). An edge e is said to be adjacent to a vertex v if v is an element of e as a set.

Neighborhood: Let G = (V, E) be a graph and let v ∈ V . The neighbors of v are the set of vertices ~~that are adjacent to v.~~

Example: The neighborhood of Vertex 1 is Vertices 2 and 4 and Vertex 1 is adjacent to these vertices.





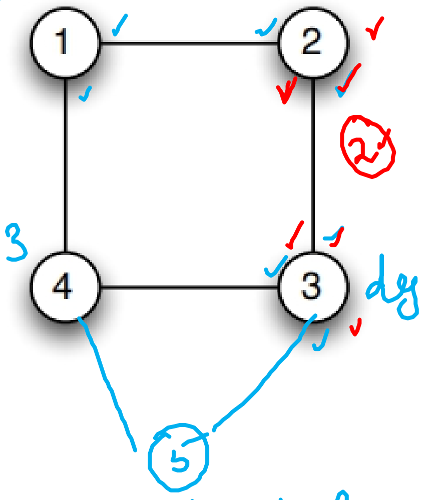
e_1 & e_2 are adjacent

Graphs, Multi-Graphs, Simple Graphs

Degree: Let G = (V, E) be a graph. The ***degree of v***, written ***deg(v)*** is the number of non-self-loop edges adjacent to v plus two times the number of self-loops defined at v.

$\deg(v) = $ No of edges incidence

Ex1

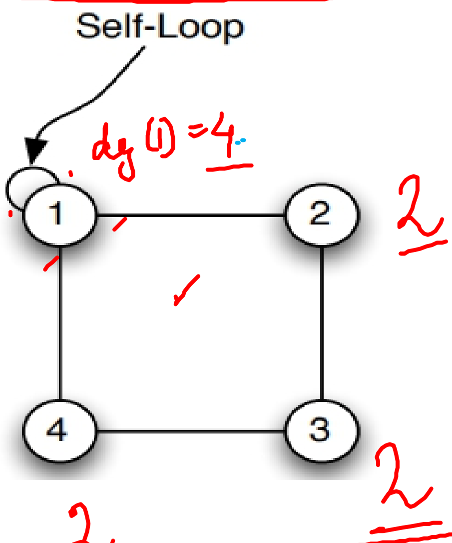


$|V| = 5$

$|E| = 6$

$\Sigma \deg(v) = 12$

$$\boxed{\Sigma \deg(v) = 2|E|}$$

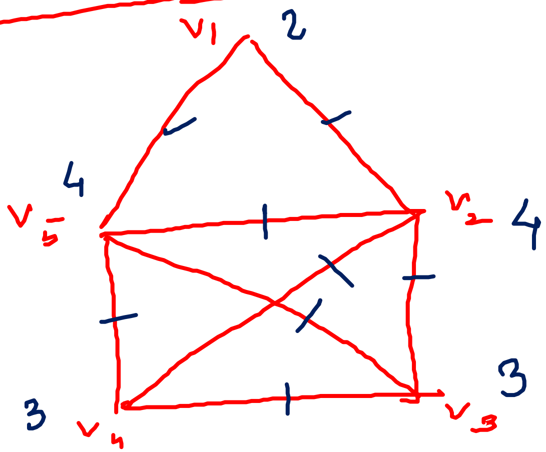


$4 = |V|$

$|E| = 5$

$\Sigma \deg(v) = 10$

$$\boxed{\Sigma \deg(v) = 2e}$$



$\Sigma \deg(v) = 16$

$e = 8$

$$\boxed{\Sigma \deg(v) = 2e}$$

Graphs, Multi-Graphs, Simple Graphs

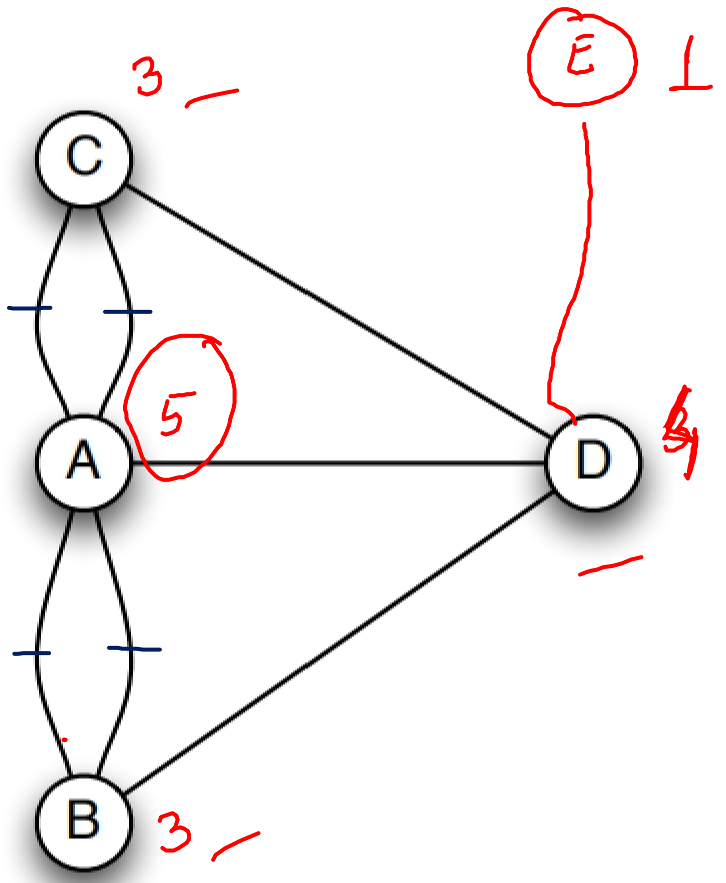
Multi-Graphs: Let $G = (V, E)$ is a ***multigraph*** if there are two edges e_1 and e_2 in E so that e_1 and e_2 are equal as sets. That is, there are two vertices v_1 and v_2 in V so that $e_1 = e_2 = \{v_1, v_2\}$.

