3D-stress transformation

Q1. The state of stress at a point P is given as σ with respect oxyz

$$\sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the stress matrix σ' with respect to new axis ox'y'z' axes.

Step-1: Find the direction cosine along x', y' and z' axes.

	x' - axis	y' - axis	z' - axis
l	$\cos\theta$	$-\sin\theta$	0
m	$\sin\theta$	$\cos\theta$	0
n	0	0	1

Let $\theta = 30^0$

$\cos 30^0 = 0.866$ and $\sin 30^0 = 0.5$

New Stress matrix

σ_x	τ_{xy}	τ_{xz}
τ_{yx}	σ_y	τ_{yz}
τ_{zx}	τ_{zy}	σ_z

For evaluating σ_x, τ_{xy} *and* τ_{xz} , evaluate traction vector on a plane whose normal vector is x'-axes.

$\{T\} = [\sigma]\{x'\}$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} \cos 30^0 \\ \sin 30^0 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} \cos 30^0 + \sin 30^0 \\ \cos 30^0 + \sin 30^0 \\ \cos 30^0 + \sin 30^0 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.866 + 0.5 \\ 0.866 + 0.5 \\ 0.866 + 0.5 \end{Bmatrix} = \begin{Bmatrix} 1.366 \\ 1.366 \\ 1.366 \end{Bmatrix}$$

(1) Normal stress $\sigma_x = \{T\}^T \{x'\}$

$$= \{1.366 \quad 1.366 \quad 1.366\} \begin{Bmatrix} \cos 30^0 \\ \sin 30^0 \\ 0 \end{Bmatrix}$$

$$= 1.366 \times \cos 30^0 + 1.366 \times \sin 30^0$$

$$= 1.87$$

(2) Shear stress $\tau_{xy} = \{T\}^T \{y'\}$

$$= \{1.366 \quad 1.366 \quad 1.366\} \begin{Bmatrix} -\sin 30^0 \\ \cos 30^0 \\ 0 \end{Bmatrix}$$

$$= 1.366 \times -\sin 30^0 + 1.366 \times \cos 30^0$$

$= 0.5$

(3) Shear stress $\tau_{xz} = \{T\}^T \{z'\}$

$$= \{1.366 \quad 1.366 \quad 1.366\}\begin{Bmatrix}0\\0\\1\end{Bmatrix}$$

$= 1.366$

$= 1.366$

For evaluating τ_{yx}, σ_y *and* τ_{yz} , evaluate traction vector on a plane whose normal vector is y'-axes.

$$\{T\} = [\sigma]\{y'\}$$

$$= \begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}\begin{Bmatrix}-\sin 30^0\\ \cos 30^0\\ 0\end{Bmatrix}$$

$$= \begin{Bmatrix}\cos 30^0 - \sin 30^0\\ \cos 30^0 - \sin 30^0\\ \cos 30^0 - \sin 30^0\end{Bmatrix}$$

$$= \begin{Bmatrix}0.866 - 0.5\\0.866 - 0.5\\0.866 - 0.5\end{Bmatrix} = \begin{Bmatrix}0.366\\0.366\\0.366\end{Bmatrix}$$

(4) Normal stress $\sigma_y = \{T\}^T \{y'\}$

$$= \{0.366 \quad 0.366 \quad 0.366\}\begin{Bmatrix}-\sin 30^0\\ \cos 30^0\\ 0\end{Bmatrix}$$

$$= 0.366 \times -\sin 30^0 + 0.366 \times \cos 30^0$$

$$= 0.134$$

(5) Shear stress $\tau_{yz} = \{T\}^T \{z'\}$

$$= \{0.366 \quad 0.366 \quad 0.366\}\begin{Bmatrix}0\\0\\1\end{Bmatrix}$$

$$= 0.366$$

$\tau_{yx} = \tau_{xy} = 0.5$

For evaluating τ_{zx}, τ_{zy} *and* σ_z , evaluate traction vector on a plane whose normal vector is z'-axes.

$$\{T\} = [\sigma]\{y'\}$$

$$= \begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}\begin{Bmatrix}0\\0\\1\end{Bmatrix}$$

$$= \begin{Bmatrix}1\\1\\1\end{Bmatrix}$$

(6) Normal stress $\sigma_z = \{T\}^T \{z'\}$

$$= \{1 \quad 1 \quad 1\}\begin{Bmatrix}0\\0\\1\end{Bmatrix} = 1$$

New Stress matrix

1.87	0.5	1.366
0.5	0.134	0.366
1.366	0.366	1