* <u>Relation</u> :- A relation from A to B is a subset of the cartesian product of ~~A×B~~ A and B i.e. $A \times B$.

* Number of relation possible from A to B is equal to the number of subset of $A \times B$.

* If $A = B$ then we call relation on A. Relation is denoted by R.

* <u>Equivalence Relation</u> :- A relation R on a set A is said to be equivalence relation if the following properties are satisfied :

① Reflexive Relation :- A relation R on a set A is said to be reflexive if ~~∀a∈A~~ aRa, $\forall a \in A$

i.e. $(a,a) \in R$, $\forall a \in A$.

② <u>Symmetric Relation</u> :- A relation R on a set A is said to be symmetric if ~~∀a,b∈A~~

$aRb \Rightarrow bRa$,

i.e. $(a,b) \in R \Rightarrow (b,a) \in R$,

for every ordered pair $(a,b) \in R$.

③ Transitive Relation :- A relation on a set A is said to be transitive if

aRb and $bRc \Rightarrow aRc$.

i.e. $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$.

~~Find the~~ Equivalence classes posses the following properties:

① $a \in [a]$

② Any two equivalence classes are either disjoint or equal i.e. either $[a] \cap [b] = \phi$ or $[a] = [b]$.

③ The set A is the union of all its disjoint equivalence classes.

Ex Find the equivalence classes for the equivalence relation $a \equiv b \pmod 5$ on the set Z of integers.

First, we consider the integer 0. Then the equivalence class [0] is given by

$$[0] = \{x : x \in Z,\ x - 0 = 5n,\ n \in Z\}$$
$$= \{x : x \in Z,\ x = 5n,\ n \in Z\}$$
$$= \{ \ldots, -15, -10, -5, 0, 5, 10, 15, \ldots \}$$

The equivalence class of 1 is given by

$$[1] = \{x : x \in Z,\ x - 1 = 5n,\ n \in Z\}$$
$$= \{x : x \in Z,\ x = 1 + 5n,\ n \in Z\}$$
$$= \{ \ldots, -14, -9, -4, 1, 6, 11, 16, \ldots \}$$

Hence the equivalence relation $a \equiv b \pmod 5$ divide the set Z into five disjoint equivalence classes & $Z = [0] \cup [1] \cup [2] \cup [3] \cup [4]$

Similarly,

$$[2] = \{ \ldots, -13, -8, -3, 2, 7, 12, \ldots \}$$
$$[3] = \{ \ldots, -12, -7, -2, 3, 8, 13, \ldots \}$$
$$[4] = \{ \ldots, -11, -6, -1, 4, 9, 14, 19, \ldots \}$$
$$[5] = \{ \ldots, -10, -5, 0, 5, 10, 15, \ldots \}$$

— — — — — — — — —

Evidently, $[5] = [0]$.

These equivalence classes are called residue classes modulo 5 or congruence classes modulo 5.

Q- To show that congruent modulo m relation R is an equivalence relation.

Ans: we verify the following properties:

① $a \equiv a \pmod{m}$ iff $m \mid a-a$

iff $a-a = 0m$, $\forall a \in \mathbb{Z}$

Hence, R is Reflexive.

② $a \equiv b \pmod{m}$ iff $m \mid a-b$

iff $a-b = km$, for some integer k

iff $b-a = (-k)m$

iff $m \mid b-a$

iff $b \equiv a \pmod{m}$

Therefore, ~~Hence~~, if $aRb \Rightarrow bRa$, (mod m)

Hence, R is symmetric.

③ If $a \equiv b \pmod{m}$ iff $m \mid a-b$, $b \equiv c \pmod{m}$ iff $m \mid b-c$

iff $a-b = km$. iff $b-c = lm$

Adding

$$(a-b) + (b-c) = \overset{\text{int}}{k}m + \overset{\text{int}}{l}m$$

$$a-c = \underbrace{(k+l)}_{\text{int}}m$$

iff $m \mid a-c$

iff $a \equiv c \pmod{m}$

Therefore, if $a \equiv b$ and $b \equiv c \pmod{m} \Rightarrow a \equiv c \pmod{m}$.

Hence transitivity holds.

Thus, R is an equivalence relation on Z.