Strain Energy in pure Bending

Objective:

(a) To learn the **strain energy** developed in beam subjected to **pure bending**.

(b) Understand the meaning of **pure bending** in beam.

Pure bending

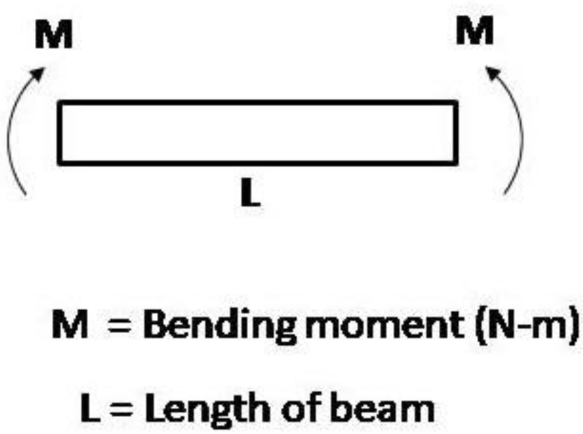
If bending moment is constant along the length of beam, then we called as beam is subjected to pure bending.

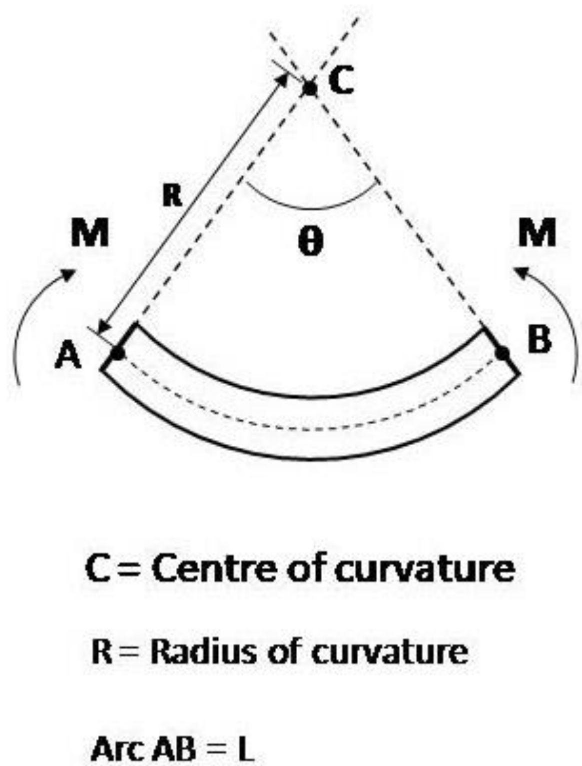
Example: A cantilever beam subjected to end moment M.

If a cantilever beam subjected to end moment M. Then bending moment is constant along the length of beam. So it is a case of pure bending.

Strain energy of bending

Consider a prismatic (find the meaning of prismatic) beam subjected to pure bending within the elastic limit of material. For such loading, the bending moment M is constant along the length L of the beam and the elastic line is a circular arc of curvature M/EI and the angle Ө subtended by this arc is





Strain Energy in pure Bending

$\theta = \frac{L}{R}$ - - - - - (1)

By flexurale formulae,

$$\frac{M}{I} = \frac{E}{R}$$

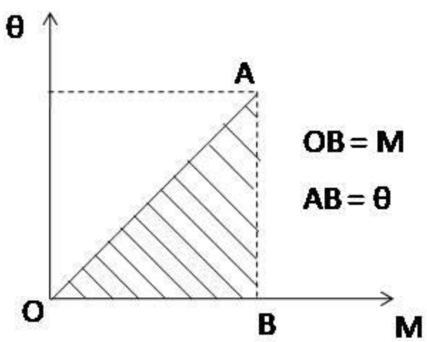
$$\frac{1}{R} = \frac{M}{EI}$$

Substitute 1/ R in equation (1)

$$\theta = \frac{ML}{EI}$$

Here θ is linear proportional to Bending moment.

Draw a graph of bending moment verses theta



Area of triangle = $\frac{1 \times OB \times OA}{2}$

$= \frac{M\theta}{2}$

Strain energy stored in beam is equal to work done by applied end moment. Area of Triangle is equal to work done by external moment M.

Strain energy due to bending moment M = $\frac{M\theta}{2}$

Strain energy due to pure bending:

$$U_{bending} = \frac{1}{2}M\theta = \frac{1}{2}M\frac{ML}{EI} = \frac{M^2L}{2EI}$$

Note: In pure bending elastic curve (neutral axis of beam) take circular shape of Radius of curvature R and angle subtends by Arc AB in above figure at centre of curvature is θ.

Strain Energy in pure Bending

$\theta = \frac{L}{R}$ and $\frac{1}{R} = \frac{d^2y}{dx^2}$ (Explore the derivation of 1/ R by yourself).