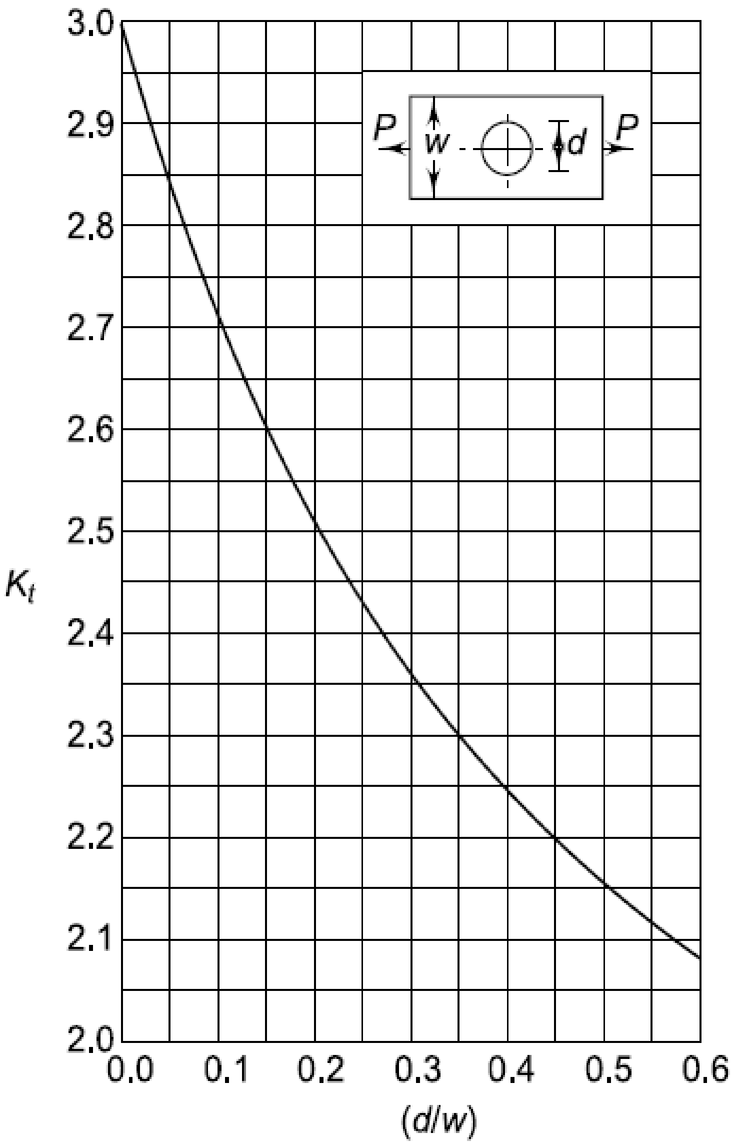


Fig. 5.2 *Stress Concentration Factor (Rectangular Plate with Transverse Hole in Tension or Compression)*

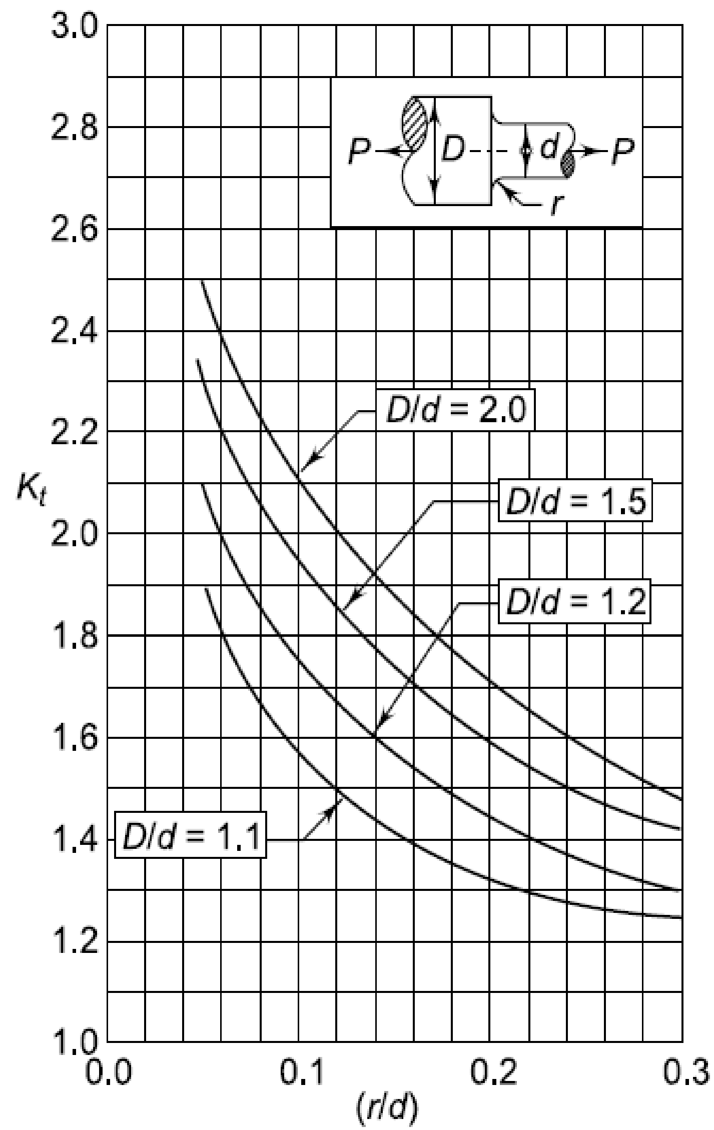


Fig. 5.4 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Tension)*

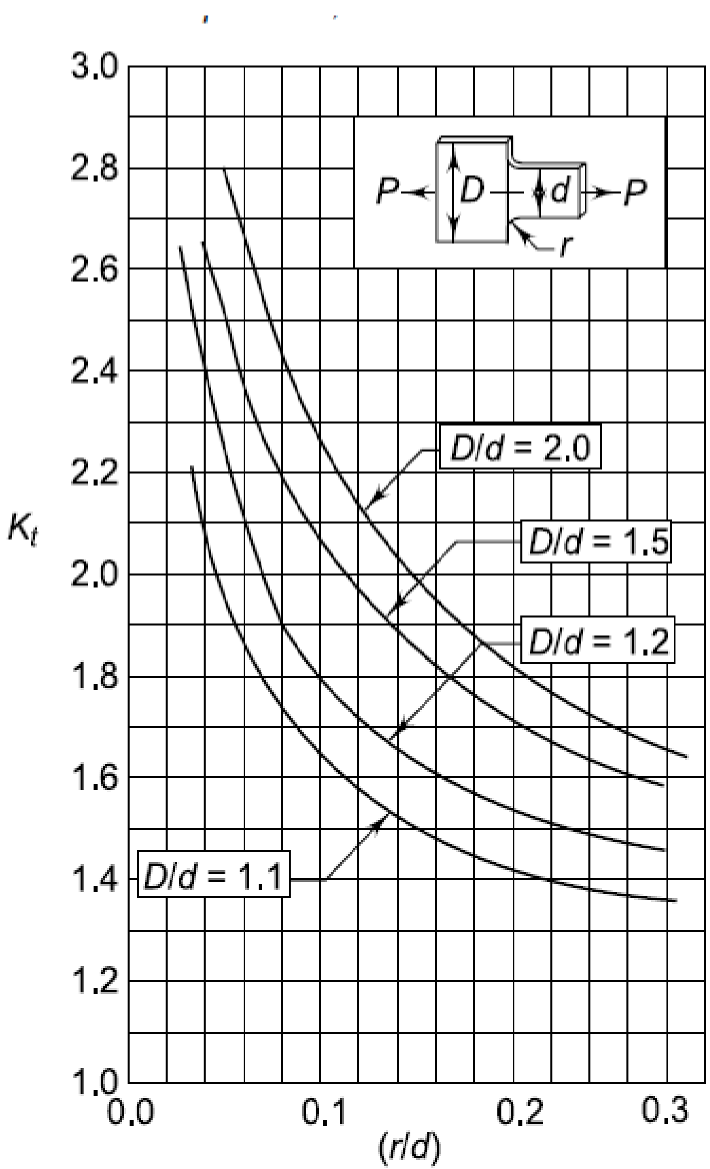


Fig. 5.3 *Stress Concentration Factor (Flat Plate with Shoulder Fillet in Tension or Compression)*

