18.1.22

for a given set of axis = one principal axis

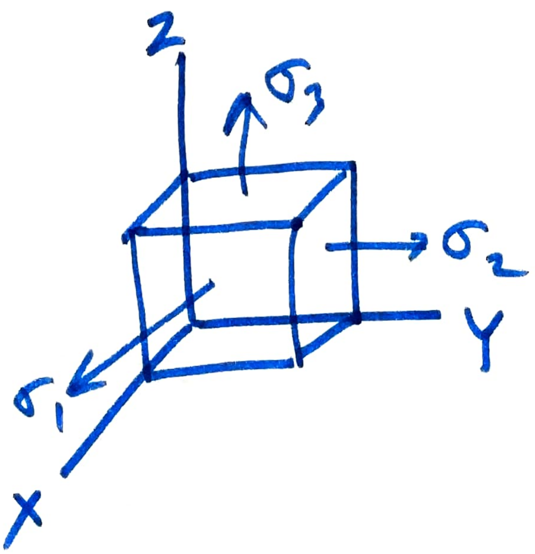
by changing the axis,
invarients shold not change

they are not funcdim of choice of axis.

Principal plane
there will be

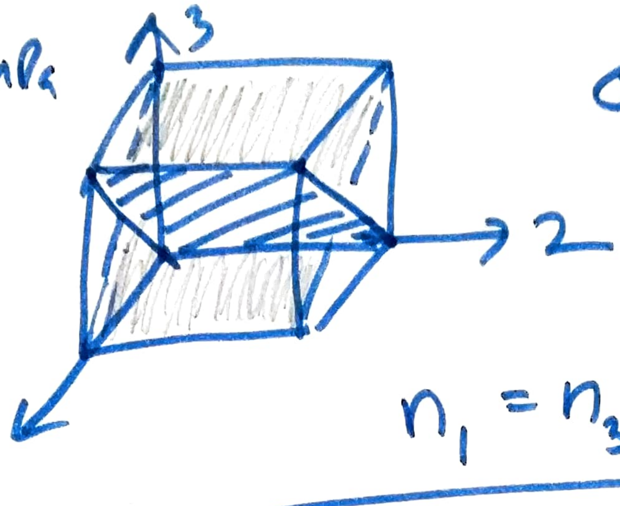
→ only normal stress
→ no shear stress

Metal deforms by shear stress



planes at an angle 45°
$\Downarrow$ will have
max^m shear stress

σ_2 500 MPa
σ_3 200
σ_1 800

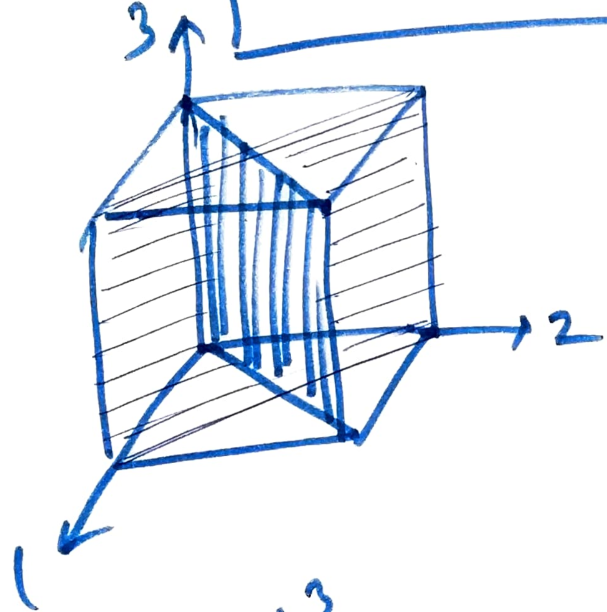


$\sigma_1 > \sigma_2 > \sigma_3$

largest — smallest

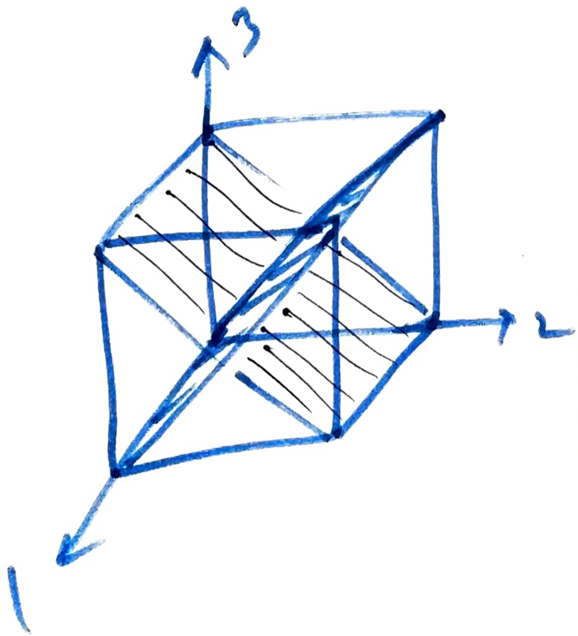
$$n_1 = n_3 = \pm\frac{1}{\sqrt{2}}$$

$$\boxed{\tau_1 = \frac{\sigma_1 - \sigma_3}{2}}$$ max^m



$$n_1 = n_2 = \pm\frac{1}{\sqrt{2}}$$

$$\boxed{\tau_2 = \frac{\sigma_1 - \sigma_2}{2}}$$



$$n_2 = n_3 = \pm\frac{1}{\sqrt{2}}$$

$$\boxed{\tau_3 = \frac{\sigma_2 - \sigma_3}{2}}$$

20.1.22

Octahedral Planes

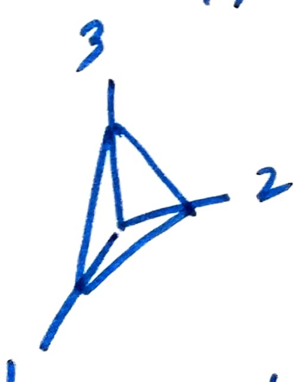
$\Downarrow$

Equally inclined
to all principal axes

$$n_1 = n_2 = n_3 = \frac{1}{\sqrt{3}}$$

Principal stresses

σ_1 , σ_2 , σ_3



Find
What are O.H. stresses
on the plane ?

Find normal stress,
shear stress
Total stress

$$S_i = \sigma_{ij} n_j$$

$$S_1 = \sigma_{11} n_1 + \sigma_{12} n_2 + \sigma_{13} n_3$$

$$S_2 = \sigma_{21} n_1 + \sigma_{22} n_2 + \sigma_{23} n_3$$

$$S_3 = \sigma_{31} n_1 + \sigma_{32} n_2 + \sigma_{33} n_3$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$S_1 = \frac{\sigma_1}{\sqrt{3}}, \quad S_2 = \frac{\sigma_2}{\sqrt{3}}, \quad S_3 = \frac{\sigma_3}{\sqrt{3}}$$

