Bresenham's circle drawing algorithm:

It is not easy to display a continuous smooth arc on the computer screen as our computer screen is made of pixels organized in matrix form. So, to draw a circle on a computer screen we should always choose the nearest pixels from a printed pixel so as they could form an arc.

Now, how to calculate the next pixel location from a previously known pixel location (x, y). In Bresenham's algorithm at any point (x, y) we have two option either to choose the next pixel in the east i.e. (x+1, y) or in the south east i.e. (x+1, y-1).

This can be decided by the decision parameter d.

- If d <= 0, then N(X+1, Y) is to be chosen as next pixel.
- If d > 0, then S(X+1, Y-1) is to be chosen as the next pixel.

Let's say our circle is at some random pixel P whose coordinates are (xk, yk). Now we need to find out our next pixel.

The shortest of d1 and d2 will help us Decide our next pixel.

note- $x_{k+1} = x_k + 1$

As x_{k+1} is the next consecutive pixel of x_k

similarly $y_{k-1} = y_k - 1$

Equation of Circle, Radius r

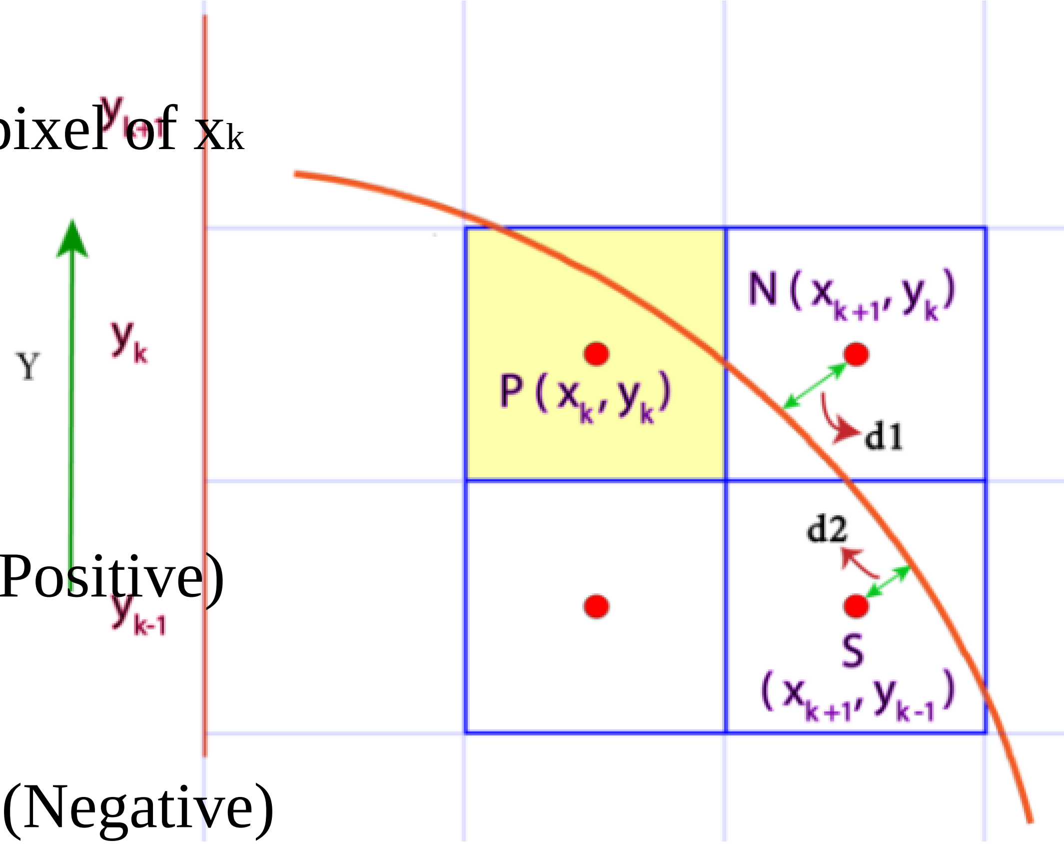
$x^2 + y^2 = r^2$

Function of Circle at N

$F(N) = (x_{k+1})^2 + (y_k)^2 - r^2$ (Positive)