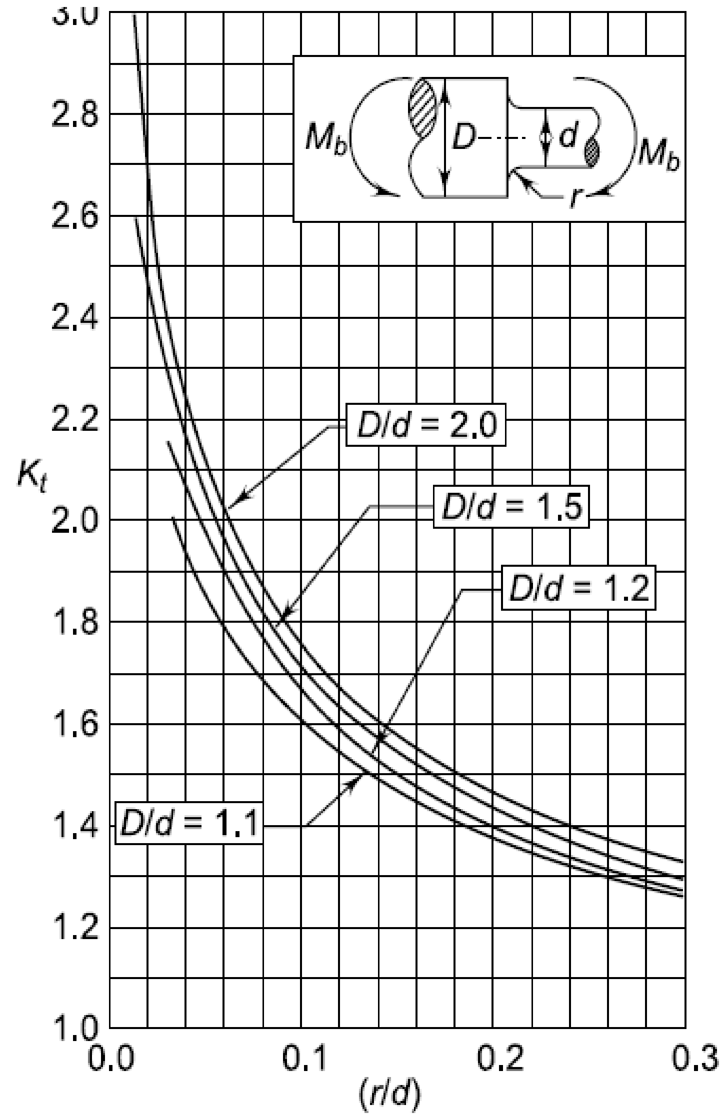


Fig. 5.5 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Bending)*

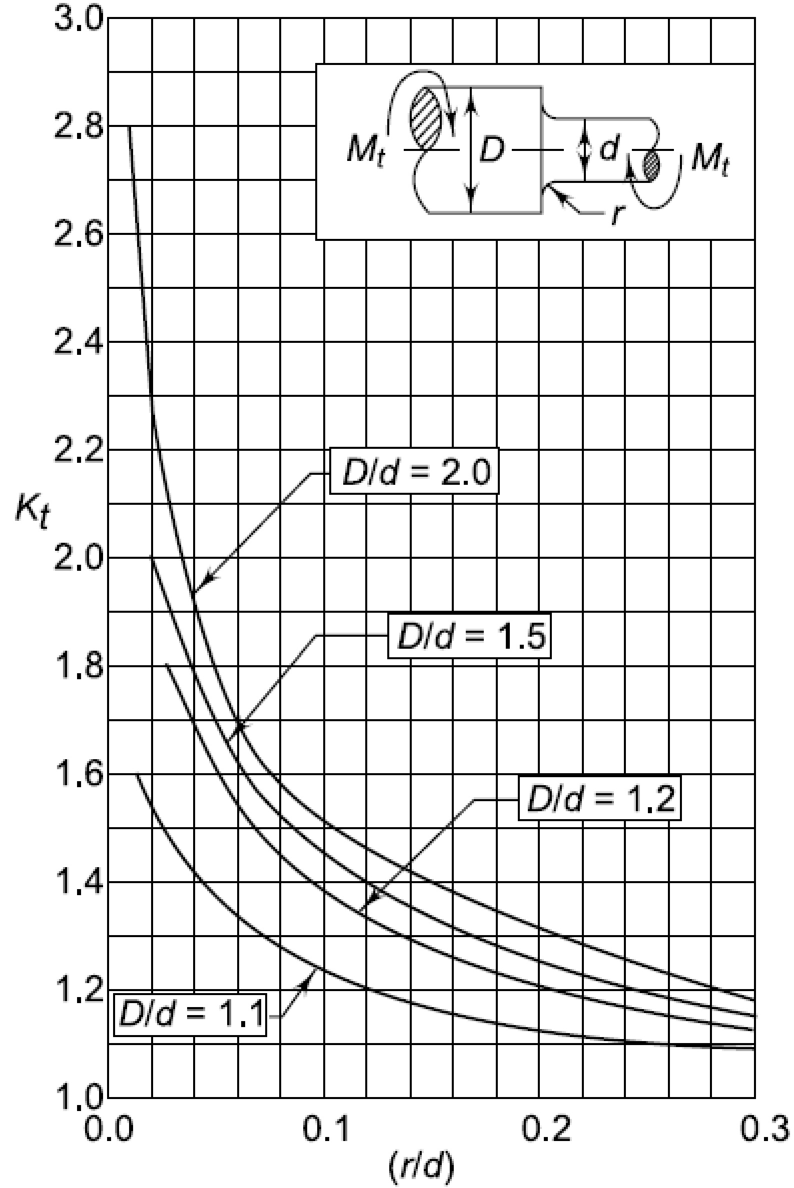
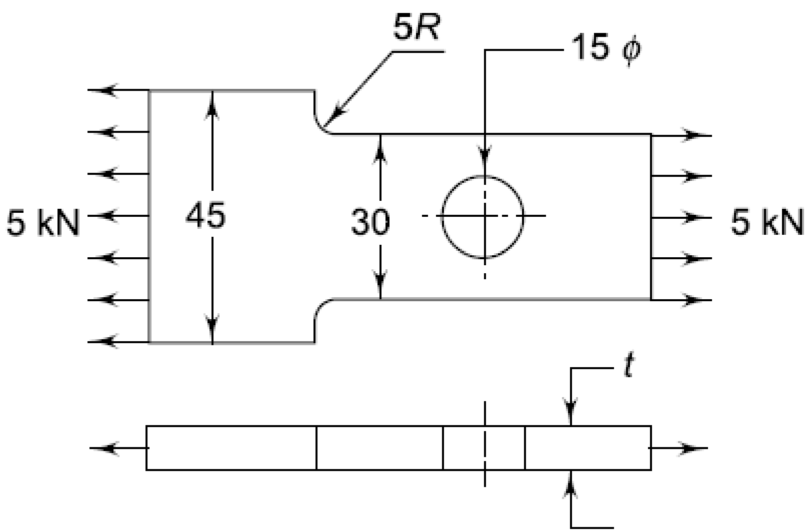


Fig. 5.6 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Torsion)*

A fl at plate subjected to a tensile force of 5 kN is shown in Fig.. The plate material is grey cast iron FG 200 and the factor of safety is 2.5. Determine the thickness of the plate.



Solution

Given $P = 5$ kN $\quad S_{ut} = 200$ N/mm² $(fs) = 2.5$

Step I *Calculation of permissible tensile stress*

$$\sigma_{max.} = \frac{S_{ut}}{(fs)} = \frac{200}{2.5} = 80 \text{ N/mm}^2$$

Step II Tensile stress at fillet section

The stresses are critical at two sections—the fillet section and hole section. At the fillet section,

$$\sigma_o = \frac{P}{dt} = \left(\frac{5000}{30t}\right)$$

$$\frac{D}{d} = \frac{45}{30} = 1.5 \text{ and } \frac{r}{d} = \frac{5}{30} = 0.167$$

From Fig. 5.3, $K_t = 1.8$

$$\therefore \sigma_{max.} = K_t \sigma_o = 1.8\left(\frac{5000}{30t}\right) = \left(\frac{300}{t}\right) \text{N/mm}^2 \quad \text{(i)}$$

Step III Tensile stress at hole section

$$\sigma_o = \frac{P}{(w-d)t} = \frac{5000}{(30-15)t} \text{ N/mm}^2$$

$$\frac{d}{w} = \frac{15}{30} = 0.5$$

From Fig. 5.2,

$$K_t = 2.16$$

$$\sigma_{max.} = K_t \sigma_o = 2.16\left[\frac{5000}{(30-15)t}\right] = \left(\frac{720}{t}\right) \text{N/mm}^2 \quad \text{(ii)}$$

Step IV Thickness of plate

From (i) and (ii), it is seen that the maximum stress is induced at the hole section.

Equating it with permissible stress, we get

$$\left(\frac{720}{t}\right) = 80$$

or $t = 9$ mm

A non-rotating shaft supporting a load of 2.5 kN is shown in Fig. The shaft is made of brittle material, with an ultimate tensile strength of 300 N/mm2. The factor of safety is 3. Determine the dimensions of the shaft.

Solution

Given $P = 2.5$ kN $S_{ut} = 300$ N/mm^2 $(fs) = 3$

Step I *Calculation of permissible stress*

$$\sigma_{max.} = \frac{S_{ut}}{(fs)} = \frac{300}{3} = 100 \text{ N/mm}^2$$

Step II Bending stress at fillet section

Due to symmetry, the reaction at each bearing is 1250 N. The stresses are critical at two sections—(i) at the centre of span, and (ii) at the fillet. At the fillet section,

$$\sigma_o = \frac{32 M_b}{\pi d^3} = \frac{32(1250 \times 350)}{\pi d^3} \text{ N/mm}^2$$

$$\frac{D}{d} = 1.1 \text{ and } \frac{r}{d} = 0.1$$

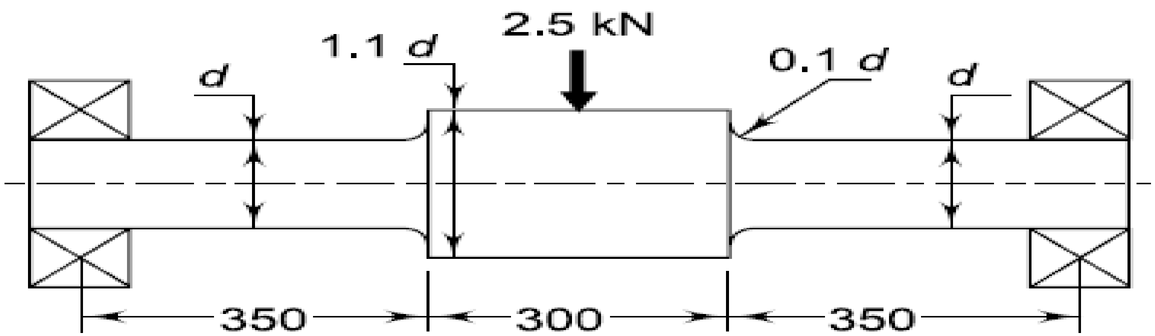


Fig. 5.14

From Fig. 5.5, $K_t = 1.61$

$$\therefore \quad \sigma_{max.} = K_t \sigma_o = 1.61\left[\frac{32(1250 \times 350)}{\pi d^3}\right]$$

$$= \left(\frac{7\,174\,704.8}{d^3}\right) \text{N/mm}^2 \qquad \text{(i)}$$

Step III   Bending stress at centre of the span

$$\sigma_o = \frac{32 M_b}{\pi d^3} = \frac{32(1250 \times 500)}{\pi (1.1d)^3}$$

$$= \left(\frac{4\,783\;018.6}{d^3} \right) \text{N/mm}^2 \qquad \text{(ii)}$$

Step IV   Diameter of shaft

From (i) and (ii), it is seen that the stress is maximum at the fillet section. Equating it with permissible stress,

