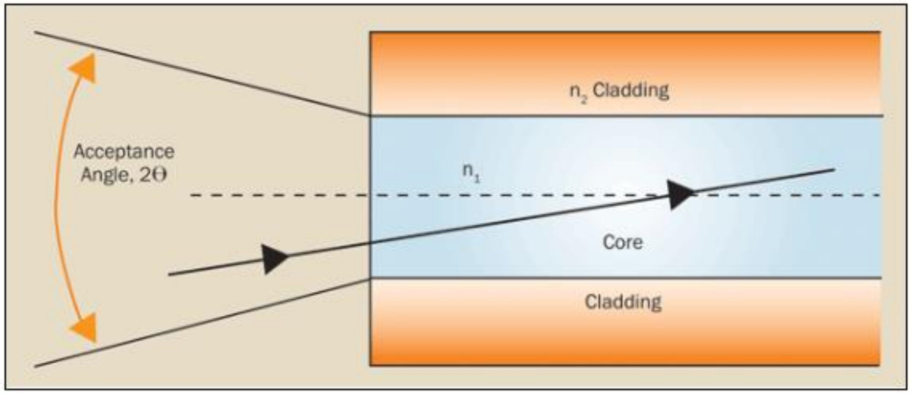
OPTICAL FIBER WAVEGUIDE-II



Optical Fiber Wave guiding

- To understand transmission mechanisms of optical fibers with dimensions approximating to those of a human hair;
  - Necessary to consider the optical waveguiding of a cylindrical glass fiber.

- Fiber acts as an open optical waveguide – may be analyzed using simple ray theory – **Geometric Optics**
  - Not sufficient when considering all types of optical fibers

- **Electromagnetic Mode Theory** for Complete Picture

Cylindrical Fiber

- **Exact solution of Maxwell's Eqns. for a cylindrical dielectric waveguide-** ***very complicated & complex*** results
- In common with planar waveguide, TE and TM modes are obtained within dielectric cylinder.
  - A cylindrical waveguide is bounded in two dimensions, therefore, two integers, l and m to specify the modes.

    $\mathbf{TE}_{lm}$ and $\mathbf{TM}_{lm}$ modes

  These modes from meridional rays propagation within guide
- Hybrid modes where $\mathbf{E}_z$ and $\mathbf{H}_z$ are nonzero – results from skew ray propagation within the fiber. Designated as

  $\mathbf{HE}_{lm}$ and $\mathbf{EH}_{lm}$ depending upon whether the components of **H** or **E** make the larger contribution to transverse field

Modes in Cylindrical Fibers

- Analysis simplified by considering fibers for communication purposes.
  - **Satisfy, weakly guided approximation , Δ<<1, small grazing angles θ**

- Approximate solutions for full set of HE, EH, TE and TM modes may be given by two linearly polarized (LP) components
  - Not exact modes of fiber except for fundamental mode, however, as Δ is very small, HE-EH modes pairs occur with almost identical propagation constants $\Rightarrow$ **Degenerate modes**
  - The superposition of these degenerating modes characterized by a common propagation constant corresponds to particular LP modes regardless of their HE, EH, TE or TM configurations.
  - *This linear combination of degenerate modes produces a useful simplification in the analysis of weakly guiding fibers.*