$$S_n = S_1 n_1 + S_2 n_2 + S_3 n_3$$

$$S_n = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$S_{total}^2 = S_1^2 + S_2^2 + S_3^2$$

$$S_t^2 = \frac{\sigma_1^2}{3} + \frac{\sigma_2^2}{3} + \frac{\sigma_3^2}{3} \qquad S_t^2 \checkmark$$

$$S_{total}^2 = S_n^2 + S_{shear}^2$$

$$S_{shear}^2 = S_{total}^2 - S_n^2$$

(2)

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{pmatrix}}_{\text{Hydrostatic component}} + \underbrace{\begin{pmatrix} (\sigma_{11}-\sigma_m) & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & (\sigma_{22}-\sigma_m) & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & (\sigma_{33}-\sigma_m) \end{pmatrix}}_{\text{Stress Deviator}}$$

$$\sigma_m = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} = \frac{I_1}{3}$$

Hydrostatic Component

⇓

All principal stresses are same

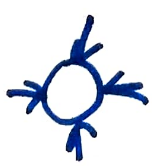
change axis → get same thing

No off diagonal elements

or

any plane stress is only normal stress

ex- Inside the sea



Stress Deviator

State of pure shear

⇓

→ Sum of diagonal elements is zero

→ $\sigma_1 + \sigma_2 + \sigma_3 = 0$

Invarient

→ No normal stress

→ Only shear stress

→ needed for plasticity or deformation

Mohr's Circle

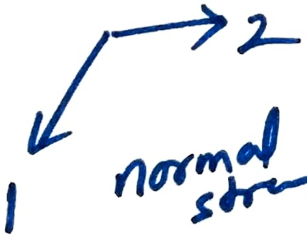
$\Downarrow$

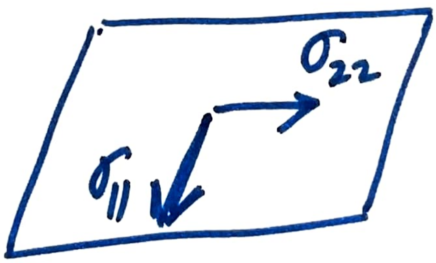
Graphical method to find out Principal planes & Principal stresses.