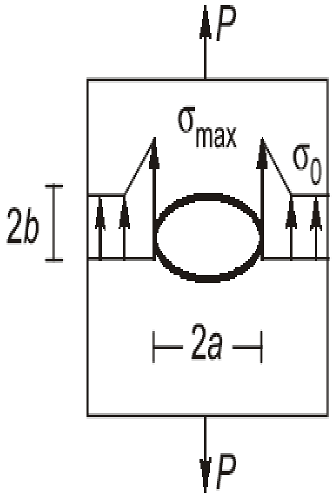
$$\left(\frac{7\;174\;704.8}{d^3} \right) = 100$$

or $$d = 41.55 \text{ mm}$$

Stress Concentration Near an Elliptical Hole

The stress at the joints away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole.

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

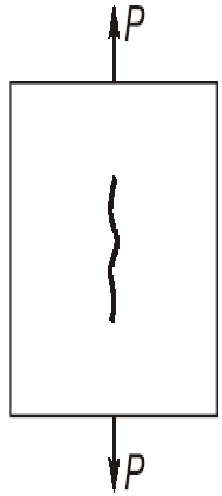


$$k_t = \frac{\sigma_{max}}{\sigma_0} = \left(1 + \frac{2a}{b}\right)$$

For circular hole $a = b$

$$k_t = 3$$

(a) Elliptical Hole

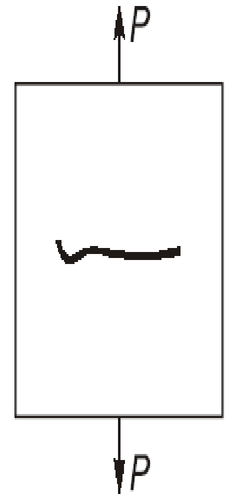


$$a = 0$$

$$k_t = \frac{\sigma_{max}}{\sigma_0} = 1$$

$$\sigma_{max} = \sigma_0$$

(b) Crack parallel to load

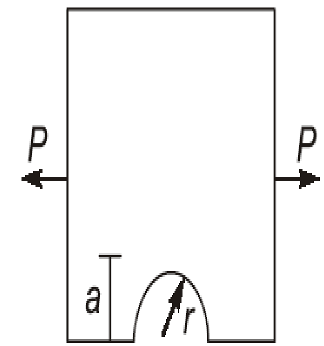


$$b = 0$$

$$k_t = \frac{\sigma_{max}}{\sigma_0} = \infty$$

$$\sigma_{max} = \infty$$

(c) Crack perpendicular to load



a - depth of notch

r - radius of notch

$$k_t = \frac{\sigma_{max}}{\sigma_0} = 1 + \frac{2a}{r}$$

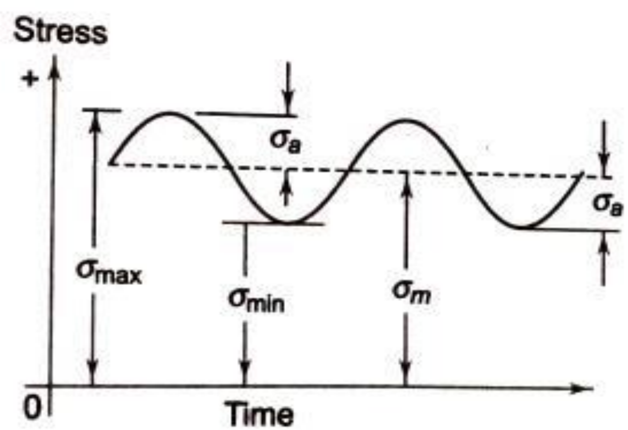
(d) Circular Notch

Fluctuating Stress

It is observed that about 80% of failure of mechanical components are due to fatigue failure .
There are three types of cyclic stress.
1.Fluctuating or alternating stress
2.Repeated stress
3.Reversed stress

Fluctuating Stress

- Varies in sinusoidal manner with respect to time
- It Fluctuate between two limits max. and min.
- The stress can be Tensile or Compressive or Partially Tensile and Compressive



$$Mean\ stress:\ \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$alternative\ or\ variable\ stress\ \sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

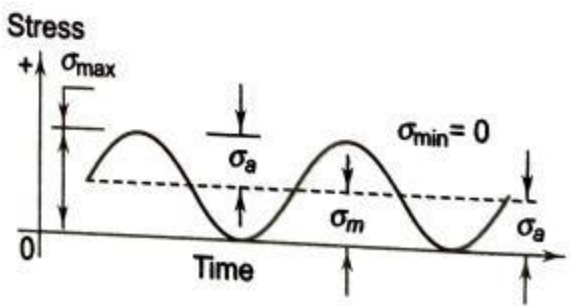
Repeated stress:
Varies in sinusoidal max. value
The half portion of stress

Mean stress,

$$\sigma_m = \frac{\sigma_{max}}{2}$$

Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max}}{2}$$



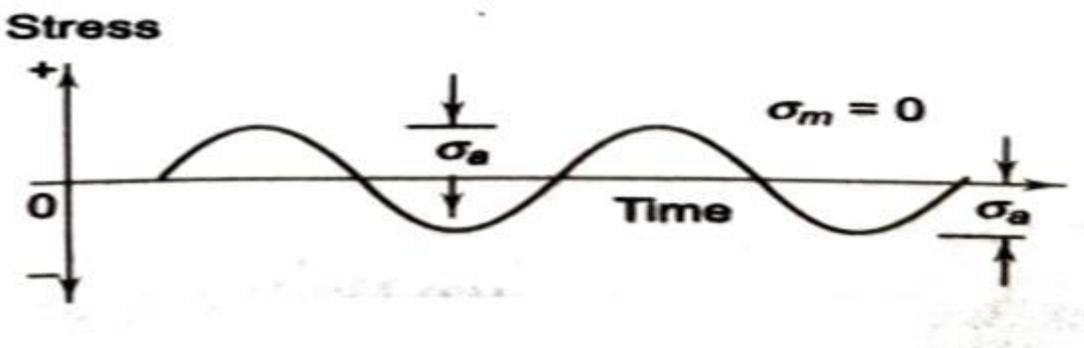
$$\sigma_{min} = 0$$

$$Mean\ stress:\ \sigma_m = \frac{\sigma_{max}}{2}$$

$$Reversed\ or\ alternative\ or\ variable\ stress \sigma_v = \frac{\sigma_{max}}{2}$$

Cyclic or Reversed stress

Completely reversal from tension to compression between two values

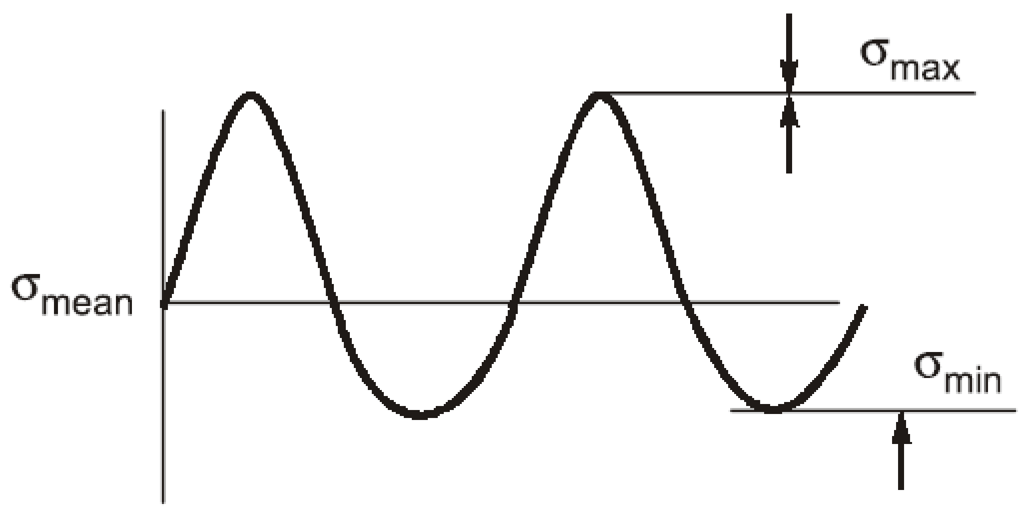


$$Mean\ stress:\ \sigma_m = 0$$

$$Reversed\ or\ alternative\ or\ variable\ stress\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Alternating Fatigue Stress

The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called alternating fatigue stresses.



