Perceptron learning Convergence Theorem:-

Let $x(n)$ be the input vectors & $W(n)$ be the weight vectors, b is bais, y be the actual response d be the desired output & η be the learning rate parameter $0 < \eta \leq 1$.

① ~~Initili~~ Initialization. Set $W(0) = 0$. Then perform the following computations for time step $n = 1, 2, \ldots$

② Activation: At time step n, activate the perceptron by applying continuous valued input vector $x(n)$ & desired response $d(n)$.

③ Computation of Actual response of perceptron

$$y(n) = \mathrm{sgn}[W^T(n)\, x(n)]$$

signum function.

④ ~~Adaption~~ Adaptation of Weight Vector:
update the weight vector of perceptron:

$$W(n+1) = W_{ij}(n) + \eta[d(n) - y(n)]\, x(n)$$
$$b(n+1) = b(n) + \eta[d(n) - y(n)]$$

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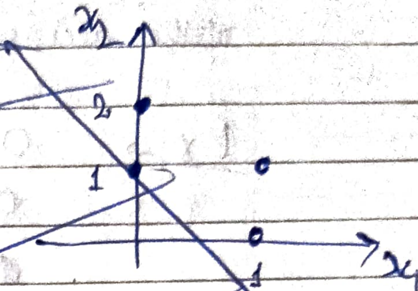
Where, $d(n) = \begin{cases} +1, & \text{if } X(n) \text{ belongs to class I} \\ -1, & \text{if } X(n) \text{ belongs to class II} \end{cases}$

(5) Continuation: Increment time step n by one & go back to (3).

Example:—

	X_1	X_2	Class d	
x_1	0	2	C_1	1
x_2	0	1	C_1	1
x_3	1	0	C_2	-1
x_4	1	1	C_2	-1

$\eta = 1$



(1) initial weight vector $W(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Iteration 1: $W^T(0)\, x(1) = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = 0$

Ex → $\quad w_{11} = 1, \quad w_{12} = 1, \quad b = -1$

$$a = w_i^T x_i + b$$

$$1 \cdot x_1 + 1 \cdot x_2 + b = 0$$

$$x_1 + x_2 - 1 = 0$$

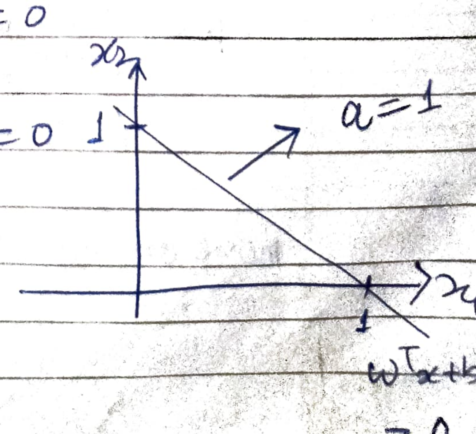
To draw the line, we can find the points, where it intersects the x_1 & x_2

if $x_1 = 0$, $\Rightarrow$ $0 + x_2 - 1 = 0$

$x_2 = 1$

if $x_2 = 0$ $\Rightarrow$ $x_1 + 0 - 1 = 0$

$x_1 = 1$



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Be Positive......

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$$x = \begin{bmatrix}2\\0\end{bmatrix}, \quad a = \text{hardlim}(w^T x + b)$$

$$= \text{hardlim}\left(\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix} - 1\right) = 1$$

Output will be 1, for the region above and to the right of the decision boundary.

Ex →

0	0	0
0	1	0
1	0	0
1	1	1

(1) design the decision boundary.

(2) choose a weight vector that is orthogonal to the decision boundary.