[5] PLANAR AND DUAL GRAPHS

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An abstract graph G1 can be defined as

$$G1 = (V, E, \psi)$$

where the set V consists of the five objects named a, b, c, d, & e that is

$$V = \{a, b, c, d, e\}$$

& the set E consists of seven object (none of which is in set V) named 1, 2, 3, 4, 5, 6, 7, that is

$$E = \{1, 2, 3, 4, 5, 6, 7\}$$

and the relationship b/w two sets is defined by the mapping ψ, which consists of

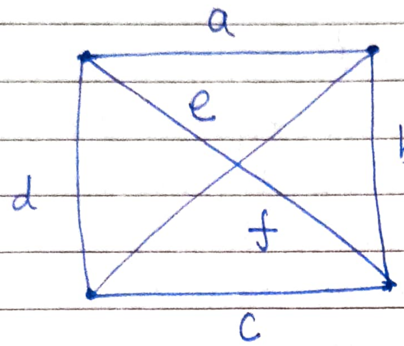
$$\psi = \begin{array}{l} 1 \longrightarrow (a, c) \\ 2 \longrightarrow (c, d) \\ 3 \longrightarrow (a, d) \\ 4 \longrightarrow (a, b) \\ 5 \longrightarrow (b, d) \\ 6 \longrightarrow (d, e) \\ 7 \longrightarrow (b, e) \end{array}$$

PLANAR GRAPHS :- A graph G is said to be planar if there exists some geometric representation of G which can be drawn on a plane such that no two of its edges intersect. (Meeting of edges at vertex is not considered an intersection).

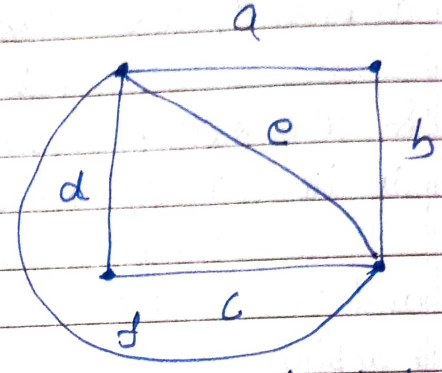
→ A graph that cannot be drawn on a plane without a crossover b/w its edges is called nonplanar.

→ A drawing of geometric representation of a graph on any surface such that no edges intersect is called embedding.

→ A geometric graph G is ~~pb~~ planar if there exists a graph isomorphic to G that is embedded in a plane. Otherwise G is planar.



non planer



planar (No intersection)

KURATOWSKI'S TWO GRAPHS:-

Theorem:- The complete graph of five vertices is nonplanar.

Proof:- Let the five vertices in the complete graph be named v_1, v_2, v_3, v_4, v_5.

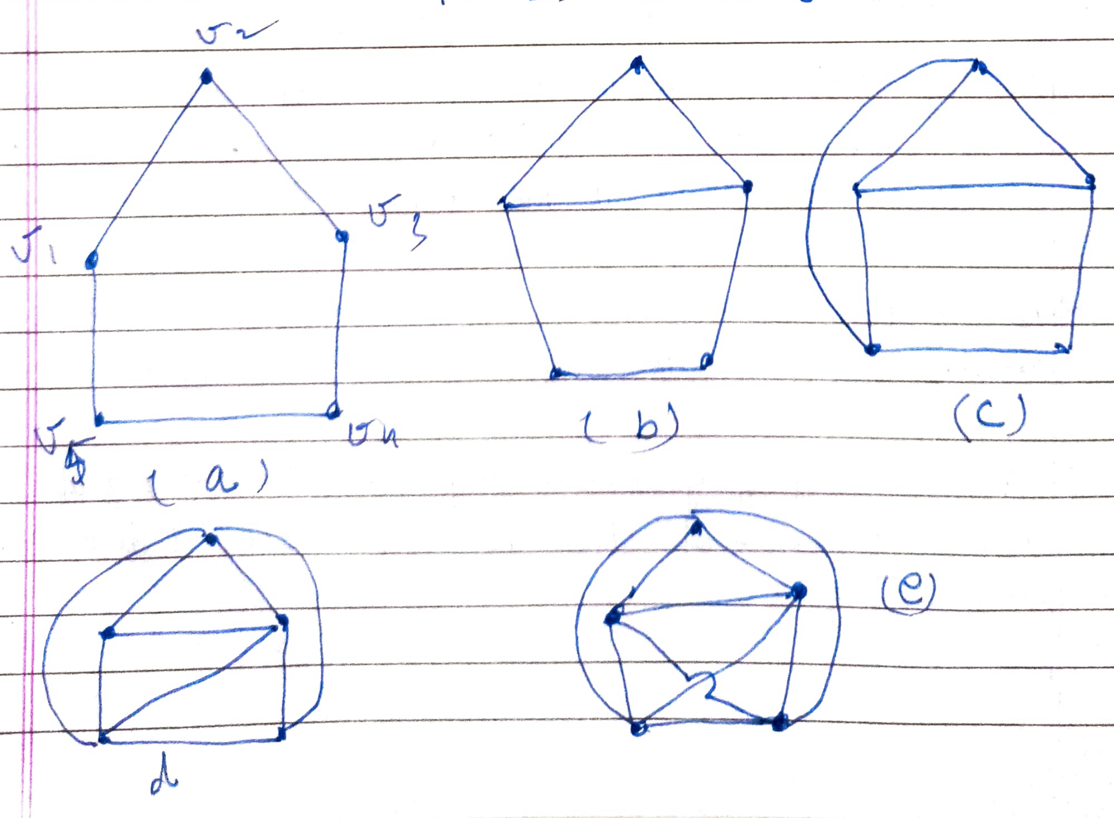
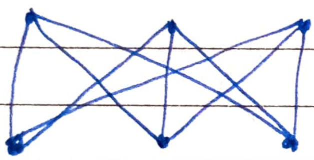