Function of Circle at S

$F(S) = (x_{k+1})^2 + (y_{k-1})^2 - r^2$ (Negative)



Now we need a decision parameter which help us decide the next pixel

Say D_k

And , $D_k = F(N)+F(S)$

So if $D_k < 0$ that means the negative F(S) is bigger then the positive F(N), that implies Point N is closer to the circle than point S. So we will select pixel N as our next pixel.

and if $D_k > 0$ that means positive F(N) is bigger and S is more closer as F(S) is smaller. So we will Select S as our next pixel.

Now lets find D_k

$D_k = (x_{k+1})^2 + (y_k)^2 – r^2 + (x_{k+1})^2 + (y_{k-1})^2 – r^2$

(replacing x_{k+1} with $x_k + 1$ and y_{k-1} with $y_k -1$)

$= 2(x_k + 1)^2 + (y_k)^2 + (y_k -1)^2 – 2r^2$ ----- (i)

Now lets find D_{k+1} (Replacing every k with k+1 in eq. i)

$D_{k+1} = 2(x_{k+1} + 1)^2 +(y_{k+1})^2 + (y_{k+1} -1)^2 – 2r^2$

(Replacing x_{k+1} with $x_k + 1$ but now we can't replace y_{k+1} because we don't know the exact value of y_k)

$= 2(x_k+2)^2 + (y_{k+1})^2 + (y_{k+1} -1)^2 – 2r^2$ ----- (ii)

Now to find out the decision parameter of next pixel i.e. D_{k+1} We need to find $D_{k+1} - D_k$ = (ii) - (i)

$= 4x_k + 2(y_{k+1})^2 - 2y_{k+1} - 2(y_k)^2 + 2y_k + 6$

$D_{k+1} = D_k + 4x_k + 2(y_{k+1})^2 - 2y_{k+1} - 2(y_k)^2 + 2y_k + 6$ ----- (iii)

- If ($D_k < 0$) then we will choose N point as discussed. i.e. (xk+1, yk) next y coordinate is y_k i.e. $y_{k+1} = y_k$, putting y_k in (iii)

now, $D_{k+1} = D_k + 4x_k + 2(y_k)^2 - 2y_k - 2(y_k)^2 + 2y_k + 6$

$= D_k + 4x_k + 6$

If ($D_k > 0$) then we will choose S point. i.e. (x_{k+1}, y_{k+1})

Now we know $y_{k+1} = yk – 1$

$$D_{k+1} = D_k + 4(x_k - y_k) + 10$$

Now to find initial decision parameter means starting point that is (0,r) ,value of y is r Putting (0,r) in (i)

$D_k = 2(x_k + 1)^2 + (y_k)^2 + (y_k -1)^2 – 2r^2$

$D0 = 2(0 + 1)^2 + r^2 + (r-1)^2 – 2r^2$

$= 3-2r$

Step 1: Get the Radius of Circle R And Coordinates of centre of circle (Xc,Yc)

Step 2: X and Y are going to be plotted points Set X=0 and Y=R

Step 3: D = 3-2R (Initial decision Parameter)

Step 4: Plot Circle (Xc,Yc,X,Y)