• Means stress,	$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$
• Stress amplitude,	$\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$
• Stress range,	$\sigma_r = \sigma_{max} - \sigma_{min}$
• Stress ratio,	$R = \dfrac{\sigma_{max}}{\sigma_{min}}$
• Amplitude ratio,	$A = \dfrac{\sigma_a}{\sigma_m}$

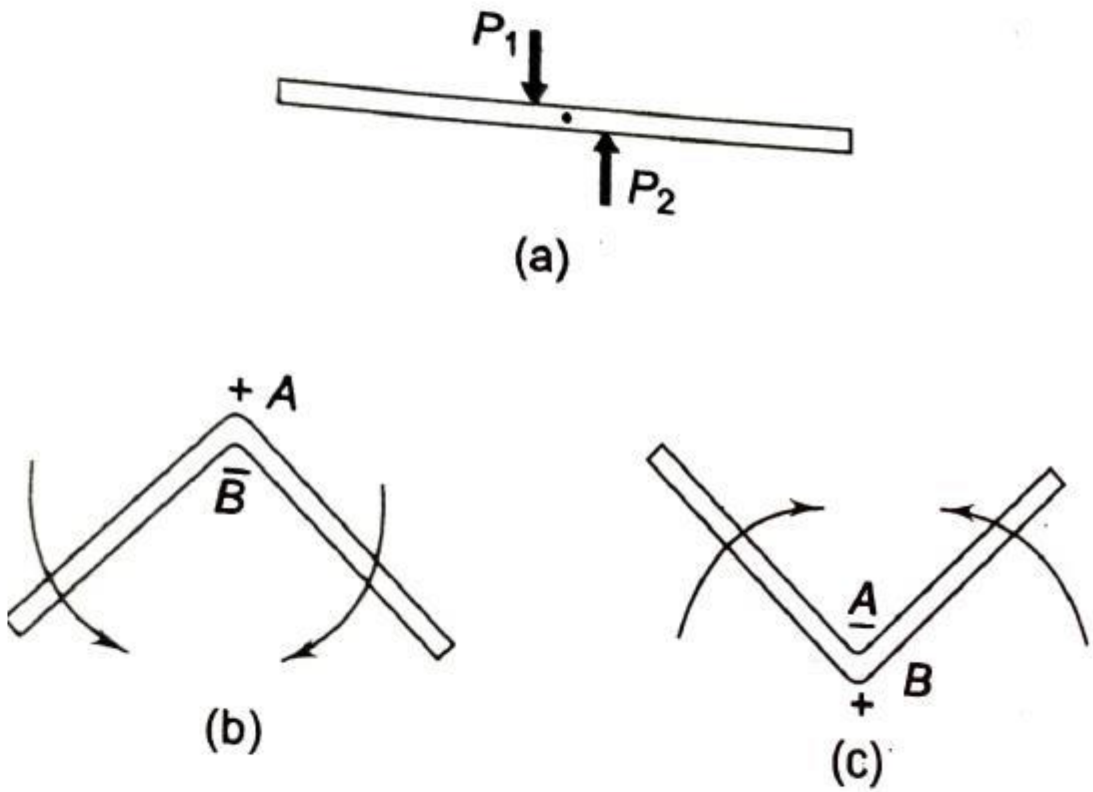
here σ_{max} = Maximum stress value during complete cycle

σ_{min} = Minimum stress value during complete cycle

FATIGUE

- When a material is subjected to repeated stresses (repeated loads/alternating loads), it fails at a stress far below the yield point stress. Such type of failure is referred to as fatigue.
- The failure is caused by means of a progressive crack formation which are usually fine and microscopic size
- Fatigue failure is always brittle and catastrophic in nature with no visible warning prior to failure .
- It is observed that about 80 % of failures of mechanical components are due to fatigue failure resulting from fluctuating stresses.
- The decreased resistance of the materials to cyclic stresses is the main characteristics of fatigue failure .
- Fatigue failure is defined as the time delayed fracture under cyclic loading
- Transmission shafts ,connecting rods ,gears ,vehicle Suspension springs, ball bearings are subjected to fatigue failure.

  - ✓ Automobiles in Mechanical Engineering
  - ✓ Bridges in Civil Engineering
  - ✓ Aircrafts in Aeronautical Enginnering
  - ✓ Ship hull in Marine Engineering
  - ✓ Pressure vessels in Chemical Engineering
  - ✓ Tractors involving Agricultural Engineering

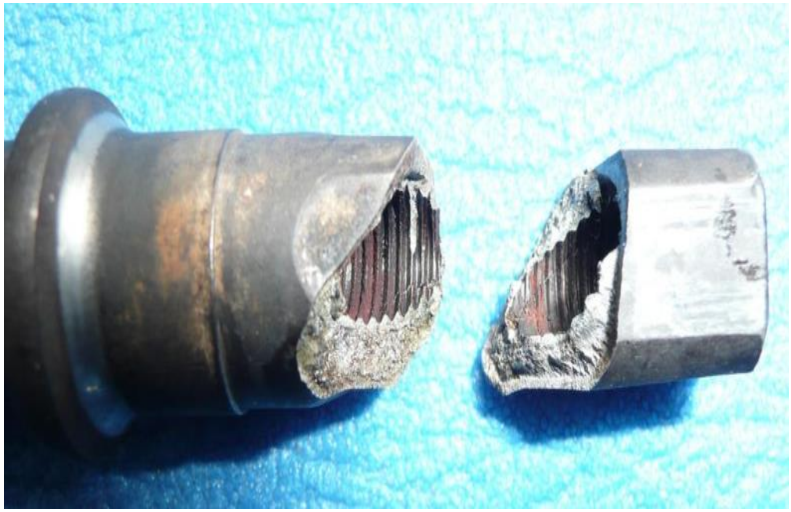


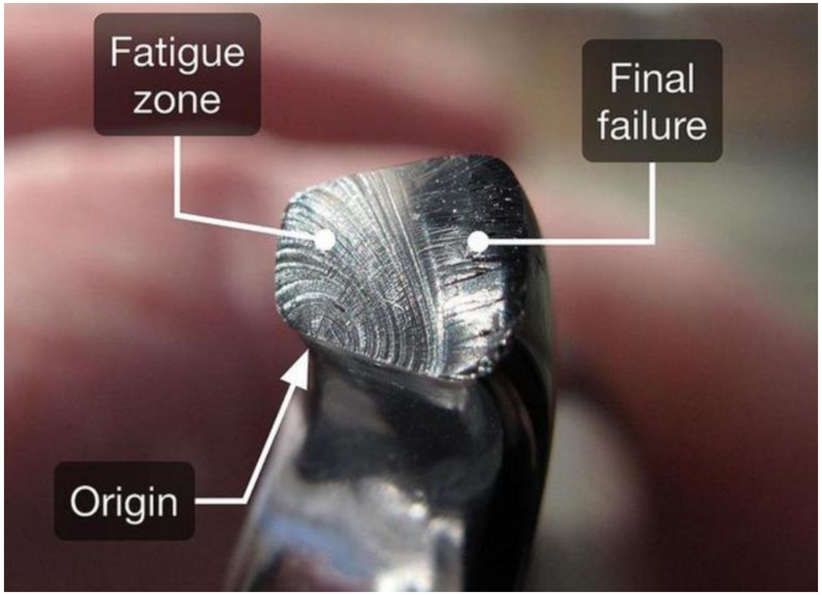
P1
P2
(a)
+ A
B
(b)
A
B
+
(c)

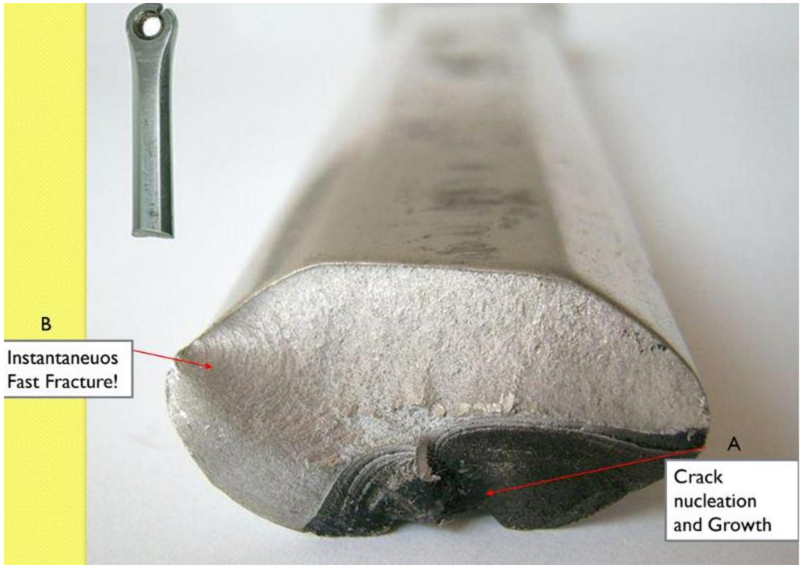
FACTORS TO BE CONSIDERED TO AVOID FATIGUE FAILURE

- Variation in size of the component should be as gradual as possible. Holes , notches and other stress raisers should be avoided.
- Proper stress deconcentrators such as fillets and notches should be provided wherever necessary.
- Components should be protected from corrosion.
- Provide smooth finish on the outer surface of the component ,thereby increasing fatigue life.
- Materials with high fatigue strength should be selected.
- Residual compressive stresses over the parts surface increases its fatigue strength


