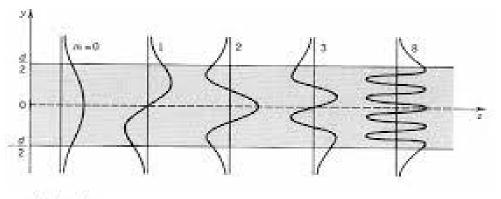
OPTICAL FIBERS WAVEGUIDE-I

Guided Modes in a Planar Waveguide



m: Mode order

Only discrete values of are allowed in a waveguide

Optical Fiber Waveguide

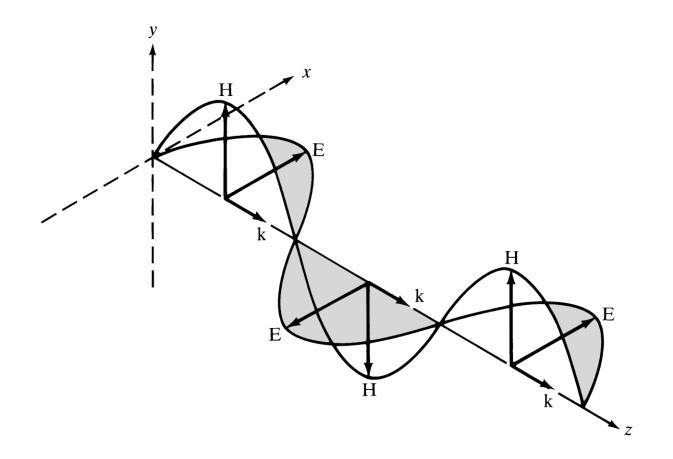
- □ To understand transmission mechanisms of optical fibers with dimensions approximating to those of a human hair;
 - Necessary to consider the optical waveguiding of a cylindrical glass fiber.
- Fiber acts as an open optical waveguide may be analyzed using simple ray theory **Geometric Optics**
 - > Not sufficient when considering all types of optical fibers
- Electromagnetic Mode Theory for Complete Picture

ELECTROMAGNETIC THEORY

- To obtain an detailed understanding of propagation of light in an optical fiber
 - Light as a variety of EM vibrations E and H fields at right angle to each other and perpendicular to direction of propagation.
 - Necessary to solve Maxwell's Equations
 - Very complex analyses *Qualitative aspects only*

Field distributions in plane E&H waves

• Light as a variety of EM vibrations **E** and **H** at right angle to each other and perpendicular to direction of propagation.



Maxwell's Equations

 Assuming a linear isotropic dielectric material having no currents and free charges

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$
$$\nabla \cdot \mathbf{D} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$

where $D = \epsilon E$ and $B = \mu H$.

Maxwell's Equations

Substituting for **D** and **B** and taking curl of first equation

$$abla imes (
abla imes \mathbf{E}) = -\mu \frac{\partial}{\partial t} (
abla imes \mathbf{H}) = -\epsilon \mu \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Using vector identity

$$abla imes (
abla imes \mathbf{E}) =
abla (
abla \cdot \mathbf{E}) -
abla^2 \mathbf{E}$$

We get

$$\nabla^2 \mathbf{E} = \epsilon \mu \, \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

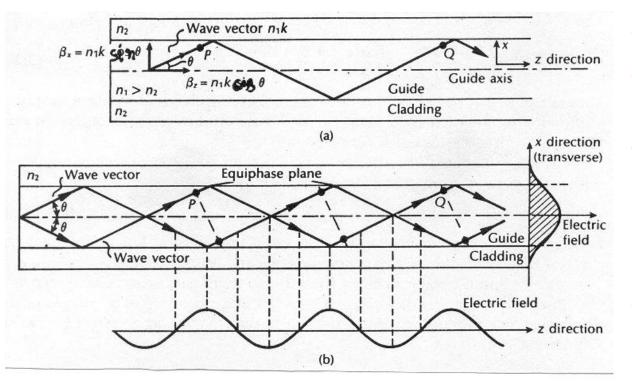
Similarly

$$\nabla^2 \mathbf{H} = \epsilon \mu \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

 \Box Wave equations for each component of the field vectors **E** & **H**.