fig (a) is pentagon. This divide the plane of a paper into two regions, one inside & the other outside.

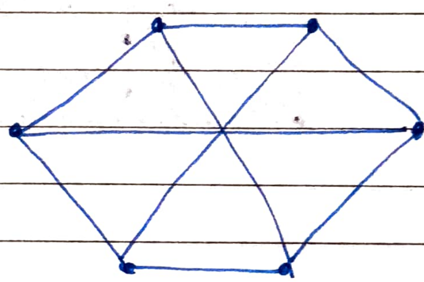
Since vertex v_1 is to be connected to v_3, suppose the line is inside the pentagon.

from last fig (e), draw an edge b/w v_1 & v_4. This edge cannot be placed inside or outside the pentagon without a crossover. Thus the graph cannot be embedded in a plane. #

The second graph of KURATOWSKI is a regular connected graph with six vertices & nine edges,



(a)



(b)

Theorem:- Kuratowski's second graph is also non-planar.

~~como~~ Common properties of two graphs of Koratowski,

1. Both are regular graphs.
2. Both are nonplanar
3. Removal of one edge or a vertex makes each ~~of~~ a planar graph.
4. Kuratowski's first graph is the nonplanar graph with the smallest no. of vertices, and Kuratowski's second graph is the nonplanar graph with the smallest no. of edges.

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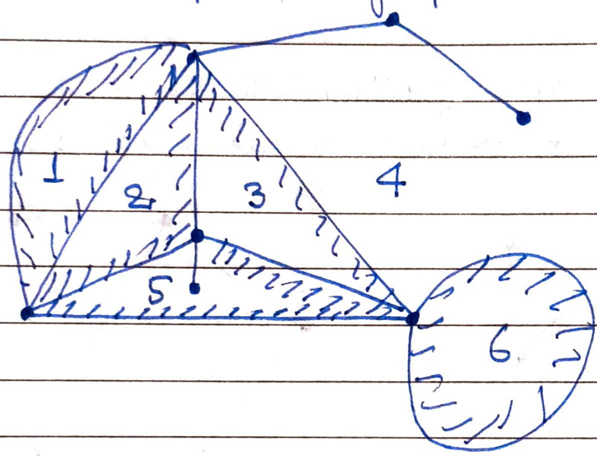
Kuratowski's first graph is denoted by K_5.
& second graph by $K_{3,3}$ ~~KR~~,

REGION :- A plane representation of a graph divides the plane into regions (also called windows, faces, or meshes). A region is characterized by the set of edges (or the set of vertices) forming its ~~bouand~~ boundry.

→ Region is not defined in a ~~f~~ nonplanar graph or even in a plannar graph not embedded in a plane.

→ ~~Ex~~



→ Infinite Region :-