Break
End of keyway




Fatigue zone
Final failure
Origin


B
Instantaneuos Fast Fracture!
A
Crack nucleation and Growth

How to Improve Fatigue Strength

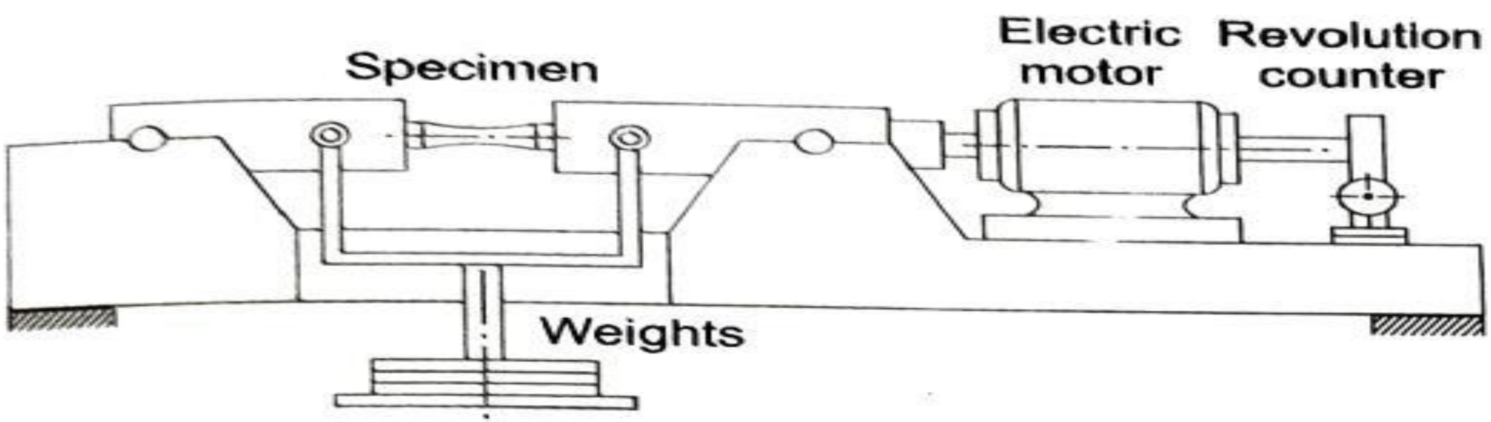
- By residual compressive stresses. Because fatigue failure is always a tensile failure. Therefore residual compressive stresses will counter it. This is done by the process called **shot peening**.
- By providing fillets, we can change its macrostructure thus we can basically increase its mechanical property that is fatigue strength.
- **Hammering:** The hammering process has been used to improve the fatigue resistance of the components.
- **Cold rolling:** Cold rolling gives excellent surfinish and increases material strength due to work hardening.
- **Burnishing:** In this metal is plastically deformed and convert the uneven surface of workpiece at normal temperature to smooth surface so as to change the surface structure, mechanical properties, shape and size. It improves the fatigue and wear strength of the workpiece.

Endurance Limit and Endurance Strength

- The fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure.

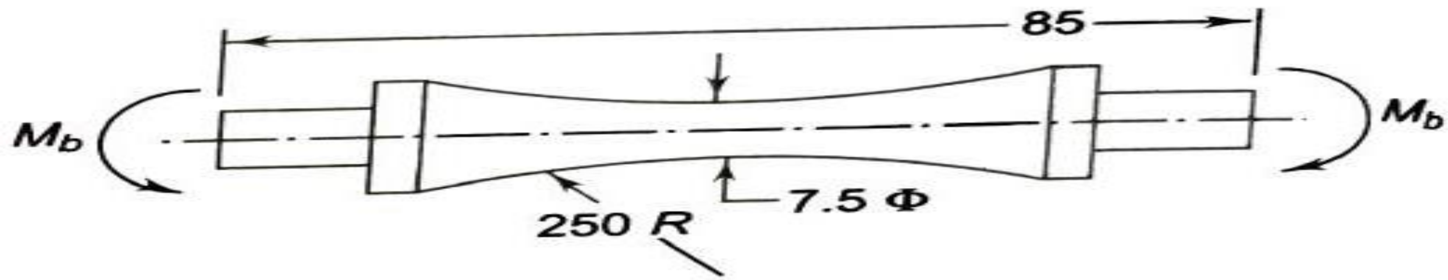
- It is the strength of a material to resist the propagation of cracks under stress reversals.

Endurance Limit Is the stress value below which an infinite number of cycles will not cause failure



Rotating beam fatigue testing machine

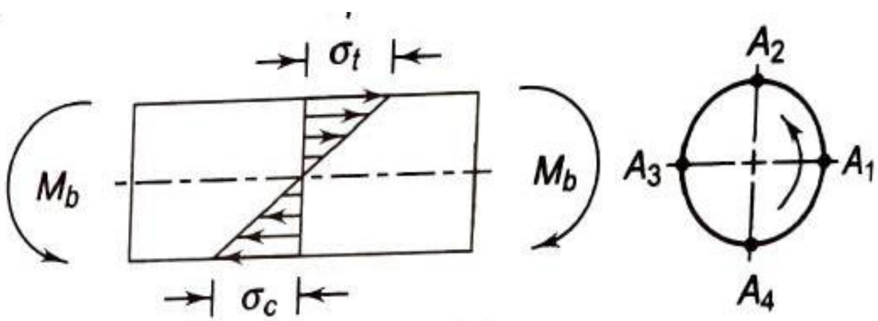
- In order to study the effect of fatigue of a material, a rotating mirror beam method is used called R.R. Moore rotating beam machine
- In this method, a standard specimen machined and polished as shown in Fig.
- The final polishing is done in axial direction in order to avoid circumferential scratches



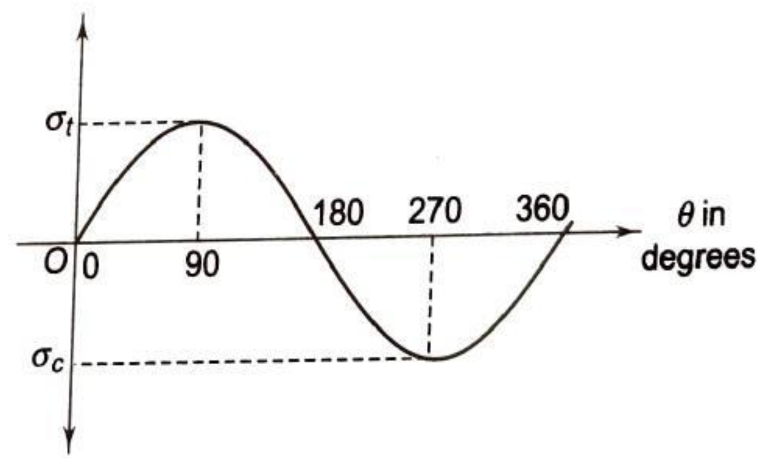
Specimen for fatigue test

Specimen is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibers varies from maximum compressive to maximum tensile while the bending stress at the lower fibers varies from maximum tensile to maximum compressive.

In other words, the specimen is subjected to a completely reversed stress cycle .The number of revolutions before the appearance of the first fatigue crack is recoded on a revolution counter. In each test. Two readings are taken viz. stress amplitude S and umber of stress cycles (N). These readings are used as two co-ordinates for plotting a point on S-N diagram. This point is called failure point. To determine the endurance limit of a material, a number of tests are to be carried out.

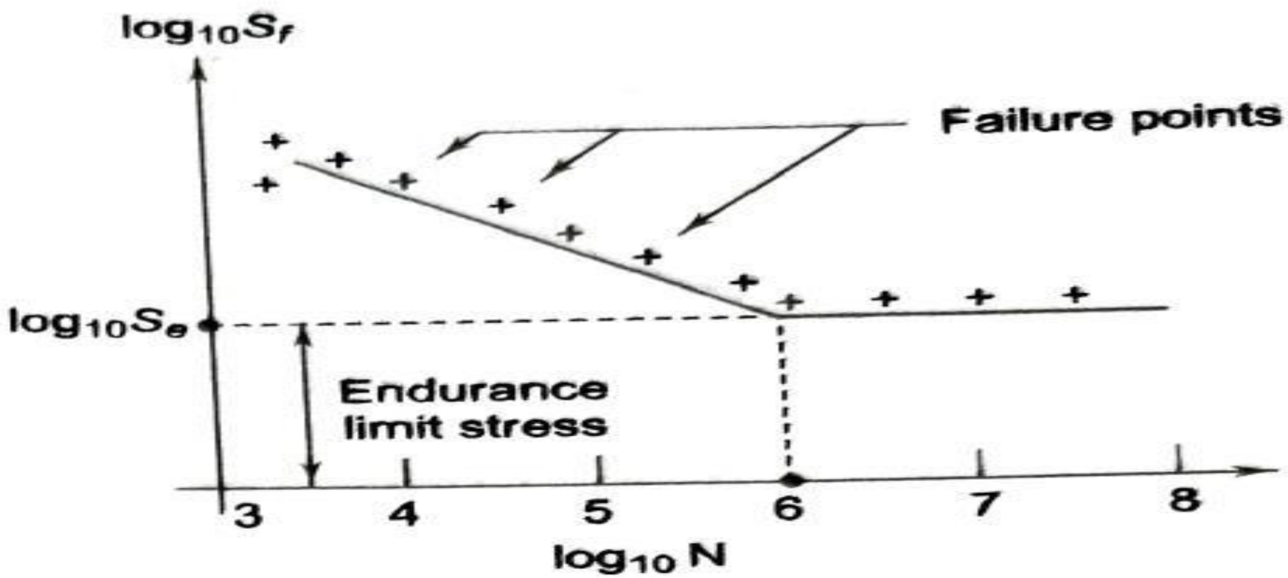


(a)



S-N curve

The results of these tests are plotted by means of a S-N curve. The S-N curve is the graphical representation of stress amplitude (S versus the number of stress cycles (N) before the fatigue failure on a log-log graph paper



The endurance limit, is not exactly a property of material like ultimate tensile strength. It is affected by factors such as the size of the component, shape of component, the surface finish, temperature, and the notch sensitivity of the material

