EULER GRAPHS:- If some closed walk in a graph contains all the edges of the graph, then the walk is called an Euler line and the graph is called Euler graph. Euler graphs do not have any isolated vertices and are therefore connected.

→ Eulerian trail:- An Eulerian trail is a trail that visits every edge of the graph once & only once. It can end on a vertex different from the one on which it began. A graph of this kind is said to be traversable.

→ Eulerian Circuit:- An Eulerian circuit is an Eulerian trail that is a circuit. That is, it begins & ends on the same vertex.

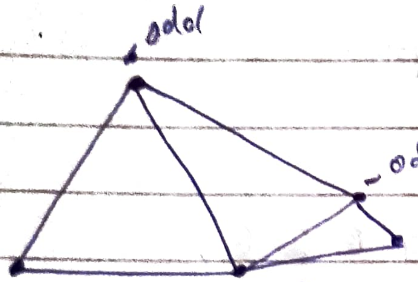
→ Eulerian Graph:- A graph is called Eulerian when it contains an Eulerian circuit.

→ A vertex is odd if its degree is odd & even if its degree is even.

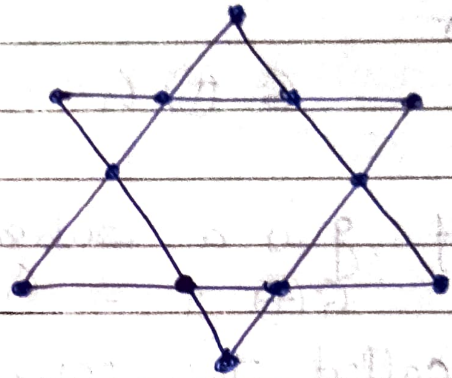
Theorem:- A connected graph G is an Euler graph if and only if all vertices of G are of even degree.

Theorem:- In a connected graph G with exactly 2k odd vertices, there exist k edge-disjoint subgraphs s.t they together contain all edges of G & that each is a unicursal graph.

→ All vertices have even degrees, so there is an Eulerian circuit, which is also Eulerian trail.
→ two vertices of odd degrees, so there is an Eulerian trail but no Eulerian circuit.
→ Each vertex has degree $n-1$ which is odd, Hence there is neither an Eulerian circuit nor an Eulerian trail.



unicursal graph



Euler graph.

Operations on Graphs :-

→ Union :- Let $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ is another graph G_3 ($G_3 = G_1 \cup G_2$) whose vertex $V_3 = (V_1 \cup V_2)$ & edge $E_3 = (E_1 \cup E_2)$.

→ Intersection :- $G_1 \cap G_2 = G_4$, G_4 consisting only those vertices & edges that are in both G_1 & G_2

→ Ring Sum :- $G_1 \oplus G_2$ is a graph consisting of the vertex set $V_1 \cup V_2$ & of edges that are either in G_1 or G_2 but not both.

$G_1 \cup G_2 = G_2 \cup G_1$,
$G_1 \cap G_2 = G_2 \cap G_2$
$G_1 \oplus G_2 = G_2 \oplus G_1$

→ If G_1 & G_2 are edge disjoint, then $G_1 \cap G_2$ is a null graph & $G_1 \oplus G_2 = G_1 \cup G_2$.

→ If G_1 & G_2 are vertex disjoint then $G_1 \cap G_2$ is empty.

$$G \cup G = G \cap G = G$$

&

$$G \oplus G = \text{a null graph.}$$

→ If g is a subgraph of G.

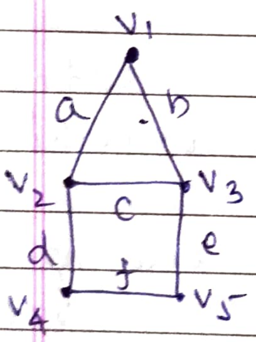
$$G \oplus g = G - g, \quad g \subseteq G.$$

called the complement of g in G.