Two Dimension (2-D)

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

plane stress





normal stress

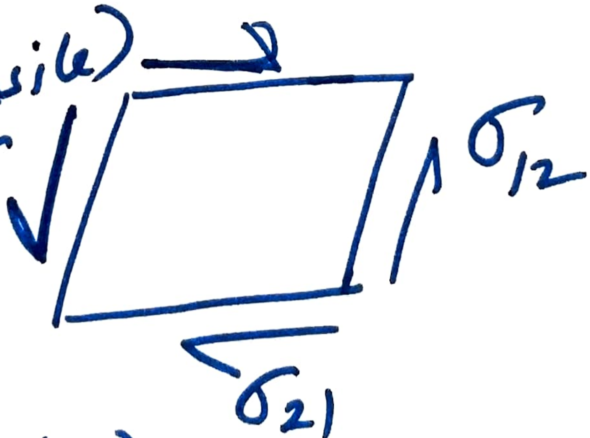
Pulling stress

+ve (tensile stress)

Pushing stress

-ve

(compressive stress)



for shear stress

Sign convention

$\Downarrow$

Clockwise

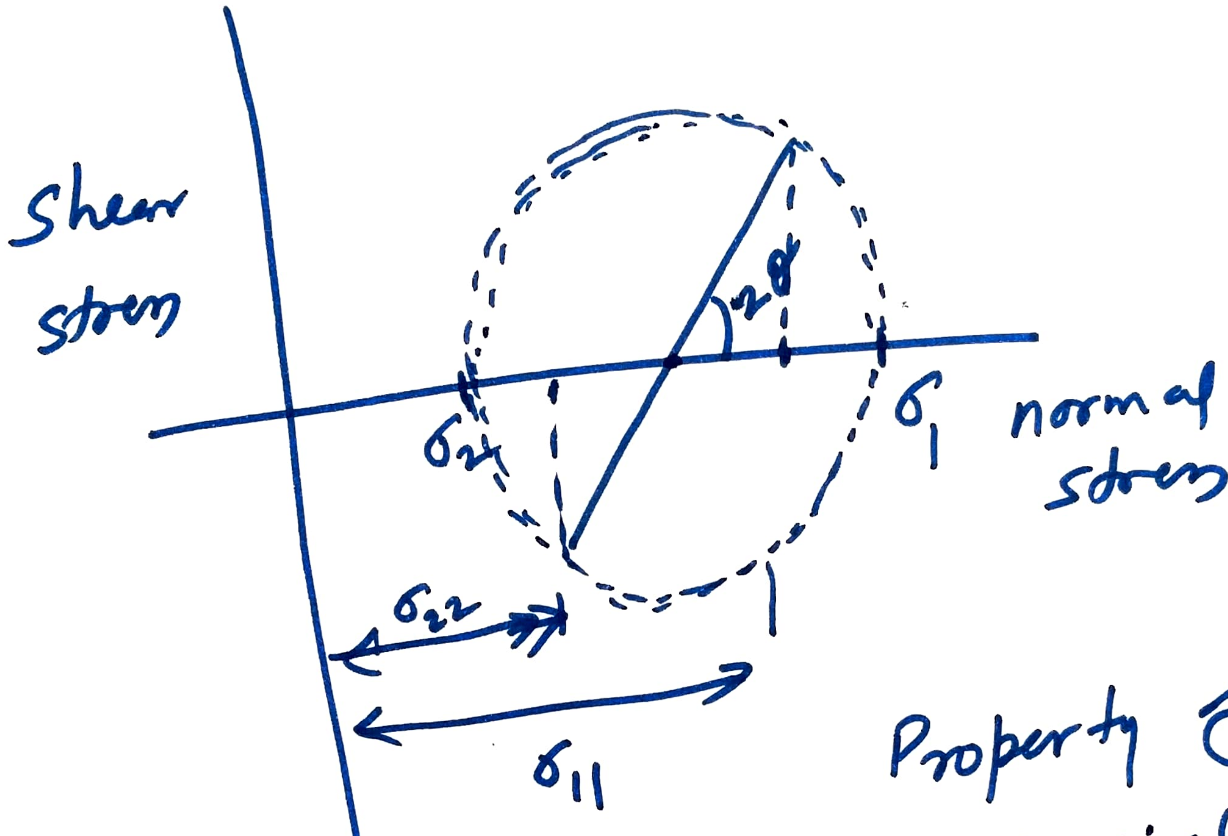
+ve

Anticlockwise

-ve

X axis → normal stress

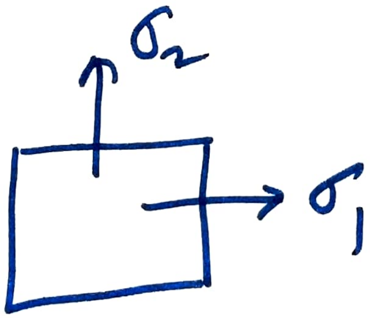
Y axis → Shear stress



