Mid-Point Line Generation Algorithm

Given coordinate of two points A(x1, y1) and B(x2, y2) such that x1 < x2 and y1 < y2. The task to find all the intermediate points required for drawing line AB on the computer screen of pixels. Note that every pixel has integer coordinates.

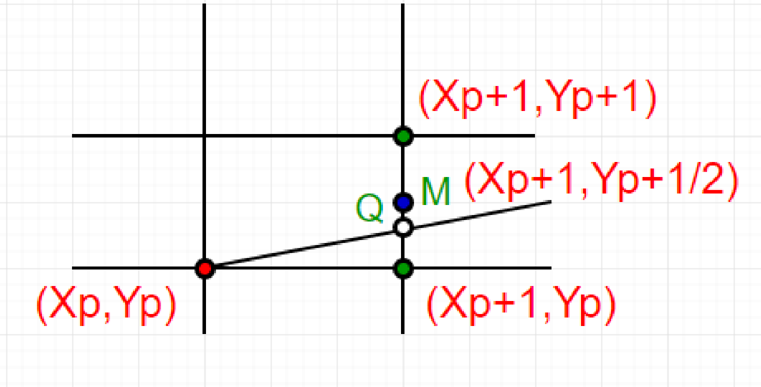
for any given/calculated previous pixel $P(X_k, Y_k)$, there are two candidates for the next pixel closest to the line, Right(X_k+1, Y_k) and Left(X_k+1, Y_k+1)

In Mid-Point algorithm we do following.

Find middle of two possible next points. Middle of $R(X_k+1, Y_k)$ and $L(X_k+1, Y_k+1)$ is $M(X_{k+1}, Y_k+1/2)$.

If M is above the line, then choose R as next point.

If M is below the line, then choose L as next point.



How to find if a point is above a line or below a line?

Below are some assumptions to keep algorithm simple.

- We draw line from left to right.
- x1 < x2 and y1< y2
- Slope of the line is between 0 and 1. We draw a line from lower left to upper right.

Let us consider a line $y = mx + B$.

We can re-write the equation as :

$y = (dy/dx)x + B$ or

$(dy)x + B(dx) - y(dx) = 0$

Let **F(x, y)** $= (dy)x - y(dx) + B(dx)$ -----(1)

Let we are given two end points of a line (under above assumptions)

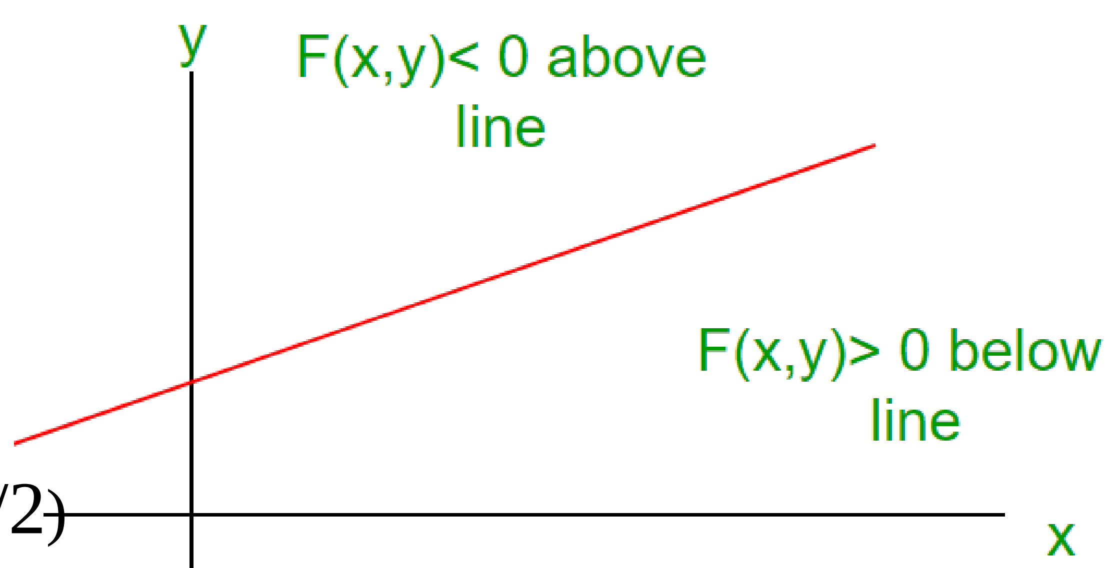
- For all points (x,y) on the line,
  the solution to F(x, y) is 0.
- For all points (x,y) above the line,
  F(x, y) result in a negative number.
- And for all points (x,y) below the line,
  F(x, y) result in a positive number.