Concept of Modes

A plane monochromatic wave propagating in direction of ray path within the guide of refractive index n₁ sandwiched between two regions of lower refractive index n₂



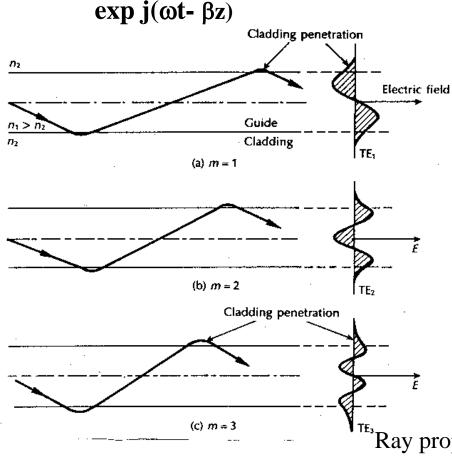
- Wavelength $= \lambda/n_1$
- Propagation constant $\beta = n_1 k$
- Components of β in z and x directions
 - $\beta_z = n_1 k \cos \theta$
 - $\beta_x = n_1 k \sin \theta$
- Constructive interference occurs and standing wave obtained in x-direction
- (a) A plane wave propagating in the guide (b) Interference of plane wave in the guide (forming lowest order mode m=0)

Concept of Modes

- Components of plane wave in x-direction reflected at corecladding interface and interfere
 - **Constructive**: when *total phase change* after two reflection is equal to $2m\pi$ radians; m an integer **Standing wave in x-direction**
 - The optical wave is confined within the guide and the electric field distribution in the x-direction does not change as the wave propagate in the z-direction – Sinusoidally varying in z-direction
- * The stable field distribution in the x-direction with only a periodic zdependence is known as a MODE.
 - Specific mode is obtained only when the angle between the propagation vectors or rays and interface have a particular value **Discrete modes** typified by a distinct value of θ
 - Have periodic z-dependence of $exp(-j\beta_z z)$ or commonly $exp(-j\beta z)$
 - Have time dependence with angular frequency ω , i.e. exp (j ω t)

Higher Order Modes

• For monochromatic light fields of angular frequency ω , a mode traveling in positive z-direction has a time and z-dependence given by



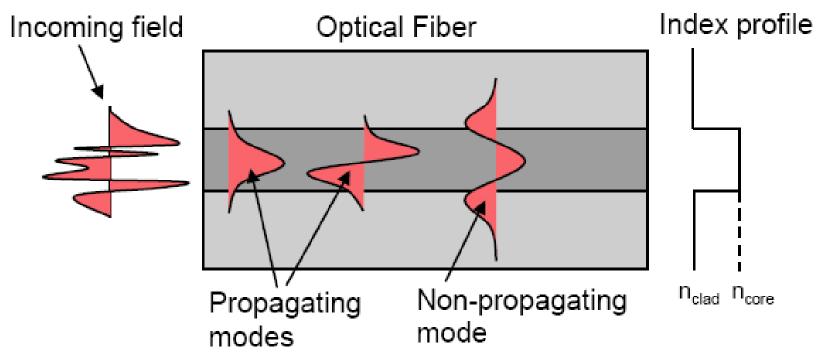
Dominant modes propagating in z-direction with electric field distribution in x-direction formed by rays with m=1,2,3

m denotes number of zeros in this transverse pattern.

It also signifies the order of the mode and is known as **mode number**.

^aRay propagation and corresponding TE field patterns of three lower order modes in planar guide.