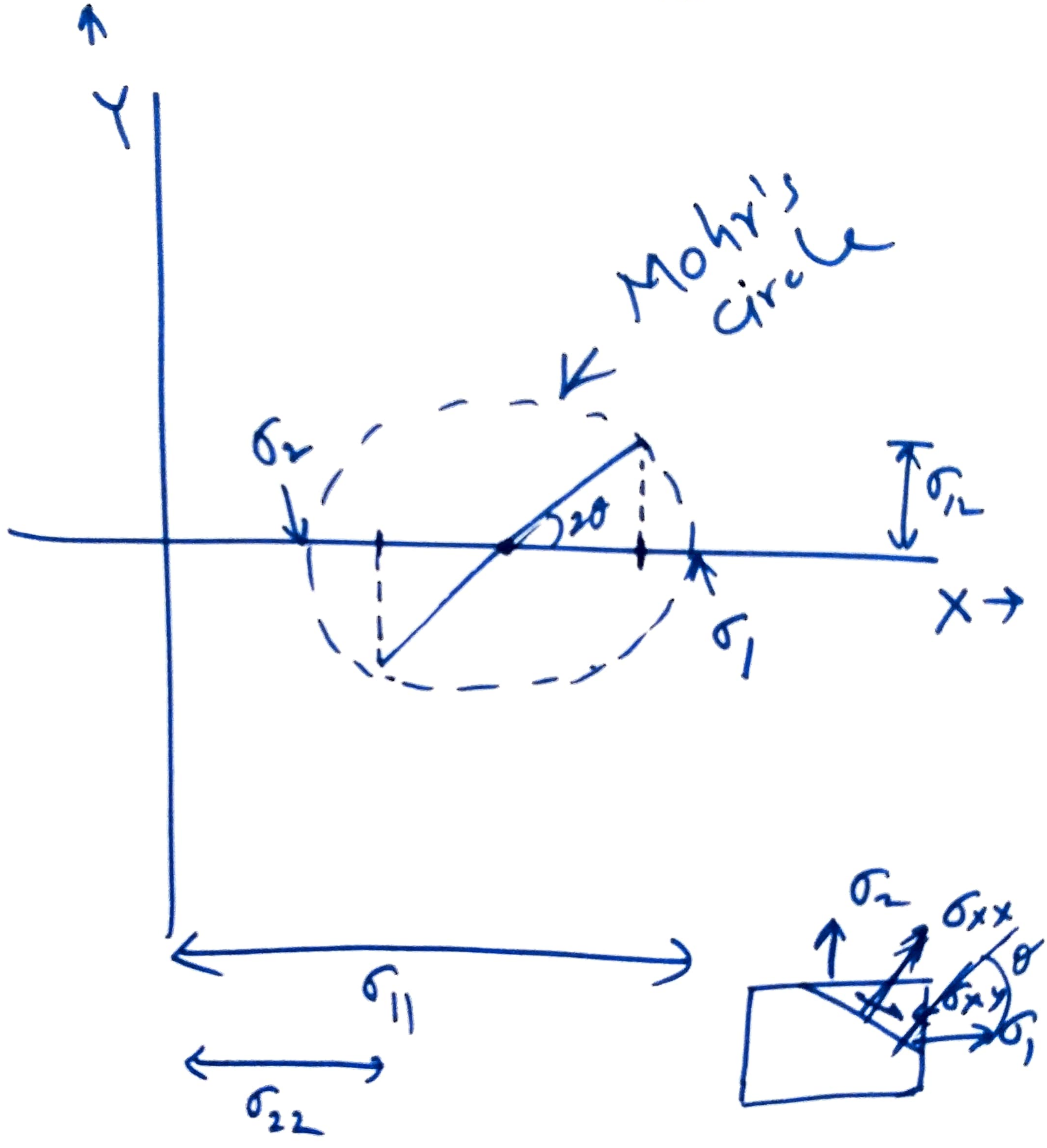
Property of mohr's circle

⇓

each point on circle represent stress on plane.



σ_{11} σ_{22} σ_{12}



σ_1, σ_2 $\Rightarrow$ Principal stress

$$\sigma_1 > \sigma_2$$

21.7.22

A rectangular sheet is loaded in plane stress to principal stresses of 100 MPa & 50 MPa.

Draw a Mohr's circle & find the values of the normal stress & shear stress on a plane with its normal inclined at an angle of 22.5 degrees to the major principal axis.

