Stress invariants

Q1. Find the stress invariants at a point P at which state of stress is given as

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Step -1: Find the characteristics equation of stress matrix

Det $(\sigma - \lambda I) = 0$

$$\begin{vmatrix} \sigma_x - \lambda & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \lambda & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \lambda \end{vmatrix} = 0$$

$$\lambda^3 - (\sigma_x + \sigma_y + \sigma_z)\lambda^2 + (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{zx}^2 - \tau_{yz}^2)\lambda - (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \tau_{xy}^2\sigma_z - \tau_{zx}^2\sigma_y - \tau_{yz}^2\sigma_z) = 0$$

Let $\boldsymbol{I_1} = \sigma_x + \sigma_y + \sigma_z$

$\boldsymbol{I_2} = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{zx}^2 - \tau_{yz}^2$

$\boldsymbol{I_3} = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \tau_{xy}^2\sigma_z - \tau_{zx}^2\sigma_y - \tau_{yz}^2\sigma_z$

Where I_1 , $I_2, and\ I_3$ are called **first**, **second**, **third** stress invariants.

Let roots of above cubic equation are $\lambda_1, \lambda_2 and\ \lambda_3$.

$$\lambda_1 + \lambda_2 + \lambda_3 = -\frac{b}{a} = \sigma_x + \sigma_y + \sigma_z$$

$$\lambda_1\ \lambda_2\ \lambda_3 = \frac{d}{a} = -(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \tau_{xy}^2\sigma_z - \tau_{zx}^2\sigma_y - \tau_{yz}^2\sigma_z)$$

- Sum of roots (sum of principal stress) is equal to trace of stress matrix.
- Product of roots is equal to determinate of stress matrix.