Wave picture of waveguides



- The step-index profile provide focusing just like lenses and GRIN materials
- The guides modes of the fiber are those that propagate without changing their profile
- The guided modes are those intensity profiles, for which the focusing, due to the index profile, exactly matches the diffraction
- In the core is small, only one such mode exists (single mode fiber)

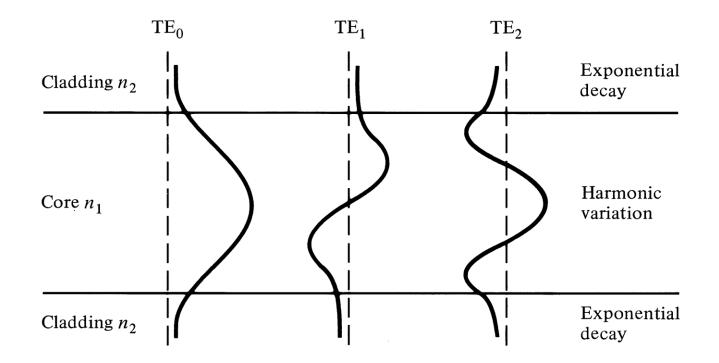
TE and TM modes

• **Transverse Electric mode (TE)**: When electric field is perpendicular to the direction of propagation, i.e. $E_z=0$, but a

corresponding component of the magnetic field **H** is in the direction of propagation.

- Transverse Magnetic (TM) mode: When a component of E field is in the direction of propagation, but H_z=0.
- Transverse ElectroMagnetic (TEM) : When total field lies in the transverse plane in that case both E_z and H_z are zero.

Low-order TE or TM mode fields



Phase and Group Velocity

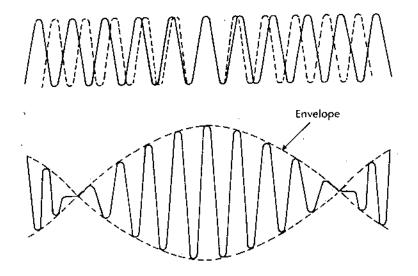
• **Phase Velocity**: For plane wave, there are points of constant phase, these constant phase points forms a surface, referred to as a **wavefront**.

> As light wave propagate along a waveguide in the z-direction, wavefront travel at a phase velocity ; $\mathbf{v}_{\mathbf{p}} = \omega / \beta$

• Non-monochromaticity leads to group of waves with closely similar frequencies – Wave Packet

Wave packet observed to move at a group velocity, $v_g = \delta \omega / \delta \beta$

 V_g is of great importance in study of TCs of optical fibers as it relates to the propagation characteristics of observable wave groups



Formation of wave packet from combination of two waves of nearly equal frequencies

Group Velocity

- Considering propagation in an infinite medium of R.I. n₁,
- Propagation constant: $\beta = n_1 k = n_1 \frac{2\pi}{\lambda} = n_1 \frac{\omega}{c}$ Phase velocity: $v_p = \frac{c}{n_1}$ Group velocity: $v_g = \frac{c}{\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right)} = \frac{c}{N_g}$
 - Parameter N_g is known as the *group index* of the guide

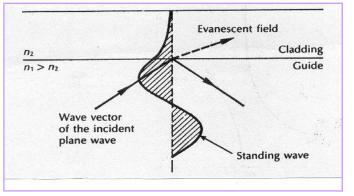
Evanescent Field

Another phenomenon of interest under conditions of TIR is the form of the electric field in the cladding of the guide.

The transmitted wave field in the cladding is of the form

$$\mathbf{B} = \mathbf{B}_0 \exp(-\xi_2 \mathbf{x}) \exp \mathbf{j}(\omega \mathbf{t} - \beta \mathbf{z})$$

The amplitude of the field in the cladding is observed to decay exponentially in the x-direction.- *Evanescent Field*



Exponentially decaying evanescent field in the cladding

- A field of this type stores energy and transports it in the direction of propagation (z) but does not transport energy in the transverse direction (x).
- Indicates that optical energy is transmitted into the cladding.

Cladding Material

* The evanescent field gives rise to the following requirements for the choice of cladding material

- > The cladding should be transparent to light at the wavelengths over which the guide is to operate.
- Ideally, the cladding should consist of a solid material in order to avoid both damage to the guide and the accumulation of foreign matter on the guide walls.
- The cladding thickness must be sufficient to allow the evanescent field to decay to a low value or losses from the penetrating energy may be encountered.
- Therefore, the most widely used optical fibers consist of a core and cladding, both made of glass. Although, it give a lower NA for fiber, but provides a far more practical solution.

Step Index / Graded Index fiber