- Step 5: if D < 0 Then D = D + 4X + 6

X=X+1

Y=Y

- D >=0 then D=D+4(X-Y)+10

X=X+1

Y=Y-1

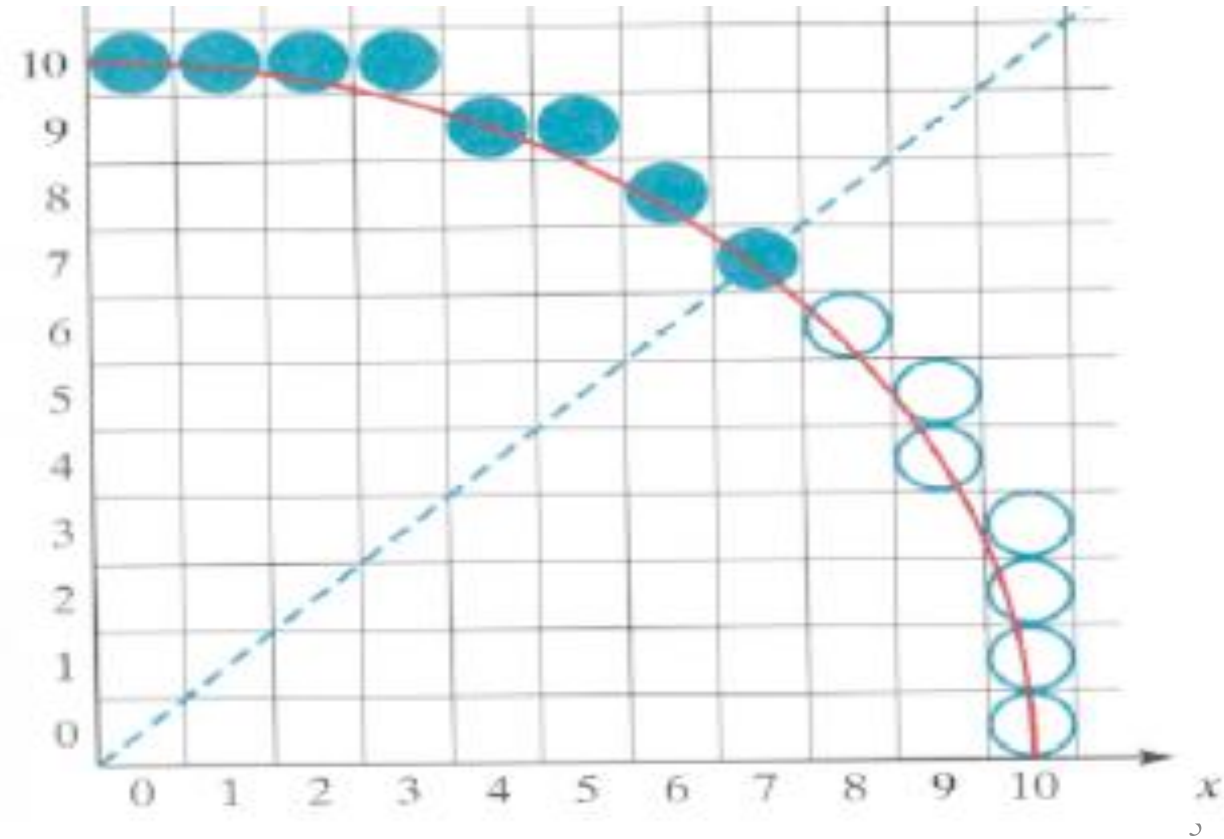
Step 6: Check, if X=Y : Stop/Exit

Advantages

- It is a simple algorithm.
- It can be implemented easily
- It is totally based on the equation of circle i.e. $x^2 + y^2 = r^2$

Disadvantages

- There is a problem of accuracy while generating points.
- This algorithm is not suitable for complex and high graphic images



Example): Given a circle radius r = 10, demonstrate the Bresenham's circle algorithm by determining positions along the circle octant in the first quadrant from x = 0 to x = y.

Step 1: Set X=0 and Y=R
Step 2: D = 3-2R
(Initial decision Parameter)
Step 3: Plot Circle (0,r)

- if D < 0 Then
  D = D + 4X + 6
  X=X+1
  Y=Y
- D >=0 then
  D=D+4(X-Y)+10
  X=X+1
  Y=Y-1

K	P_k	x_{k+1}	y_{k+1}
		0	10
0	– 17	1	10
1	– 11	2	10
2	– 1	3	10
3	13	4	9
4	– 5	5	9
5	17	6	8
6	11	7	7