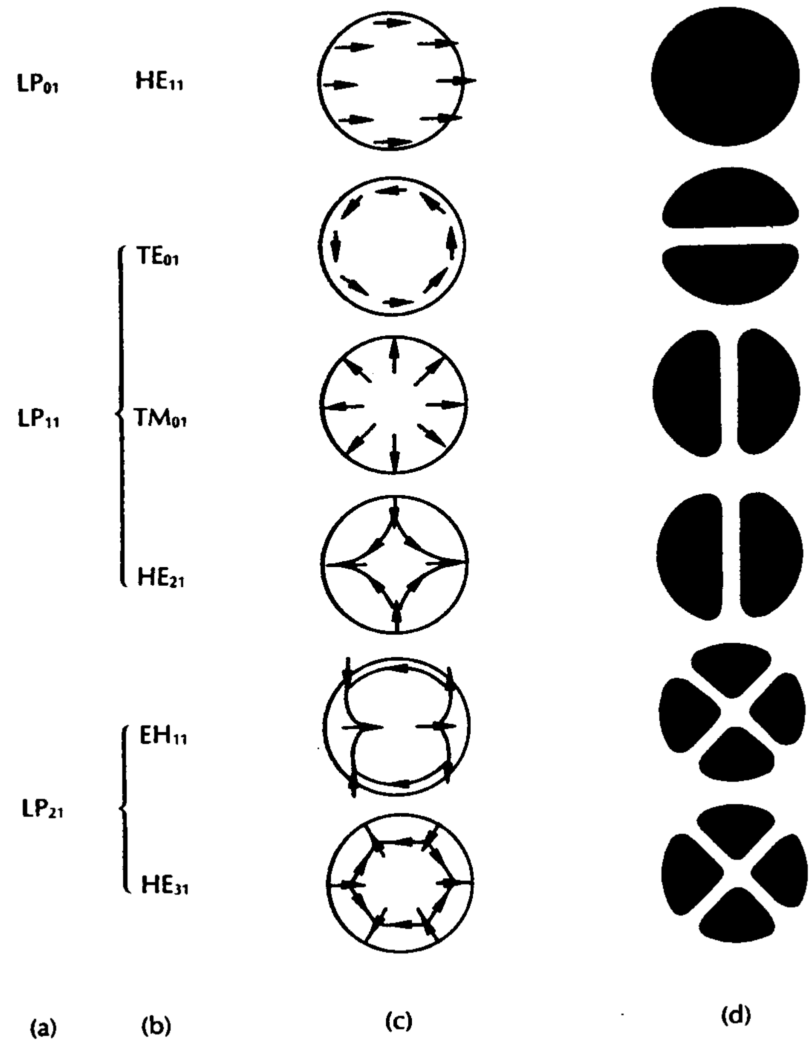
Correspondence between the lower order in linearly polarized modes and the traditional exact modes from which they are formed.

Linearly polarized	Exact
LP_{01}	HE_{11}
LP_{11}	HE_{21}, TE_{01}, TM_{01}
LP_{21}	HE_{31}, EH_{11}
LP_{02}	HE_{12}
LP_{31}	HE_{41}, EH_{21}
LP_{12}	HE_{22}, TE_{02}, TM_{02}
LP_{lm}	HE_{2m}, TE_{0m}, TM_{0m}

Intensity Profiles

- **Electric field configuration for the three lowest LP modes in terms of their constituent exact modes:**
    - (a) LP mode designations;
    - (b) exact mode designations;
    - (c) electric field distribution of the exact modes;
    - (d) intensity distribution of E_x for exact modes indicating the electric field intensity profile for the corresponding LP modes.

❖ **Field strength in the transverse direction is identical for the modes which belong to the same LP mode.**



Solutions of Wave Equation

- The scalar wave equation for homogeneous core waveguide under weak guidance conditions is

$$\frac{d^2\Psi}{dr^2}+\frac{1}{r}\frac{d\Psi}{dr}+\frac{1}{r^2}\frac{d^2\Psi}{d\phi^2}+\left(n_1^2k^2-\beta^2\right)\Psi=0$$

Ψ is the field (**E** or **H**).

- The propagation constant for the guided modes β lie in the range

$$n_2k< \beta <n_1k$$

- Solution of wave equation for cylindrical fiber have the form

$$\Psi=E(r)\left\{\begin{matrix}\cos l\phi \\ \hline \sin l\phi\end{matrix}\exp(\omega t-\beta z)\right\}$$

Here, Ψ Represents the dominant transverse electric field component. The periodic dependence on ϕ gives a mode of radial order l.

