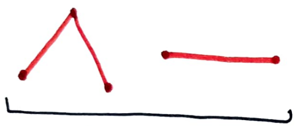
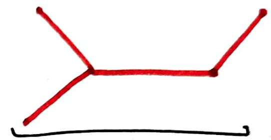
# TREE



* A graph having no ~~cyle~~ cycle is acyclic graph.
* A forest is an acyclic graph; a tree is a connected acyclic graph.
* A leaf (or pendent vertex) is a vertex of degree 1.



forest



tree

## Properties of Tree –

1. Every tree with atleast one edge has atleast two leaves.
   * If the minimum degree of a graph is atleast 2, then that graph must contain a cycle.
2. Every tree on $n$ vertices has exactly $(n-1)$ edges.
   * A forest on $n$ vertices has ($n$ – no. of components) edges

## Equivalent definitions of trees :-

Let T be a graph with n vertices. Then the following statements are equivalent -

(i) T is tree.

(ii) T contains no cycle and has $(n-1)$ edges.

(iii) T is connected and has $(n-1)$ edges.

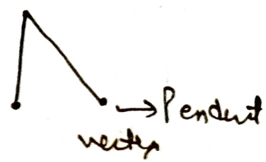
(iv) T is connected, and every edge is a cut-edge.

(v) Any two vertices of T are connected by exactly one path.

OR

A graph is a tree iff it is minimally connected.

→ **Pendent Vertex** – A vertex of degree 1.




* In any tree (with two or more vertices), there are atleast two pendent vertices.

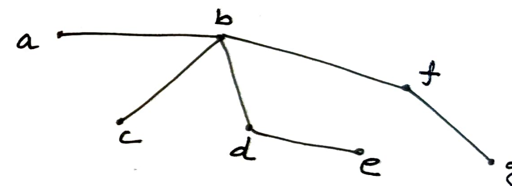
→ **Eccentricity of vertex** – $ecc(v) = \max_{x \in V_G} \{ d(v, x) \}$

* A central vertex of a graph is vertex with minimum eccentricity.

* The center of a graph G, denoted $Z(G)$, is the subgraph induced on the set of central vertices of G.

~~Now all that things applied on Tree~~ ..

Ex →




$ecc(a) = \max\{ d(a,b), d(a,c), d(a,d), d(a,e), d(a,f), d(a,g)\}$

$= \max\{1, 2, 2, 3, 3, 3\} = 3$

$ecc(b) = 2$, $ecc(c) = 3$, $ecc(d) = 3$, $ecc(e) = 4$, $ecc(f) = 3$, $ecc(g) = 4$

central vertex = $\min\{ecc(v)\} = \min\{ecc(a), ecc(b), ecc(c), ecc(d), ecc(e), ecc(f), ecc(g)\} =$

$= \min\{3, 2, 3, 3, 4, 3, 4\} = \underline{2}$

So that b is the central vertex of the graph.

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**Lemma:-** Let T be a tree with atleast 3 vertices

* If $v$ is a leaf of T & $w$ is its neighbour then

$$ecc(w) = ecc(v) - 1$$

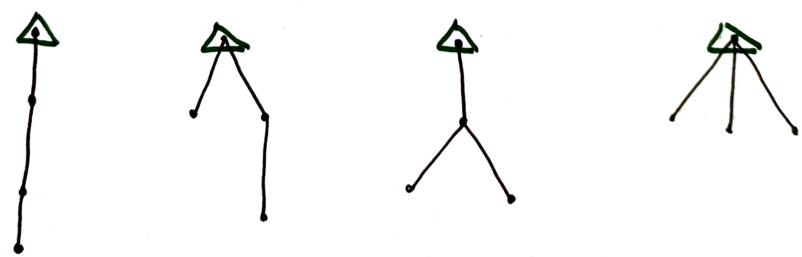
* If $u$ is a central vertex of T, then

$$\deg(u) \geq 2$$

Result

## BINARY TREE

* A tree in which one vertex (called the root) is distinguished from all others is called a rooted tree.
* the term tree means tree without root.



~~Rooted~~

→ A special class of rooted trees, called binary tree.

→ A binary tree is defined as a tree in which there is exactly one vertex of degree 2, and each of remaining vertices is of degree 1 or 3.

Two ~~post~~ properties of binary trees follows directly from the definition:

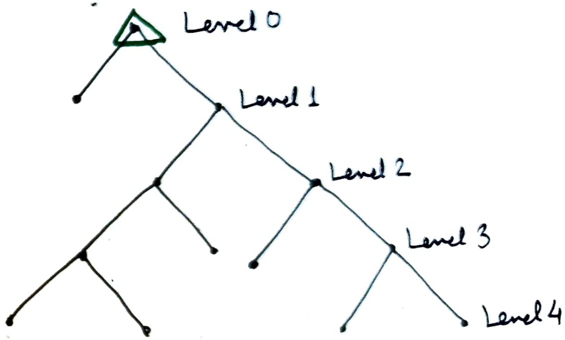
(a) The no. of vertices $n$ in a binary tree is always odd.

(b) Let $p$ be the no. of pendent vertex in a binary tree T. Then $(n-p-1)$ is the no. of vertex of degree 3.

No. of edges $= (n-1)$

$$\frac{1}{2}[p + 3(n-p-1) + 2] = n-1$$

$$\boxed{p = \frac{n+1}{2}}$$




4-level binary tree.

The maximum number of vertices possible in k-level binary tree is

$$2^0 + 2^1 + 2^2 + \cdots + 2^k \geq n$$

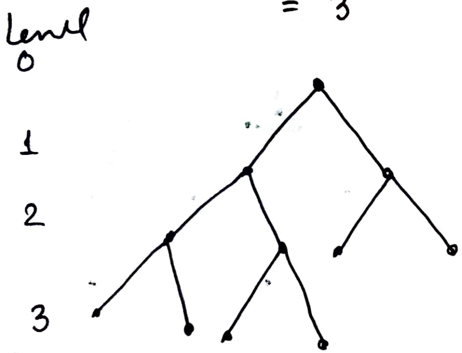
maximum possible height of n-vertex binary tree is $= \lceil \log_2 (n+1) - 1 \rceil$
min $l_{max}$ = ↳ smallest integer

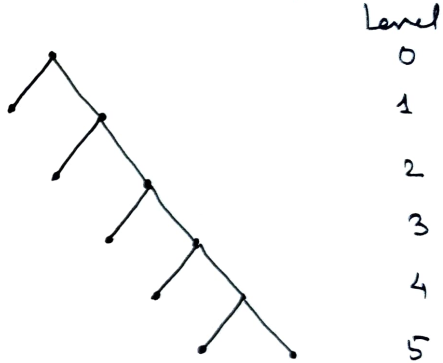
on the other hand max $l_{max} = \frac{n-1}{2}$

for ex → $n = 11$,

min $l_{max} = \lceil \log_2 12 - 1 \rceil$
$= 3$

max $l_{max} = \frac{n-1}{2} = 5$




## Counting Labelled Tree –

The no. of n-vertex labeled tree is $n^{n-2}$, for $n \geq 2$ & is known as Cayley's formula.