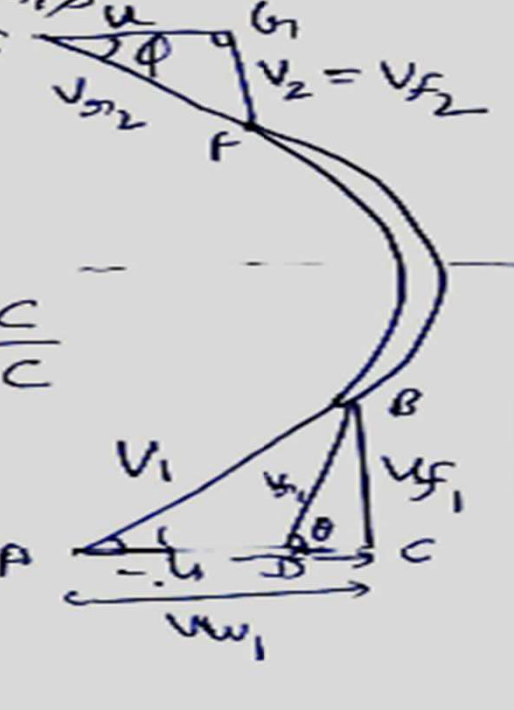
A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.

$V_1 = 40 m/s$

$u_1 = u_2 = u = 20 m/s$

$\alpha = 30°$



$\beta = 90°$

$V_{w_2} = 0$

$V_{f_1} = V_1 \sin 30$

$= 20$

Δ BDC

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u} = \frac{BC}{DC}$$

Δ A B C

$V_{w_1} = V_1 \cos\alpha$

$= 40 \cos 30$

$= 34.64 m/s$

$DC = AC - AD = 34.64 - 20 = 14.6 m/s$

$\theta = 53.87°$ Vane angle

$$\sin\theta = \frac{V_{f_1}}{V_{w_1} = u V_{r_1}}$$

$$V_{r_1} = \frac{20}{\sin 60^\circ}$$

$$= 24.75 \text{ m/s} = V_{r_2}$$

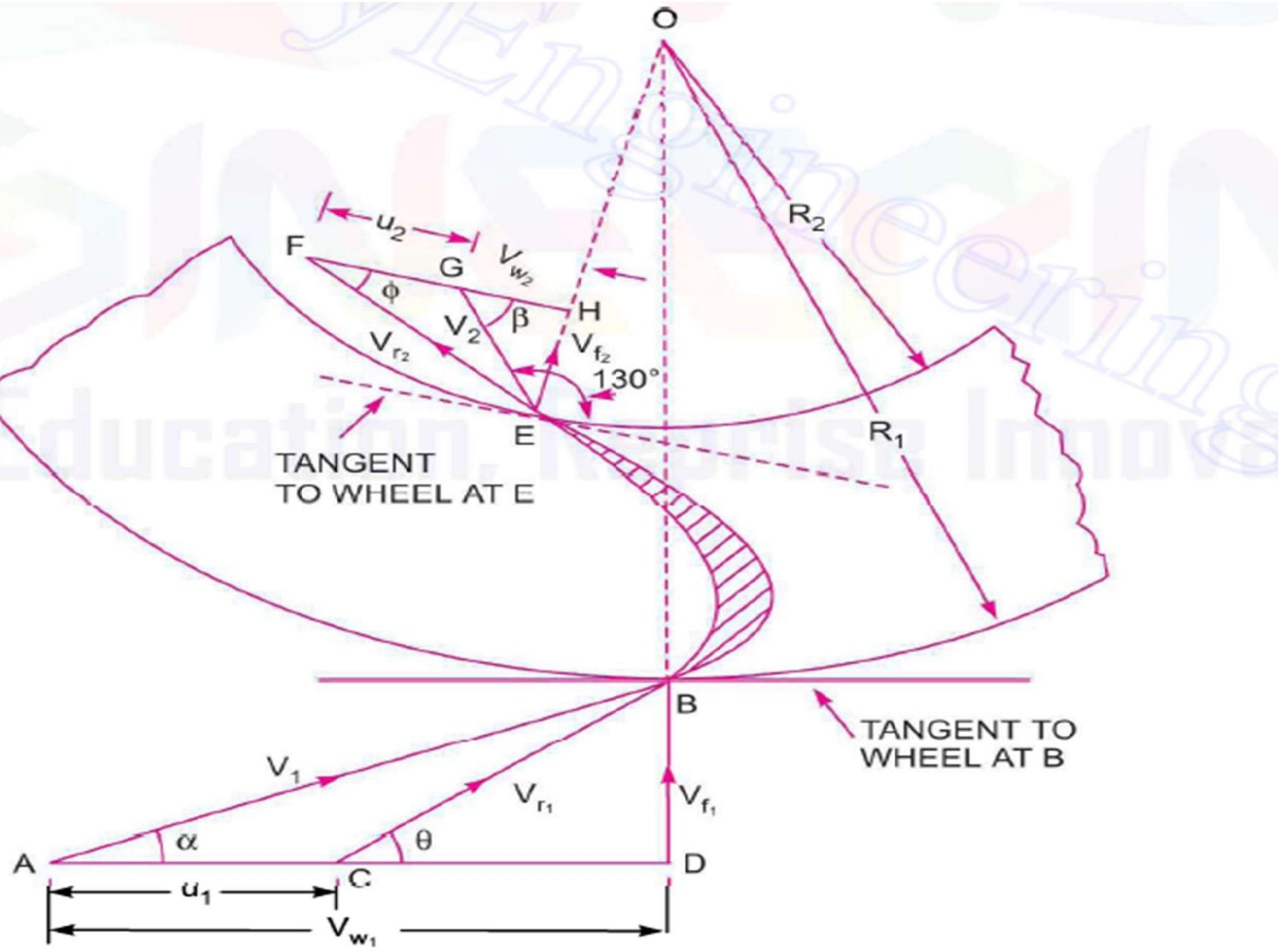
$$\cos\phi = \frac{u}{V_{f_2}} = \frac{20}{24.75}$$