Introducing the solution to wave equation results in a differential equation

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + \left[\left(n_1^2k^2 - \beta^2\right) - \frac{l^2}{r^2}\right]E = 0$$

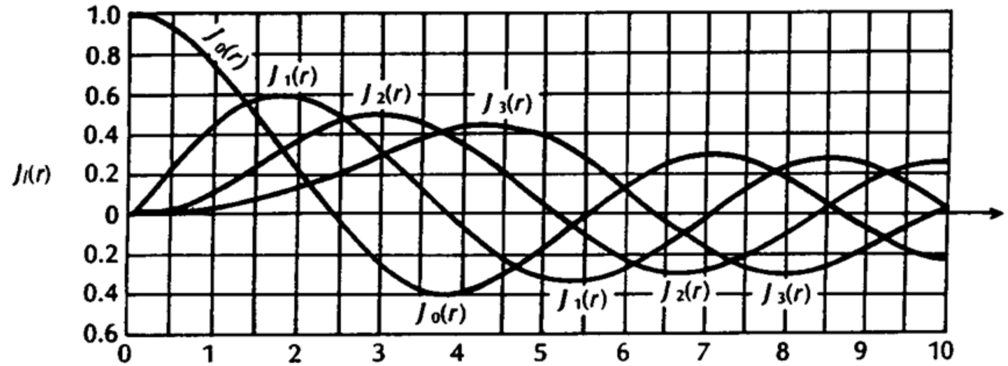
- For a SI fiber with constant RI core, it is a Bessel differential equation and the solutions are cylinder functions. In the core region the solutions are ***Bessel functions*** denoted by J_l (Gradually damped oscillatory functions w.r.t. r)

- Important to note is that the field is finite at r =0 and is represented by the ***Zero order Bessel function*** J_0. However, the field vanishes as r goes to infinity and the solutions in the cladding are therefore ***modified Bessel functions*** denoted by K_l – These modified functions decay exponentially w.r.t. r.

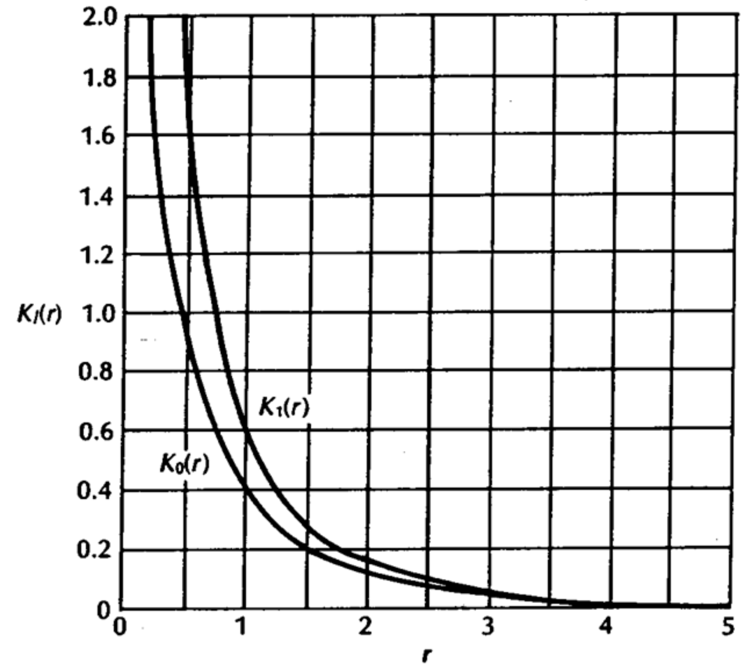
Figures Showing

(a) Variation of the Bessel function $J_l(r)$ for l = 0, 1, 2, 3 (first four orders), plotted against r.

(b) Graph of the modified Bessel function $K_l(r)$ against r for l = 0, 1.