$\sigma_1 = 100$ MPa

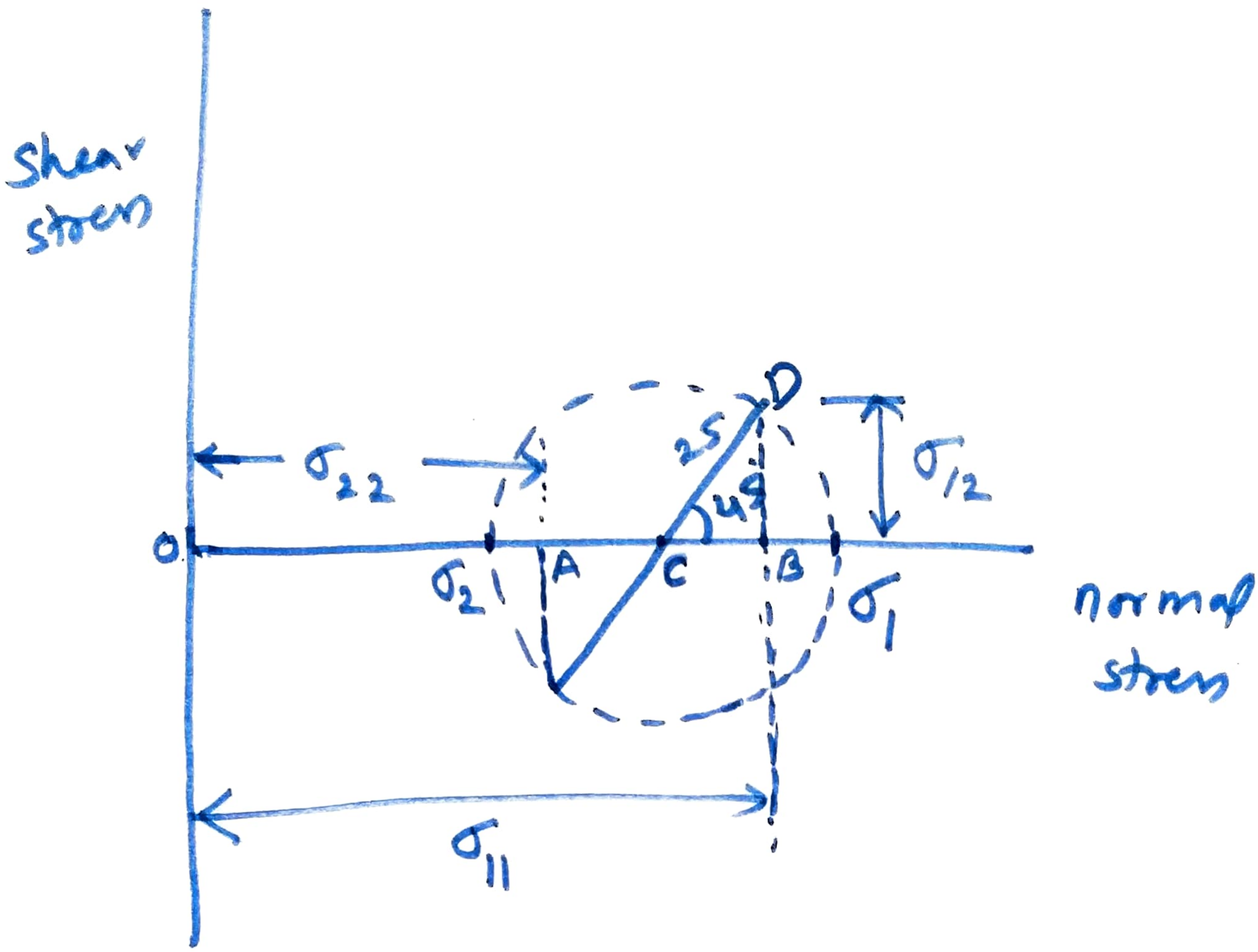
$\sigma_2 = 50$ MPa

$\left.\begin{array}{l}\sigma_{11} = ? \\ \sigma_{22} = ?\end{array}\right\}$ normal stress

$\left.\sigma_{12} = ?\right\}$ shear stress

$\theta = 22.5°$

Scale 1cm = 10 MPa



$\sigma_{11} = OB = O\sigma_2 + \sigma_2 C + CB$

$= 50 + 25 + 25 \cos 45°$

$= 75 + 25 \times \frac{1}{\sqrt{2}}$

$\sigma_{22} = OA = O\sigma_2 + \sigma_2 A$

$= O\sigma_2 + (\sigma_2 C - AC)$

$= 50 + (25 - 25 \cos 45°)$

$= 50 + (25 - 25 \times \frac{1}{\sqrt{2}})$

$\sigma_{12} = BD = 25 \sin 45°$

28

Find the principal stresses & the orientation of the axes of principal stress with the x, y axes for the following situations.

Use Mohr's circle only.

(a)

$\sigma_{11} = 5000$ MPa

$\sigma_{22} = 500$ MPa

$\sigma_{12} = -800$ MPa

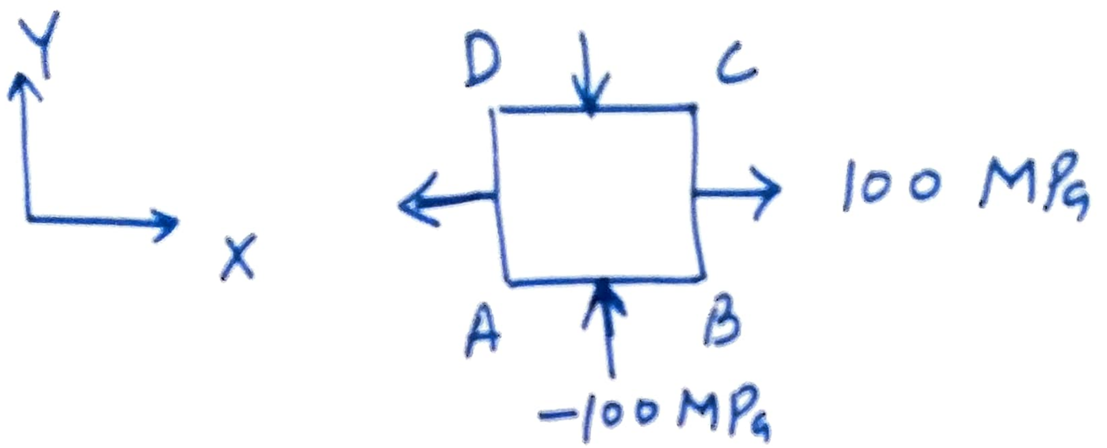
(b)