This relationship is used to determine the relative position of M

$M = (Xp+1, Yp+1/2)$

So our **decision parameter d** is,

$d = F(M) = F(Xp+1, Yp+1/2)$



How to efficiently find new value of d from its old value?

For simplicity, let as write F(x, y) as ax + by + c.

Where a = dy

b = -dx

c = B*dx, We got these values from above equation (1)

Case 1: If R is chosen then for next point :

$d_{k+1} = F(X_p+2, Y_{p+1}/2)$

$= a(X_p+2) + b(Y_{p+1}/2) + c$

$d_k = a(X_{p+1}) + b(Y_{p+1}/2) + c$

Difference (Or delta) of two distances:

$\Delta d = d_{k+1} - d_k$

$= a(X_p+2)- a(X_{p+1})+ b(Y_{p+1}/2)- b(Y_{p+1}/2)+ c-c$

$= a$

Therefore, $d_{k+1} = d_k + dy$. (as a = dy)

Case 2: If L is chosen then for next point :

$d_{k+1} = F(X_p+2, Y_p+3/2)$

$= a(X_p+2) + b(Y_p+3/2) + c$

$d_k = a(X_p+1) + b(Y_p+1/2) + c$

Difference (Or delta) of two distances:

$\Delta d = d_{k+1} – d_k$

$= a(X_p+2)- a(X_{p+1})+ b(Y_p+3/2)- b(Y_p+1/2)+ c-c$

$= a + b$

Therefore, $d_{k+1} = d_k + dy - dx$. (as a = dy, b= -dx)

Calculation For initial value of decision parameter d0:

d0 = F(X1+1 , Y1+1/2)

= a(X1 + 1) + b(Y1 + 1/2) +c

= aX1+ bY1 + c + a + b/2

= F(X1,Y1) + a + b/2

= a + b/2 (as F(X1, Y1) is on the circle so = 0)

d0 = dy – dx/2. (as a = dy, b = -dx)

PRACTICE PROBLEMS BASED ON MID POINT LINE DRAWING ALGORITHM-

Problem-01: Calculate the points between the starting coordinates (20, 10) and ending coordinates (30, 18).

Solution- Given-

Starting coordinates = $(X_0, Y_0) = (20, 10)$

Ending coordinates = $(X_n, Y_n) = (30, 18)$

Step-01:

Calculate ΔX and ΔY

$\Delta X = X_n - X_0 = 30 - 20 = 10$

$\Delta Y = Y_n - Y_0 = 18 - 10 = 8$

Step-02:

Calculate $d_{initial}$ -

$d_o = dy - dx/2 = 8 - 10/2 = 3$

Step-03:

As $d_{initial} >= 0$, so **case-02** is satisfied than $d_{k+1} = d_k + dy - dx = 1$