(a)



(b)

Bessel Function Solutions

- The electric field is given by

$$\mathbf{E}(r) = GJ_l(UR) \qquad \text{for } R<1 \ \text{(core)}$$

$$= GJ_l(U)\, K_l(WR)/K_l(W) \qquad \text{for } R>1 \text{(cladding)}$$

where G; amplitude coefficient, R=r/a; normalized radial coordinate, U & W are eigen values in the core and cladding respectively

U; radial phase parameter or radial propagation constant

W; cladding decay parameter

$$U = a(n_1^2k^2-\beta^2)^{1/2} \qquad \text{and} \qquad W= a(\beta^2-n_2^2k^2)^{1/2}$$

- The sum of squares of U & W defines a very useful quantity usually referred to as ***normalized frequency V***

$$\boldsymbol{V = (U^2+W^2)^{1/2} = ka(n_1^2-n_2^2)^{1/2}}$$

V-Number

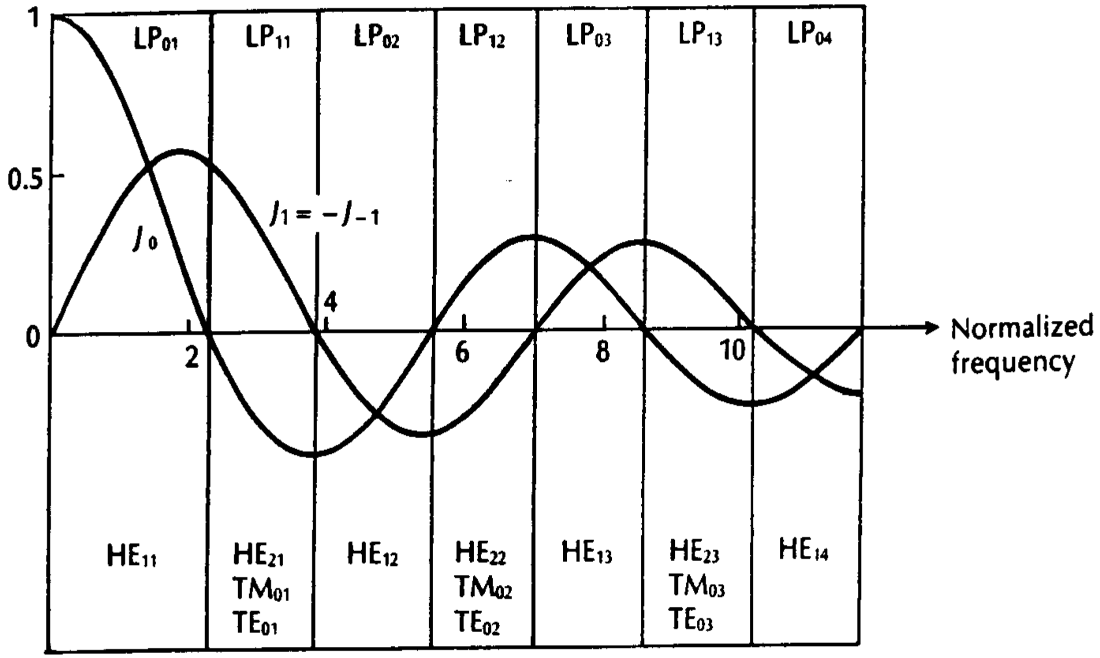
Normalized Frequency, V may be expressed in terms of NA and Δ, as

$$V = \frac{2\pi}{\lambda} a(NA) = \frac{2\pi}{\lambda} a\, n_1 (2\Delta)^{\frac{1}{2}}$$

- Normalized frequency is a dimensionless parameter and simply called ***V-number*** or ***value of the fiber***.
- It combines in a very useful manner the information about three parameters, **a**, **Δ** and **λ**.

Allowed LP modes

- **Lower order modes obtained in a cylindrical homogeneous core waveguide**



The allowed regions for the LP modes of order *l* = 0,1 against normalized frequency (*V*) for a circular optical waveguide with a constant refractive index core (step index fiber).

- Value of V, where J_0 and J_1 cross the zero gives the cutoff point for various modes.