$\sigma_{11} = -6000$ MPa

$\sigma_{22} = 500$ MPa

$\sigma_{12} = 2500$ MPa

(a) State of stress on an element of a thin plate is as follows

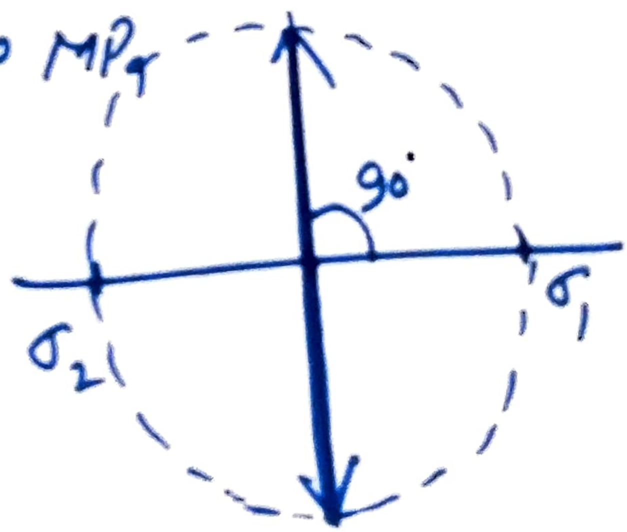


The edge AB is rotated counterclockwise by an angle 45°

Sketch the state of stress after rotation. Use Mohr's circle.

$\sigma_1 = 100$ MPa

$\sigma_2 = -100$ MPa



State of stress

Pure shear

$\tau = 100$ MPa

Give the magnitude of principal stresses & direction of principal axes for the given two states of stress using Mohr's circle. 30

(a) $\sigma_{11} = 15000$ MPa

$\sigma_{22} = 4900$ MPa

$\sigma_{12} = -6200$ MPa

(b) $\sigma_{11} = 12700$ MPa

$\sigma_{22} = 7300$ MPa

$\sigma_{12} = 7600$ MPa

At a point P in a body

$$\sigma_{xx} = 10{,}000 \text{ N cm}^{-2}$$

$$\sigma_{yy} = -5000 \text{ N cm}^{-2}$$

$$\sigma_{zz} = -5000 \text{ N cm}^{-2}$$

$$\sigma_{xy} = \sigma_{xz} = 10{,}000 \text{ N cm}^{-2}$$

$$\sigma_{yz} = 0$$

(a) Determine the normal & shear stresses on a plane that is equally inclined to all the three axes.

(b) Resolve this state of stress in hydrostatic & pure shear stress.

The stresses at a point, ~~reffer~~ referred to the principal axes, are as follows

$$\begin{matrix} 325 & 0 & 0 \\ 0 & 450 & 0 \\ 0 & 0 & 120 \end{matrix} \quad MPa$$

what is the value of the max^m shear stress and on what plane (s) is it ~~as~~ acting (show by a fig also).

$\sigma_1 > \sigma_2 > \sigma_3$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