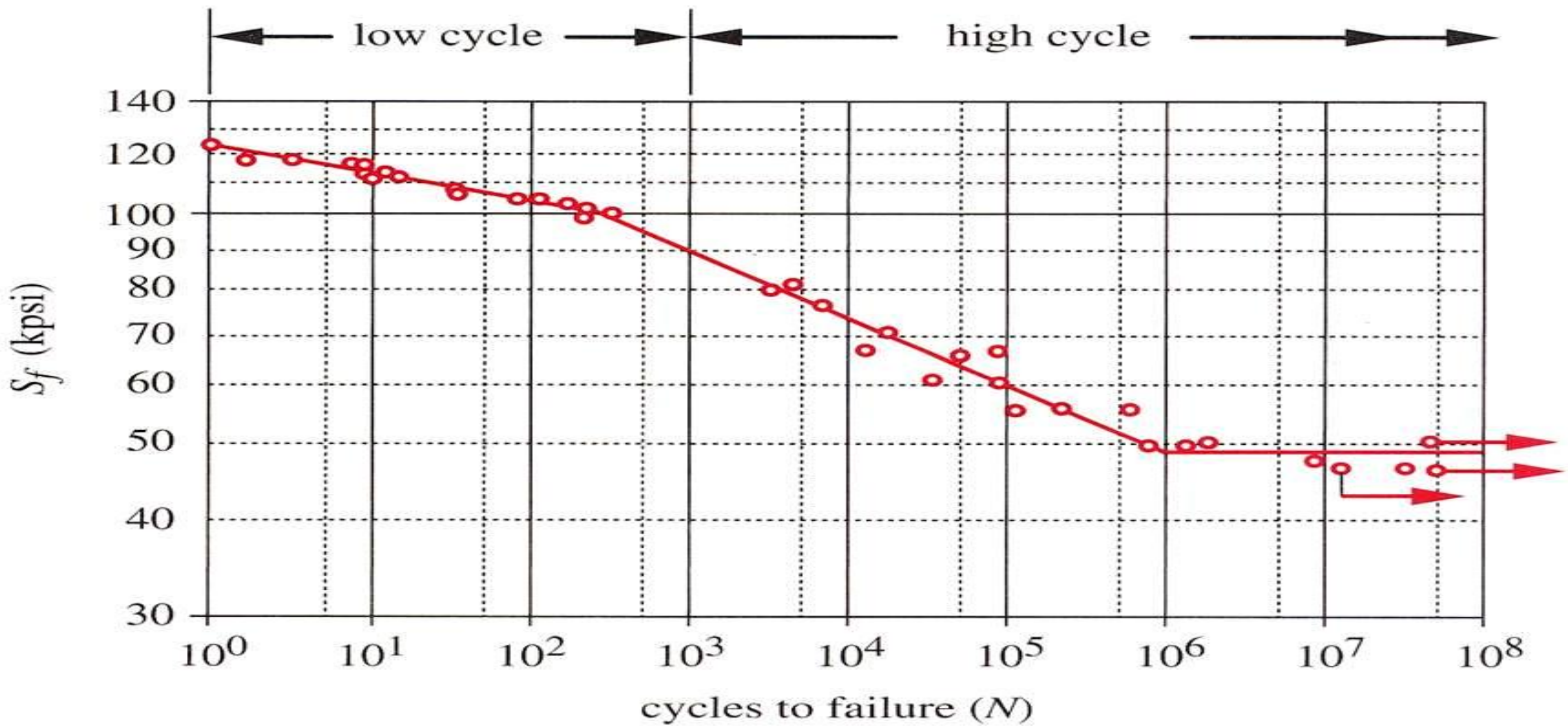
LOW-CYCLE AND HIGH-CYCLE FATIGUE

The S-N curve illustrated in Fig. is drawn from 10^3 cycles on log-log graph paper. The complete S-N curve from 100 cycle to 10^8 cycles is shown in Fig. There are two regions of this curve namely, low cycle fatigue and high-cycle fatigue.

(i)Any fatigue failure when the number of stress cycles are less than 1000, is called **low-cycle fatigue.**
(ii)Any fatigue failure when the number of stress cycles are more than 1000, is called **high-cycle fatigue.**

- Failure of studs on truck wheels, failure of setscrews for locating gears on shafts or failures of short-lived devices such as missiles are the examples of low cycle fatigue.

- The failure of machine components such as springs, ball bearings or gears that are subjected to fluctuating stresses, are the examples of high-cycle fatigue.



low cycle
high cycle
140
120
100
90
80
70
60
50
40
30
S_f (kpsi)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
cycles to failure (N)

- The low-cycle fatigue involves plastic yielding at localized areas of the components. There are some theories of low-cycle fatigue.

- In many applications the designers simply ignore the fatigue effect when the number of stress cycles is less than 1000.
- Such components are designed on the basis of ultimate tensile strength or yield strength with suitable factor of safety.

Components subjected to high-cycle fatigue are designed on the basis of endurance limit stress. S-N curve, Soderberg line, Gerber line or Goodman diagram are used in design

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as Fatigue stress concentration factor

$$K_f = \frac{Endurance\ limit\ without\ Stress\ concentartion}{Endurance\ limit\ with\ Stress\ concentartion}$$

This factor Kf is applicable to actual materials and depends upon the grain size of the material

Notch Sensitivity

Notch sensitivity is defined as the susceptibility of a material to succumb to the damaging effects of stress raising notches in fatigue loading.

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

Since σ_0 = Nominal stress as obtained by elementary equations

$\therefore$ Actual stress $= k_f \sigma_0$

Theoretical stress $= k_t \sigma_0$

Increase of actual stress over nominal stress $= (k_f \sigma_0 - \sigma_0)$

Increase of theoretical stress over nominal stress $= (k_t \sigma_0 - \sigma_0)$.

k_f - fatigue stress concentration factor

Notch sensitivity,

$$q = \frac{k_f \, \sigma_0 - \sigma_0}{k_t \, \sigma_0 - \sigma_0} = \frac{k_f - 1}{k_t - 1}$$

$$k_f = 1 + q(k_t - 1)$$

(i)When the material has no sensitivity to notches

q = 0 and $K_f = 1$

(ii)When the material is fully sensitive to notches,

q = 1 and $K_f = K_t$

(i) K_t depends on shape, size of discontinuity and type of load condition and its orientation.

(ii) K_t is independent of material behaviour.

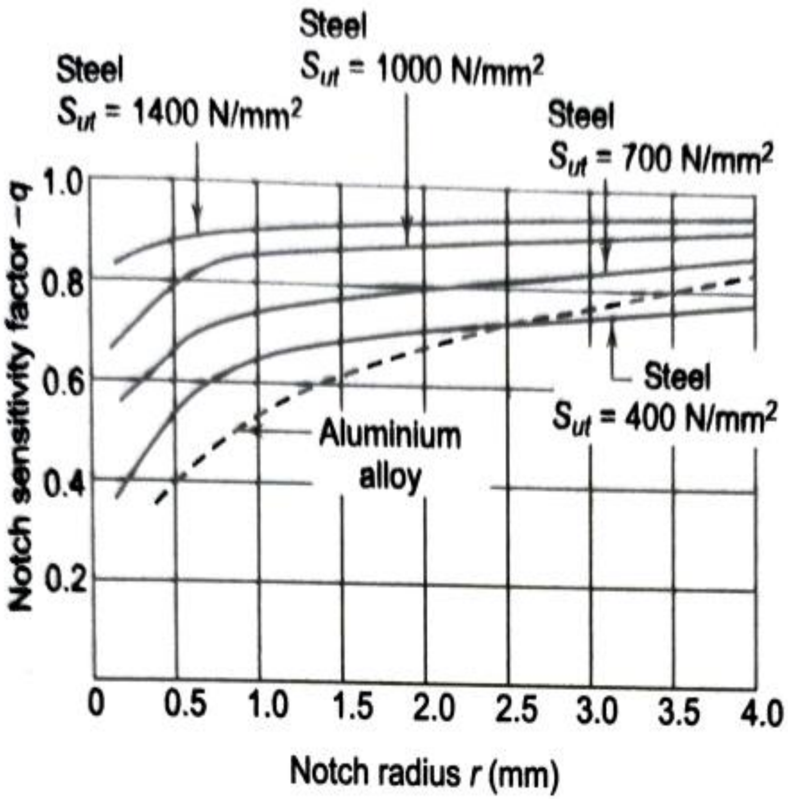
(iii) K_f depends on shape, size of orientation and type of loading condition and material of component.

(iv) Notch sensitive index (q) depends on material.

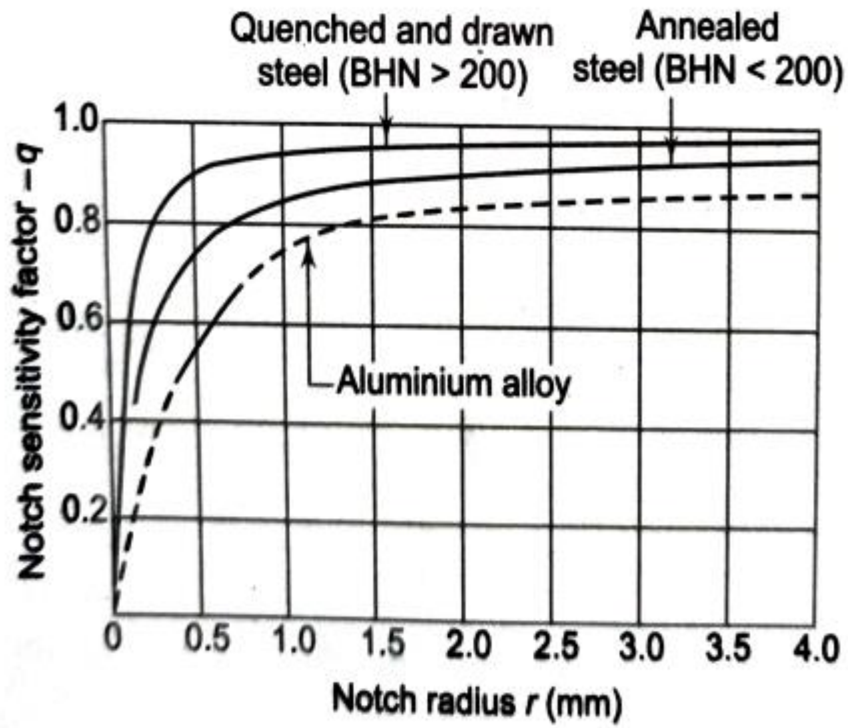
(v) If $q = 0 \Rightarrow K_f = 1 \Rightarrow$ material is insensitive to stress concentration or notch.