  $\mathbf{V = V_c}$;

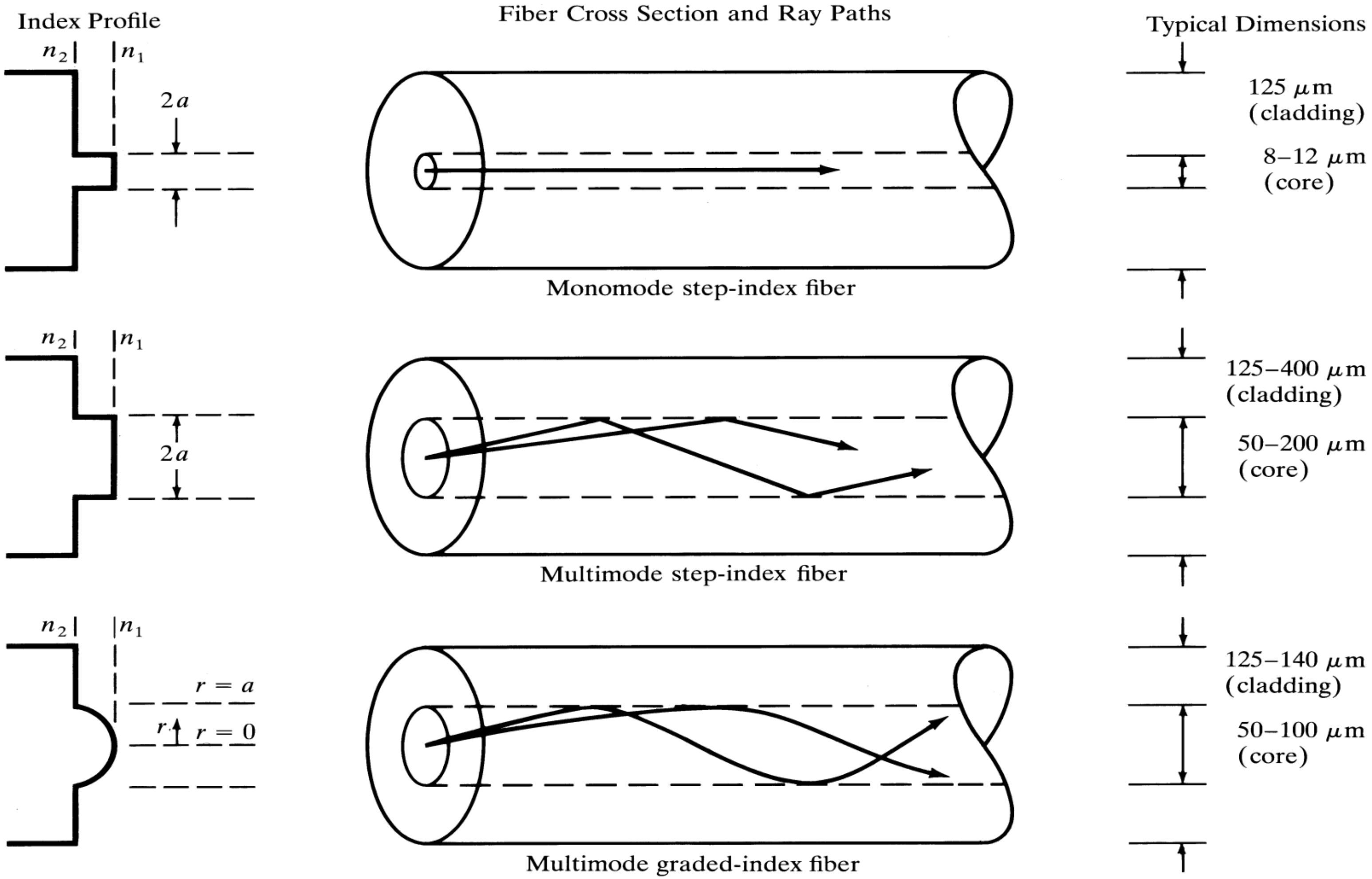
- V_c is different for different modes

  = 0 for LP_{01} mode

  = 2.405 for LP_{11}

  = 3.83 for LP_{02}

Step Index / Graded Index fiber



THANK

YOU