ASSIGNMENT - 02.

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Cauchy's law -

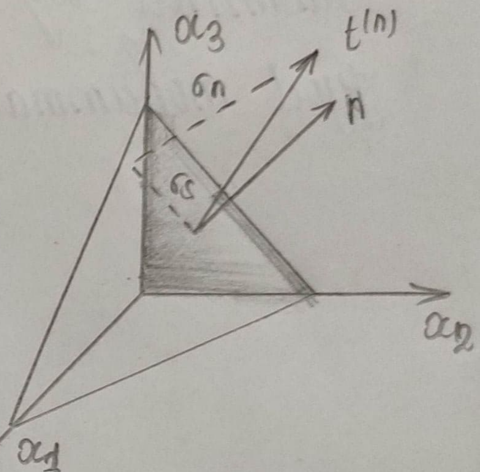
Problem-01- Solution

(a) For traction vector using Cauchy's law

$$\begin{bmatrix} t_1^{(n)} \\ t_2^{(n)} \\ t_3^{(n)} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} (2\times1 + 1\times1 + 2\times1) \\ (1\times1 + 0\times1 + -1\times1) \\ (2\times1 + (-1)\times1 + 2\times1) \end{bmatrix}$$



$$\begin{bmatrix} t_1^n \\ t_2^n \\ t_3^n \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$$

so that traction vector -

$$\boxed{t^{(n)} = \left(\frac{5}{\sqrt{3}} \hat{e}_1 + 0 + \sqrt{3}.\hat{e}_3 \right)}$$

The Component normal to the plane is

$$\sigma_n = (t)^{(n)} . n$$

$$= \left(\frac{5}{\sqrt{3}} \hat{e}_1 + \sqrt{3}.\hat{e}_3 \right) . \left(\frac{\hat{e}_1}{\sqrt{3}} + \frac{\hat{e}_2}{\sqrt{3}} + \frac{\hat{e}_3}{\sqrt{3}} \right)$$

$$\sigma_n = \left(\frac{5}{3} + 0 + 1 \right) \Rightarrow \boxed{\sigma_n = \frac{8}{3} = 2.666}$$

The Shearing Component of the traction is -

$$\sigma_s = \sqrt{(t^n)^2 - \sigma_n^2}$$

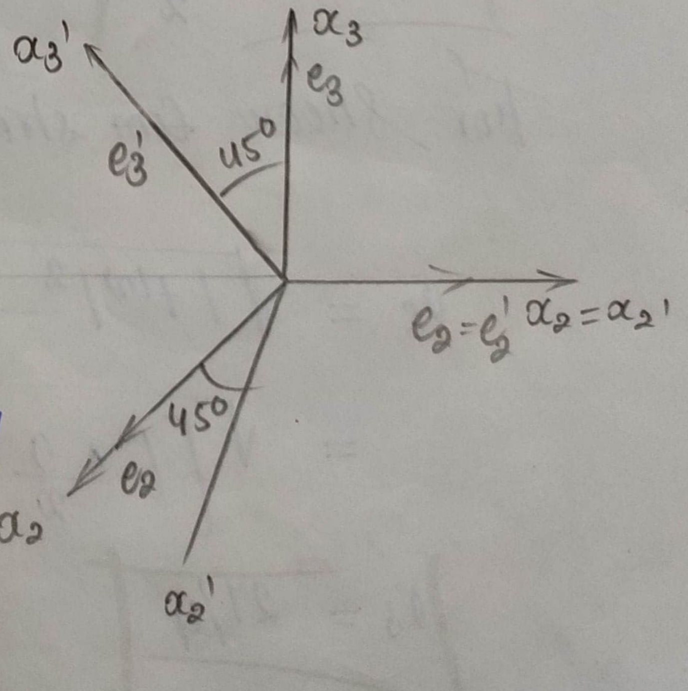
$$= \sqrt{\left[\left(\frac{5}{\sqrt{3}}\right)^2 + (\sqrt{3})^2\right] - \left(\frac{8}{3}\right)^2}$$

$$= \sqrt{(9.333 - 7.111)}$$

$$\boxed{\sigma_s = 1.490}$$

Problem-02

Two diffrent co-ordination Systems at a Point —



From the Couchy's law, the traction vector is

$$\begin{bmatrix} t_1^{(n)} \\ t_2^{(n)} \\ t_3^{(n)} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} x_2$$

$$\begin{bmatrix} t_1^{(n)} \\ t_2^{(n)} \\ t_3^{(n)} \end{bmatrix} = \begin{bmatrix} \left(1\times\frac{1}{\sqrt{2}} + 3\times 0 + 2\times\frac{1}{\sqrt{2}}\right) \\ \left(3\times\frac{1}{\sqrt{2}} + 1\times 0 + 0\times\frac{1}{\sqrt{2}}\right) \\ \left(2\times\frac{1}{\sqrt{2}} + 0\times 0 + (-2)\times\frac{1}{\sqrt{2}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 3/\sqrt{2} \\ 3/\sqrt{2} \\ 0 \end{bmatrix}$$