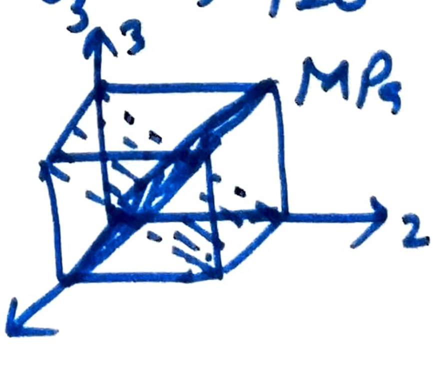
$\sigma_1 \rightarrow 450$

$\sigma_2 \rightarrow 325$

$\sigma_3 \rightarrow 120$ MPa

$$= \frac{450 - 120}{2}$$

$$\tau_{max} = 165 \text{ MPa}$$



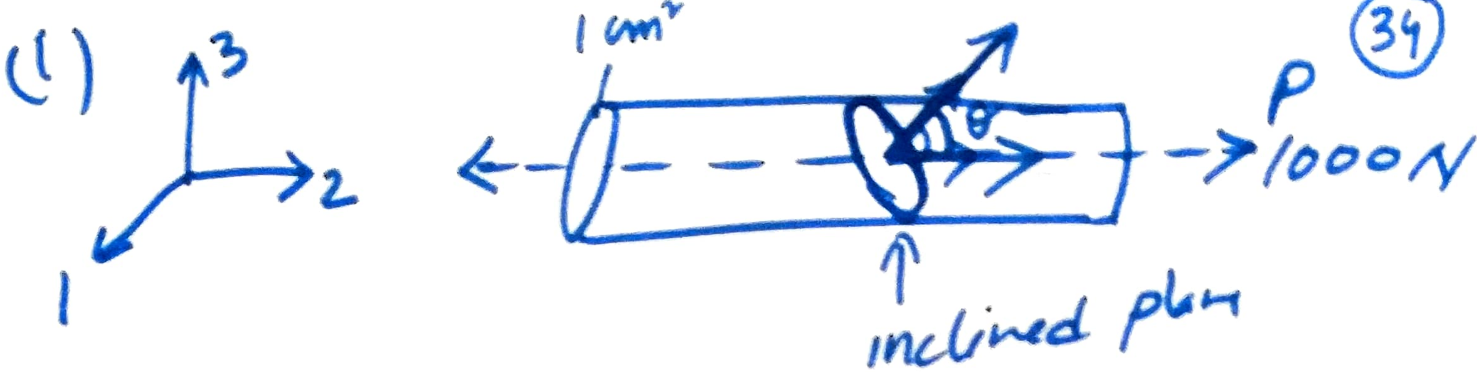
A load of 1000 N is applied along the axis of a circular bar of area 1 cm^2. Find the normal stress and the shear stress on a plane inclined at an angle θ to the axis of the bar using the following methods

(i) By resolution of the force along the given directions.

(ii) By transformation rule for the stress tensor upon change of axes.

(iii) By Drawing the Mohr's circle.

(i)



Area f inclined plane

$$= \frac{A}{\cos\theta}$$

$$\sigma = \frac{P}{A/\cos\theta} = \frac{P}{A}\cos\theta$$

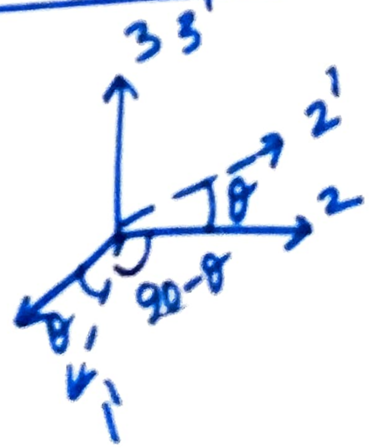
Normal stress $= \sigma\cos\theta = \underline{\frac{P}{A}\cos^2\theta}$

Shear stress $= \sigma\sin\theta$

$$= \underline{\frac{P}{A}\cos\theta\sin\theta}$$

(ii)

$$\sigma'_{ij} = a_{ik}\,a_{jl}\,\sigma_{kl}$$



$$a_{ij} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\cancel{\sigma_{11}}$

σ_{11}' σ_{21}' σ_{31}'

σ_{12}' σ_{22}' σ_{32}'

σ_{13}' σ_{23}' σ_{33}'

$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_{ij}' = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$a_{12} = \cos(90-\theta) = \sin\theta$$

new old

$$\sigma_{22} = \frac{P}{A} = \frac{1000}{1 \times 10^{-4}} = 10 \times 10^{6}\ Pa$$