(contains parallel edges)

Example: The vertex set is V = {A, B, C, D}.

The edge collection is:

E = {{A, B}, {A, B}, {A, C}, {A, C}, {A, D}, {B, D}, {C, D}}



Let $G = (V, E)$ be a graph. There are two degree values that are of interest in graph theory: the largest and smallest vertex degrees usually denoted $\Delta(G)$ and $\delta(G)$. That is:

Max-degree $\Delta(G) = \max_{v \in V} deg(v)$ $= \cancel{8}\ 5 =$

Min-degree $\delta(G) = \min_{v \in V} deg(v)$ $= \cancel{6}\ 3$ / 1

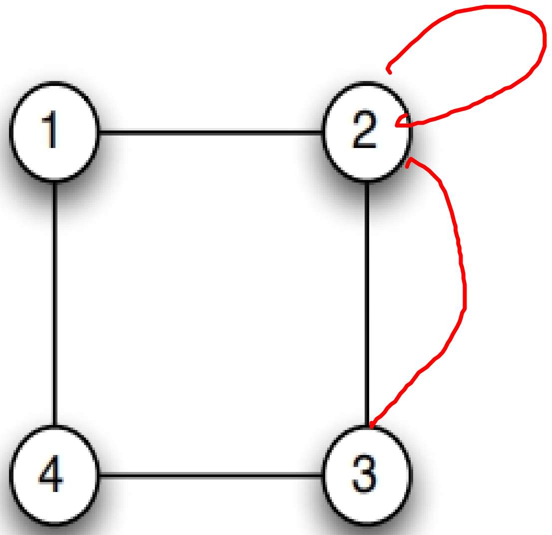
Graphs, Multi-Graphs, Simple Graphs

Simple Graph: Let G = (V, E) is a ***simple graph*** if G has **no edges** that are **self-loops** and if E is a subset of two element subsets of V ; i.e., G is not a **multi-graph**.

① No self loop & parallel edges
$\Rightarrow$ simple graph

② No parallel edge $\Rightarrow$ graph

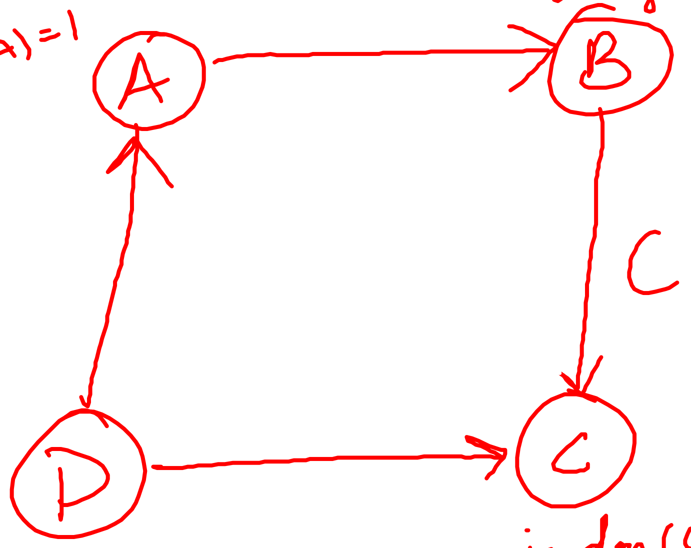
③ self loop & parallel edge $\Rightarrow$ multigraph



Directed graph →

in deg(B) = 1
out deg(B) = 1

in deg(A) = 1
out deg(A) = 1



in deg(C) = 2
out deg(C) = 0

in deg(D) = 0
out deg(D) = 2

(A, B) or (B, A)

(A, B) ✓

(B, A) ✗ Not written

in-deg

out deg