$$t^{(n)} = 3/\sqrt{2}\,\hat{e}_1 + \frac{3}{\sqrt{2}}\,\hat{e}_2 + 0.\hat{e}_3$$

normal stress component -

$$\sigma_N = t^{(n)}.n$$

$$= \left(\left(3/\sqrt{2}\right)\hat{e}_1 + \frac{3}{\sqrt{2}}\hat{e}_2\right).\left(\frac{1}{\sqrt{2}}\hat{e}_1 + \frac{1}{\sqrt{2}}\hat{e}_3\right)$$

$$\boxed{\sigma_N = \frac{3}{2}}$$

For shear ~~com~~ stress component -

$$\sigma_S = \sqrt{|t^{(n)}|^2 - \sigma_N^2}$$

$$= \sqrt{\left[\left(\frac{9}{2} + \frac{9}{2}\right) - \frac{9}{4}\right]}$$

$$\boxed{\sigma_S = 27/4}$$

The normal to the plane is equal to e_3' and so σ_N should be the same as σ_{33}' and it is. the stress σ_S should be equal to $\sqrt{(\sigma_{31}')^2+(\sigma_{32}')^2}$ and it is. The results are

traction is represent in different ways, with Components $(t_1^{(n)}, t_2^{(n)}, t_3^{(n)})$ and $(\sigma_{31}', \sigma_{32}', \sigma_{33}')$

$$\boxed{\sigma_N = \sigma_{33}' = 3/2}$$

$$\boxed{\sigma_{31}' = 1/2}$$

$$\boxed{\sigma_{33}' = -3/2}$$

Problem - 3 - Solution -

• Isotropic state of stress

Suppose the state of stress in a body is

$$\sigma_{ij} = \sigma_0 \delta_{ij}$$

$$[\sigma] = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

One fields that the application of the stress tensor transformation rule yields the very same components no ~~w~~matter what the new co-ordinate system in problem '2'. In other words, no shear stress act., no matter what the orientation of the plane through the point. This is tormed an **Isotropic state of stress**, or a spherical state of stress, one example of isotropic stress is the stressing arising in a fluid at sest, which can not support shear stress, in which case →

$$[\sigma] = -p[I]$$

where the scalar p is the fluid **hydrostatic pressure**. For this reason, an isotropic state of stress is also reffered to as a hydrostatic state of stress.

- **Problem - 04 - Solution -**
  - **The Stress Tensor**

Given,

$$\begin{bmatrix} \sigma_{11}' & \sigma_{12}' \\ \sigma_{21}' & \sigma_{22}' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} (\cos\theta.\sigma_{11} + \sin\theta \times \sigma_{21}) & (\cos\theta\, \sigma_{12} + \sin\theta\, \sigma_{22}) \\ (-\sin\theta\, \sigma_{11} + \cos\theta\, \sigma_{21}) & (-\sin\theta.\sigma_{21} + \cos\theta \times \sigma_{22}) \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

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$$= \begin{bmatrix} (\cos\theta \cdot \sigma_{11} + \sin\theta \cdot \sigma_{21})\cos\theta + (\cos\theta\, \sigma_{12} + \sin\theta \cdot \sigma_{22})\sin\theta \\ (-\sin\theta \cdot \sigma_{11} + \cos\theta \cdot \sigma_{21})\cos\theta + (-\sin\theta \cdot \sigma_{12} + \cos\theta \cdot \sigma_{22}) \cdot \sin\theta \end{bmatrix}$$

$$\begin{matrix} (\cos\theta \cdot \sigma_{11} + \sin\theta \cdot \sigma_{21}) \times -\sin\theta + (\cos\theta \cdot \sigma_{12} + \sin\theta \cdot \sigma_{22}) \cos\theta \\ (-\sin\theta \cdot \sigma_{11} + \cos\theta \cdot \sigma_{21}) \times -\sin\theta \quad + (-\sin\theta \cdot \sigma_{12} + \cos\theta \cdot \sigma_{22}) \times \cos\theta \end{matrix}$$

1 element - 1C1)

$$(\cos^2\theta \cdot \sigma_{11} + \sin^2\theta \cdot \sigma_{22} + \sin\theta \cdot \cos\theta\, \sigma_{21} + \sin\theta \cdot \cos\theta \cdot \sigma_{12}$$

element - 2C2

$$= \quad - \sin^2\theta \cdot \sigma_{21} + \cos^2\theta \cdot \sigma_{12} - \sin\theta \cdot \cos\theta \cdot \sigma_{11} + \sin\theta \cdot \cos\theta \cdot \sigma_{22}$$