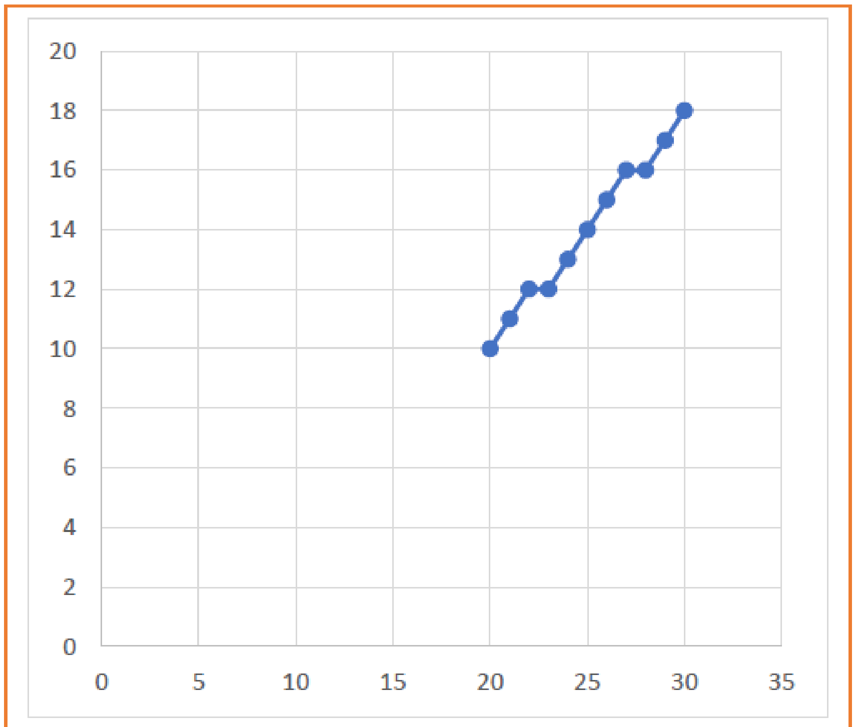
Thus,

$X_{k+1} = X_k + 1 = 20 + 1 = 21$

$Y_{k+1} = Y_k + 1 = 10 + 1 = 11$

As $d_{initial} < 0$, so **case-01** is satisfied than
$d_{k+1} = d_k + dy$.
Thus,
$X_{k+1} = X_k + 1$
$Y_{k+1} = Y_k$

$d_{k+1} = d_k + dy$.
$= -1+8$
$=7$



$D_{initial}$	D_{new}	X_{k+1}	Y_{k+1}
		20	10
3	1	21	11
1	-1	22	12
-1	7	23	12
7	5	24	13
5	3	25	14
3	1	26	15
1	-1	27	16
-1	7	28	16
7	5	29	17
5		30	18

Advantages of Mid Point Line Drawing Algorithm-

The advantages of Mid Point Line Drawing Algorithm are-

- Accuracy of finding points is a key feature of this algorithm.
- It is simple to implement.
- It uses basic arithmetic operations.
- It takes less time for computation.
- The resulted line is smooth as compared to other line drawing algorithms.

Disadvantages of Mid Point Line Drawing Algorithm-

The disadvantages of Mid Point Line Drawing Algorithm are-

- This algorithm may not be an ideal choice for complex graphics and images.
- In terms of accuracy of finding points, improvement is still needed.
- There is no any remarkable improvement made by this algorithm.

Problem-02:

Calculate the points between the starting coordinates (5, 9) and ending coordinates (12, 16)

Step-01:

Calculate ΔX and ΔY.

$\Delta X = X_n – X_0 = 12 – 5 = 7$

$\Delta Y = Y_n – Y_0 = 16 – 9 = 7$

Step-02:

Calculate $d_{initial}$

$d_o = dy - dx/2 = 7 – 7/2 = 3.5$

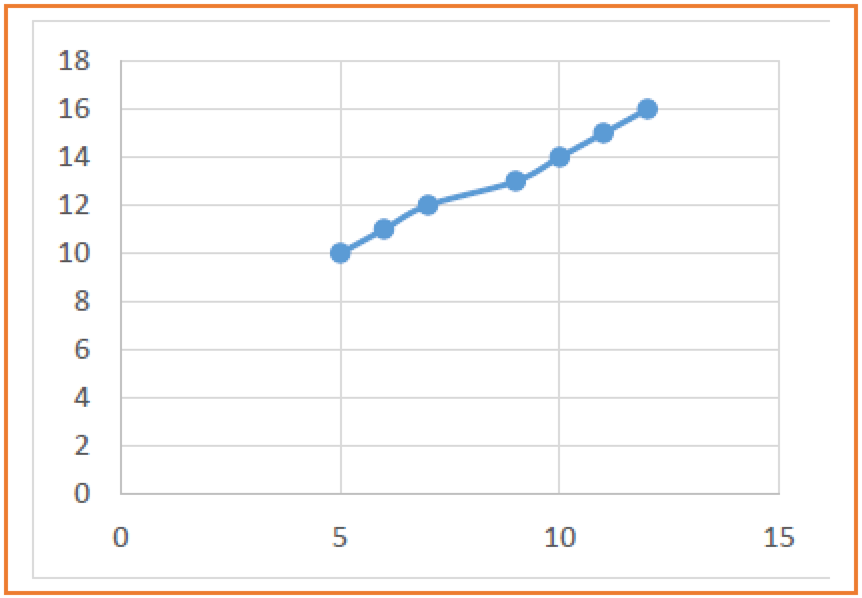
Step-03:

As $d_{initial}$ >= 0, so **case-02** is satisfied than $d_{k+1} = d_k + dy – dx = 3.5$

Thus,

$X_{k+1} = X_k + 1 = 5 + 1 = 6$

$Y_{k+1} = Y_k + 1 = 9 + 1 = 10$



$D_{initial}$	D_{new}	X_{k+1}	Y_{k+1}
		5	9
3.5	3.5	6	10
3.5	3.5	7	11
3.5	3.5	8	12
3.5	3.5	9	13
3.5	3.5	10	14
3.5	3.5	11	15
3.5		12	16