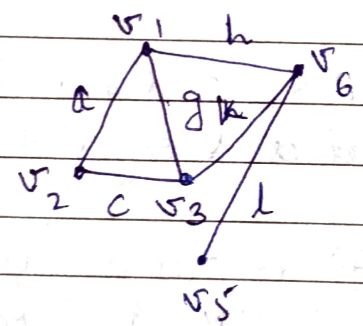
→ Decomposition :- A graph G is said to have been decomposed into two subgraphs g_1 & g_2 if

$$g_1 \cup g_2 = G.$$

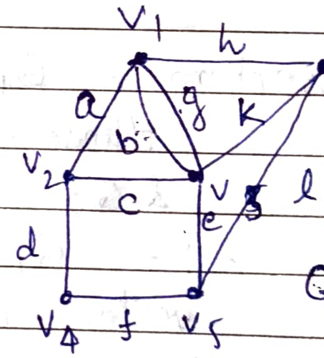
$$g_1 \cap g_2 = \text{a null graph.}$$



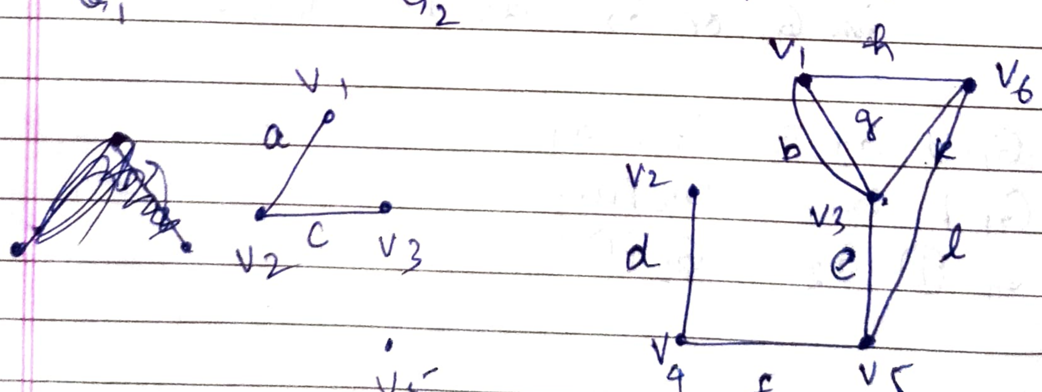
G_1



G_2



$G_1 \cup G_2$



→ A graph containing m edges $\{e_1, e_2, \ldots e_m\}$ can be ~~decomposition~~ decomposed in $2^{m-1} - 1$ different ways into pairs of subgraphs g_1 & g_2.

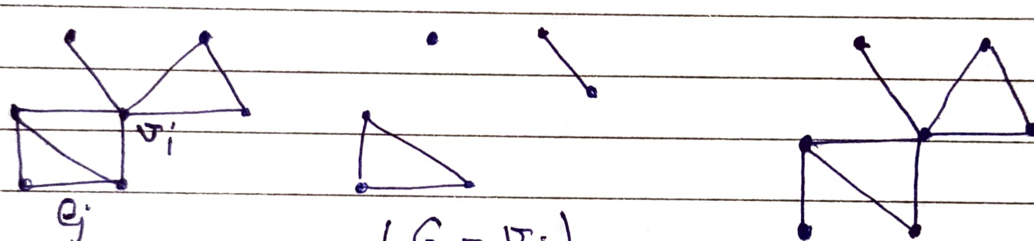
→ **Deletion :-** If vertex v_i is a vertex in graph in graph G. & e_j is an edge in G.

then

$G - v_i \Rightarrow$ a subgraph of G obtained by deleting v_i from G. it implies the deletion of all edges incident on that vertex.

$G - e_j \Rightarrow$ a subgraph of G obtained by deleting e_j from G. it does imply deletion of its end vertex.

therefor $G - e_j = G \oplus e_j$



G $(G - v_i)$ $G - e_j$

→ **Fusion :-** A pair of vertices a, b in a graph are said to be fused (merged or identified) if the two vertices are replaced by a single new vertex such that every edge that ~~to~~ was incident on either a or b or on both is incident on the new vertex. Thus ~~fust~~ fusion of two vertex does not alter the no. of edges, but it reduces the no. of vertices by one.