2nd row
element-3 C1

$$\sin^2\theta \cdot \sigma_{21} - \sin\theta \cos\theta\, \sigma_{11} + \cos^2\theta \cdot \sigma_{12} + \sin\theta \cdot \cos\theta\, \sigma_{22}$$

element-4C2

$$\sin^2\theta \cdot \sigma_{11} - \sin\theta \cos\theta \cdot \sigma_{21} + \cos^2\theta \cdot \sigma_{22} - \sin\theta \cdot \cos\theta \cdot \sigma_{12}$$

Note- Replacing 11 with xx and 22 with yy'

or we can say 1 - Replaced with x & 2 - Replaced with y for better understanding -

In 2D, the transformation eqs are from the Complex (Resultant) matrix -

$$\sigma'_{xx} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta\cdot\cos\theta$$

$$\sigma'_{yy} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\sin\theta\cdot\cos\theta$$

$$\tau'_{xy} = (\sigma_{yy} - \sigma_{xx})\sin\theta\cdot\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

- In 2-D, the principal stress orientation, θ_P, can be computted by setting $\tau'_{xy} = 0$ in the above shear equation and solving for θ to get θ_P, the principal stress angle.

$$0 = (\sigma_{yy} - \sigma_{xx})\sin\theta_P\cos\theta_P + \tau_{xy}(\cos^2\theta_P - \sin^2\theta_P)$$

This gives

$\tan(2\theta_P)$

The transformation Q is

$$Q = \begin{bmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{bmatrix}$$

~~inters~~ inserting this value ~~θ~~ for θ_P the equations for the normal stresses gives the principal values they are written as

σ_{max} and σ_{min} or alternatively as σ_1 and σ_2

$$\sigma_{max}, \sigma_{min} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

They Could also be obtained by

$$\sigma' = Q \cdot \sigma \cdot Q^T \text{ with } Q \text{ based on } \theta_P$$

- **Principal stress Notation -**

- principal stresses can be written as σ_1, σ_2 and σ_3. Only one subscript is usually used in this case to differentiate the principal stress values from the ~~m~~ normal stress components σ_{11}, σ_{22} and σ_{33}.

~~Pr~~oblem- 05 - Solution -

Given,

$$[\sigma_{ij}] = \begin{bmatrix} 5/2 & -1/2 & 0 \\ -1/2 & 5/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The principal values are the solution to the charesteristic eqn -

$$\begin{vmatrix} 5/2-\lambda & -1/2 & 0 \\ -1/2 & \frac{5}{2}-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= \left(\frac{5}{2}-\lambda\right) \cancel{\left\{\left(\frac{5}{2}-\lambda\right)(1-\lambda)-0\right\}} - \frac{1}{2}$$

$$= (1-\lambda)\left\{\left(\frac{5}{2}-\lambda\right)^2 - \frac{1}{4}\right\}$$

$$= (1-\lambda)\left(\frac{25}{4} + \lambda^2 - 5\lambda - \frac{1}{4}\right)$$

$$= (1-\lambda)(\lambda^2 - 5\lambda - 6)$$

$$\cancel{= (1-\lambda)(\lambda-3)(\lambda-2)}$$

$$= (1-\lambda)(\lambda-6)(\lambda+1)$$

Three principal values ~~ar~~ are -

~~Λ_1 or $\sigma_1 = 1$, $\sigma_2 = 6$ (Λ_2 or), $\sigma_3 = -1$ (Λ_3 or)~~

σ_1 or $\Lambda_1 = 6$

σ_3 or $\Lambda_3 = -1$

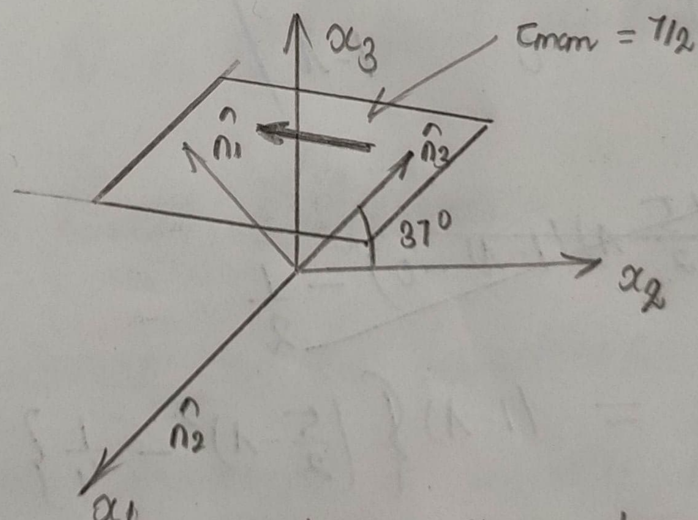
and σ_2 or $\Lambda_2 = 1$

and so the maximum shear stress is

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) =$$

$$= \frac{1}{2}(6+1)$$

$$\tau_{max} = 7/2$$



maximum shear stress direction