(vi) If $q = 1 \Rightarrow K_f = K_t \Rightarrow$ material is highly sensitive to notch.

(vii) For worst design, take $q = 1$.



Notch sensitivity charts (for reversed bending and reversed axial stresses)



Notch sensitivity charts (for reversed torsional shear stresses)

Correction Factors for Specimen's Endurance Limit

$$S_e = k_a k_b k_c k_d k_e S_e'$$

Where,

- S_e = endurance limit of component
- S_e' = endurance limit experimental(Test Specimen)
- k_a = surface finish factor
- k_b = size factor
- k_c = reliability
- k_d = modified factor to accounts for stress concentration
- k_e = operating T factor

Modifying Factor to Account for Stress Concentration

The endurance limit is reduced due to stress concentration. The stress concentration factor used for cyclic loading is less than the theoretical stress concentration factor due to the notch sensitivity of the material. To apply the effect of stress concentration, the designer can either reduce the endurance limit by (K_d) or increase the stress amplitude by (K_f). We will use the first approach. The modifying factor $\boldsymbol{Kd}$ to account for the effect of stress concentration is defined as

K_d = 1/ K_f

The endurance limit ($\boldsymbol{Sse}$) of a component subjected to fluctuating torsional shear stresses bending ($\boldsymbol{Se}$) using theories of failures.

According to the maximum shear stress theory,

Sse = 0.5 *Se*

According to distortion energy theory,

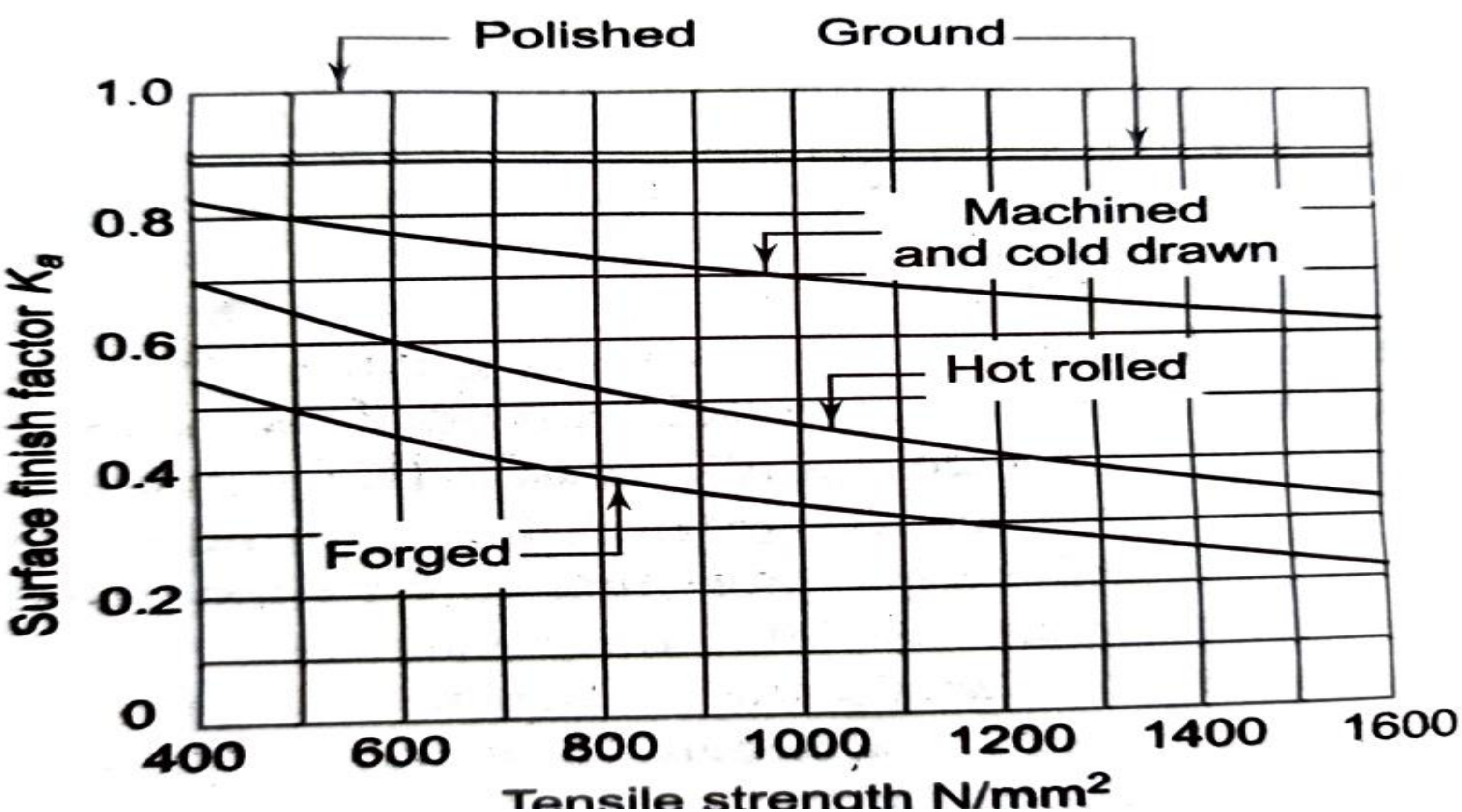
Sse = 0.577 *Se*

For axial loading,

(*Se*)*a* = 0.8 *Se*

surface finish factor :

The rotating beam test specimen has a polished surface. Most components do not have a polished surface. Scratches and imperfections on the surface act like a stress raisers and reduce the fatigue life of a part. Use either the graph or the equation with the table shown below.



- Size factor, k_b

Table 5.2 *Values of size factor*

Diameter (d) (mm)	K_b
$d \leq 7.5$	1.00
$7.5 < d \leq 50$	0.85
$d > 50$	0.75

Shigley and Mischke have suggested an exponential equation for the size factor. For bending and torsion, the equation is in the following form:

For $2.79 \text{ mm} \leq d < 51 \text{ mm}$

$$K_b = 1.24\, d^{-0.107}$$

For $51 \text{ mm} < d \leq 254 \text{ mm}$

$$K_b = 0.859 - 0.000\,873\, d$$

For axial loading, $K_b = 1$

Reliability factor

Table 5.3 *Reliability factor*

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

REVERSED STRESSES-DESIGN FOR FINITE AND INFINITE LIFE

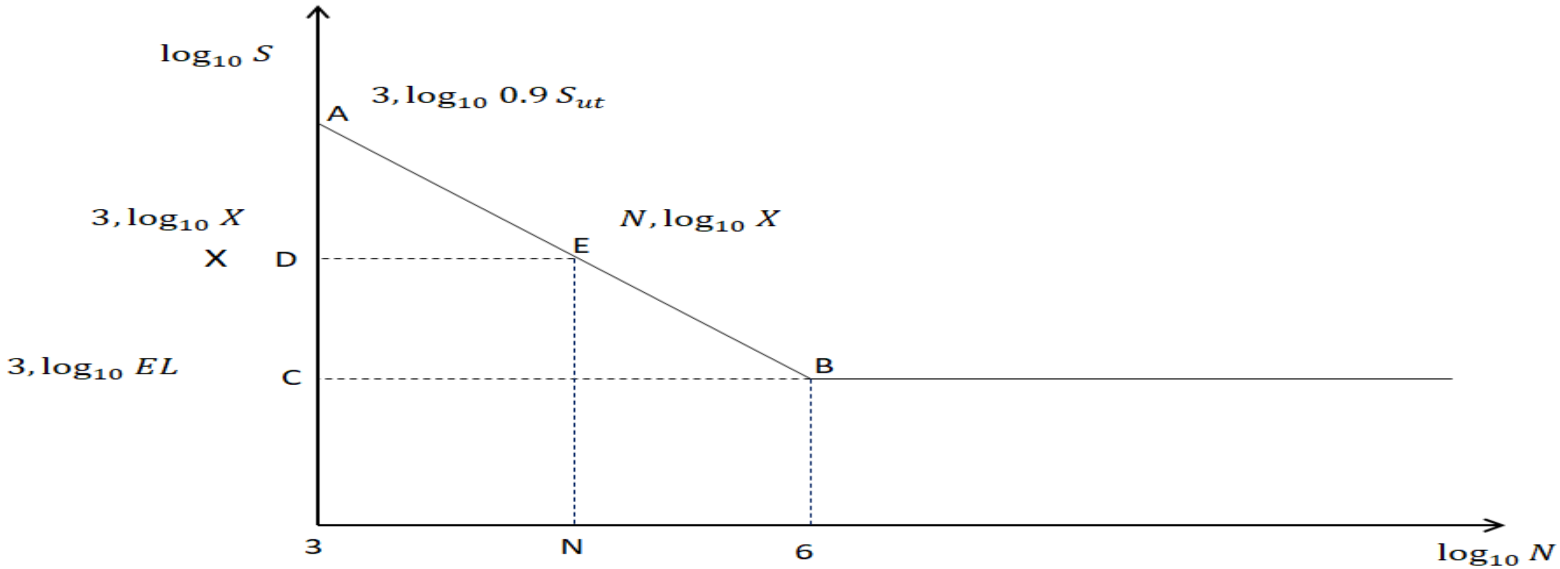
Case I: When the component is to be designed for **infinite life**, the **endurance limit** becomes the **criterion of failure**. The amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles. Such components are designed with the help of the following equations

$$\sigma_a = \frac{S_e}{(fs)}$$

$$\tau_a = \frac{S_{se}}{(fs)}$$

where (σ_a) and (τ_a) are stress amplitudes in the component and S_e and S_{se} are corrected endurance limits in reversed bending and torsion respectively.