$$= 10\ MPa$$

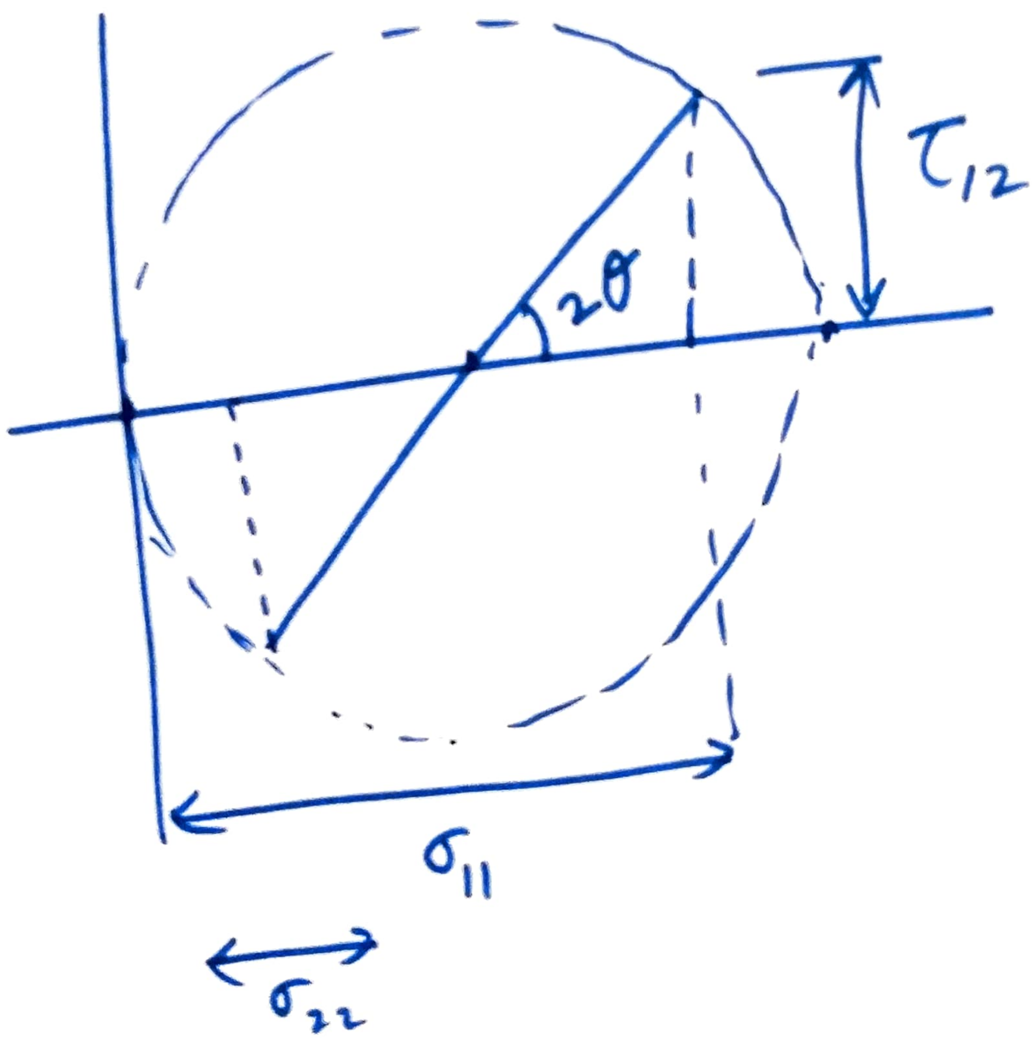
(iii)

$$\sigma_{22} = 10 MPa$$

Principal stress

$$\sigma_2 = 10 MPa$$

$$\sigma_1 = 0$$



State of stress at a point is given by 37

$$\begin{pmatrix} 8000 & 2000 & -5000 \\ 2000 & -4000 & 3000 \\ -5000 & 3000 & 6000 \end{pmatrix} \text{MPa}$$

Determine

(a) the total stress (Direction & magnitude) on plane described by direction cosine

$$n_x = \frac{1}{\sqrt{2}}, \quad n_y = \frac{1}{2},$$

$$n_z = -ve$$

(b) magnitude of normal & shear stresses on this plane.

$$n_1 = \frac{1}{\sqrt{2}} \qquad n_2 = \frac{1}{2} \qquad n_3 = -\frac{1}{2}$$

$$S_i = \sigma_{ij} \, n_j$$

S_1

S_2

S_3

$$S_{total} = \sqrt{S_1^2 + S_2^2 + S_3^2} = S$$

$$n_1' = \frac{S_1}{S}$$

$$n_2' = \frac{S_2}{S}$$

$$n_3' = \frac{S_3}{S}$$

(b)

$$S_{normal} = S_1 n_1 + S_2 n_2 + S_3 n_3$$

$$S_{total}^2 = S_{normal}^2 + S_{shear}^2$$

$$S_{shear}^2 = S_{total}^2 - S_{normal}^2$$

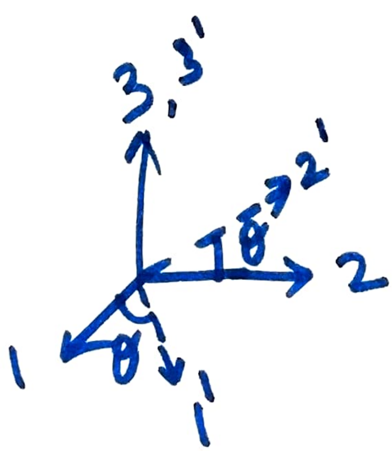
A new set of coordinate axes are obtained by rotating principal axes about the z axis by an angle θ.
All the three principal stresses are equal. Show that with reference to the new axes:

(a) $\sigma'_{11} = \sigma'_{22} = \sigma'_{33} = \sigma_1 = \sigma_2 = \sigma_3$

(b) $\sigma'_{ij} = 0$ when $i \neq j$

(c) Are the new set of axes also principal axes?

What is this state of stress called?



$a_{ij} = \begin{pmatrix} \quad & \quad & \quad \end{pmatrix}$

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\sigma_{ij}' = a_{ik}\, a_{jl}\, \sigma_{kl}$$

$\sigma_{11}' =$

$\sigma_{22}' =$

$\sigma_{33}' =$

$\sigma_{12}' =$

$\sigma_{13}' =$

$\sigma_{ij}' =$
$i \neq j$

Yes

Hydrostatic component.