$$\phi = 36.09^\circ$$

A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 r.p.m. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine :

(i) Vane angles at inlet and outlet, *(ii) Work done per unit weight of water, and*

(iii) Efficiency of the wheel.


O
R2
R1
u2
F
G
Vw2
ϕ
β
H
V2
Vr2
Vf2
130°
E
TANGENT
TO WHEEL AT E
B
TANGENT TO
WHEEL AT B
V1
Vr1
Vf1
α
θ
A
C
D
u1
Vw1

rpm = N = 200 rpm

$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.1416 \times 200}{60} \Rightarrow 20.94$ rad/s $\alpha = 20°$

$V_1 = 30$

$V_2 = 5$ $\beta = 180 - 130 = 50°$

$u_1 = r_1 \omega$

$= .5 \times 20.94 = 10.47$ m/s

$u_2 = 5.235$ m/s

ΔABC

$V_{w_1} = V \cos\alpha = 30 \cos 20$

$= 28.19$ m/s

$V_{f_1} = V \sin\alpha = 30 \sin 20 = 10.26$ m/s

$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{10.26}{28.19 - 10.47}$

$\theta = 30.07°$

$$V_{f_2} = V \sin\beta$$

$$= 5 \sin 50 = 3.83 \text{ m/s}$$

$$V_{w_2} = V \cos\beta = 3.2 \text{ m/s}$$

$$\tan\phi = \frac{V_{f_2}}{V_{w_2} + u_2}$$

$$= \frac{3.83}{3.2 + 5.235}$$

$$\phi = 24.13^\circ$$

Work done / sec / unit

$$= \frac{\rho A V_1 [V_{w_1} u_1 + V_{w_2} u_2]}{\rho A V_1 g}$$

$$= \frac{28.19 \times 10.47 + 3.12 \times 5.235}{9.81}$$

$$= 31.79 \quad \frac{Nm}{N}$$

$$\eta = \frac{2}{V_1^2} [V_{w_1} u_1 + V_{w_2} u_2] = \frac{2}{30^2} [311.90]$$

$$\eta = 69.93\%$$