Case II : When the component is to be designed for finite life, the S-N curve used

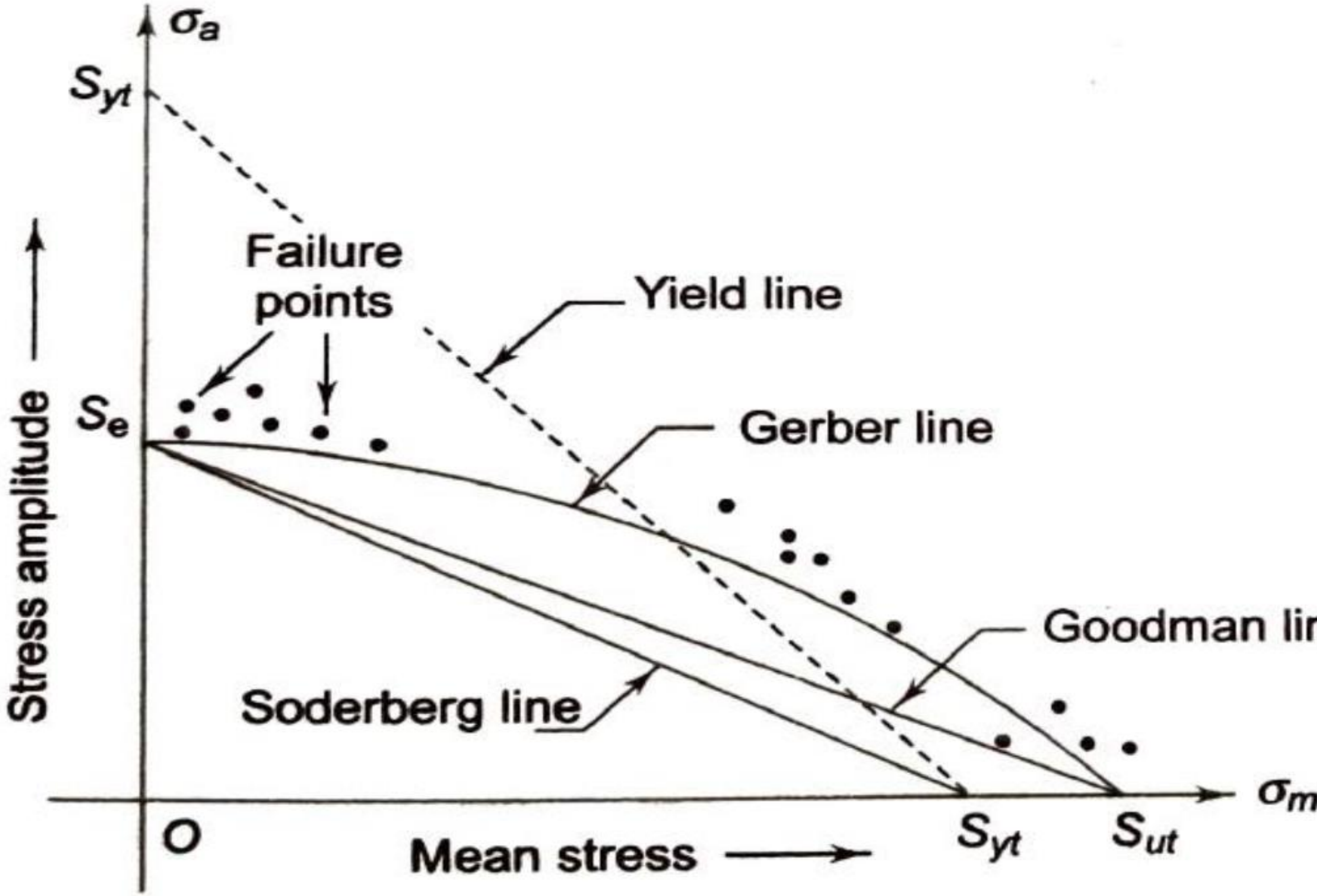


$$\frac{\log_{10} 0.9\, S_{ut} - \log_{10} EL}{\log_{10} X - \log_{10} EL} = \frac{6-3}{6-n}$$

Gerber ,SODERBERG AND GOODMAN LINES

When a component is subjected to fluctuating stresses there is mean stress as well as stress it has been observed that mean stress a component has effect on fatigue failure when it is present in combination with alternating component.

When stress amplitude is zero, the load is purely static and the criterion of failure is S_{yt} or S_{ut} These limits are plotted on the abscissa. When the mean stress is zero, the stress is completely reversing and the criterion of failure is endurance limit Se that is plotted on the ordinate.



σ_a
S_{yt}
Failure
points
Yield line
S_e
Gerber line
Goodman line
Stress amplitude
Soderberg line
σ_m
O
Mean stress
S_{yt}
S_{ut}

Soderberg line

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1 = \frac{1}{f.s.}$$

Good man line

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1 = \frac{1}{f.s.}$$

Gerber line

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1 \quad \text{or} \quad \frac{\sigma_a}{S_e} + f.s.\left(\frac{\sigma_m}{S_{ut}}\right)^2 = \frac{1}{f.s.}$$

ENDURANCE LIMIT-APPROXIMATE ESTIMATION

$s_e{}'$ = Endurance limit stress of a rotating beam specimen subjected to reversed bending stress (N/mm^2)

S_e = Endurance limit stress of a particular mechanical component subjected to reversed bending stress (N/mm^2)

There is an approximate relationship between the endurance limit and the ultimate tensile strength of the material.

steels, $S_e{}' = 0.5\ S_{ut}$

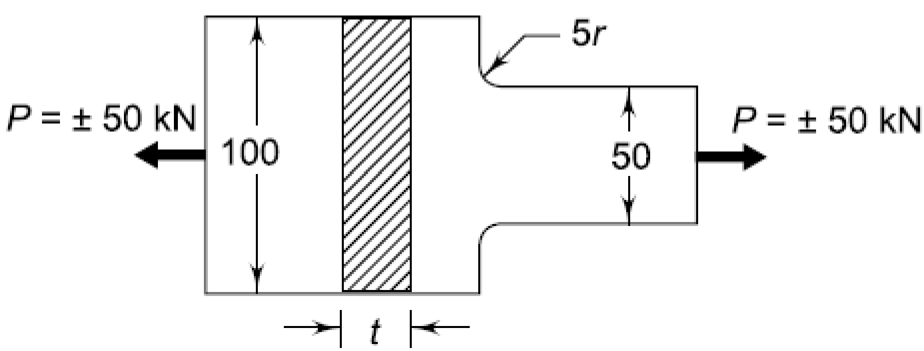
cast iron and cast steels, $S_e{}' = 0.4\ S_{ut}$

wrought aluminium alloys, $S_e{}' = 0.4\ S_{ut}$

cast aluminium alloys, $S_e{}' = 0.3\ S_{ut}$

These relationships are based on 50% reliability.

A component machined from a plate made of steel 45C8 (Sut = 630 N/mm2) is shown in Fig. 5.29. It is subjected to a completely reversed axial force of 50 kN. The expected reliability is 90% and the factor of safety is 2. The size factor is 0.85. Determine the plate thickness t for infinite life, if the notch sensitivity factor is 0.8.



Solution

Given $P = \pm 50$ kN $\quad S_{ut} = 630$ N/mm^2 $\quad (fs) = 2$

$R = 90\%$ $\quad q = 0.8$ $\quad K_b = 0.85$

Step I *Endurance limit stress for plate*

$S'_e = 0.5 S_{ut} = 0.5(630) = 315$ N/mm^2

From Fig. 5.24 (machined surface and $S_{ut} = 630$ N/mm^2),

$K_a = 0.76$

$K_b = 0.85$

For 90% reliability, $\quad K_c = 0.897$

$$\left(\frac{D}{d}\right) = \frac{100}{50} = 2$$

and

$$\left(\frac{r}{d}\right) = \frac{5}{50} = 0.1$$

From Fig. 5.3, $\quad K_t = 2.27$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.8\,(2.27 - 1) = 2.016$

$$K_d = \frac{1}{K_f} = \frac{1}{2.016} = 0.496$$

$S_e = K_a\, K_b\, K_c\, K_d\, S'_e$

$= 0.76\,(0.85)(0.897)(0.496)(315)$

$= 90.54$ N/mm^2

Step II *Permissible stress amplitude*

From Eq. (5.30),

$$(S_e)_a = 0.8S_e = 0.8(90.54) = 72.43 \text{ N/mm}^2$$

$$\sigma_a = \frac{(S_e)_a}{(fs)} = \frac{72.43}{2} = 36.22 \text{ N/mm}^2$$

Step III Plate thickness

Since $$\sigma_a = \frac{P}{(50t)}$$

$$\therefore \qquad t = \frac{P}{50\sigma_a} = \frac{(50 \times 10^3)}{50(36.22)} = 27.61 \text{ mm}$$