

Strain energy

Q1. Find the strain energy of a beam subjected to point load P and uniformly distributed load w in a cantilever beam.

Detailed solution:

Bending moment expression

$$M(x) = -Px - w \frac{x^2}{2}$$

Strain energy

$$U = \int_0^L \frac{M^2}{2EI} dx = \int_0^L \frac{\left[-Px - w \frac{x^2}{2}\right]^2}{2EI} dx = \frac{1}{2EI} \int_0^L P^2 x^2 + w^2 \frac{x^4}{4} + 2 \times Px \times w \frac{x^2}{2} dx$$

$$U = \frac{1}{2EI} \left[\frac{P^2 L^3}{3} + w^2 \frac{L^5}{20} + Pw \frac{L^4}{4} \right] = \frac{P^2 L^3}{6EI} + \frac{w^2 L^5}{40EI} + \frac{PwL^4}{8EI}$$

Deflection of beam at free end

$$\delta = \frac{\partial U}{\partial P} = \frac{PL^3}{3EI} + 0 + \frac{wL^4}{8EI}$$

$$\frac{PL^3}{3EI} \rightarrow \text{deflection due to point load}$$

$$\frac{wL^4}{8EI} \rightarrow \text{deflection due to uniformly distributed load}$$