Fuzzy membership functions

MCA 4002

Subject: Soft Computing

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Fuzzy membership functions

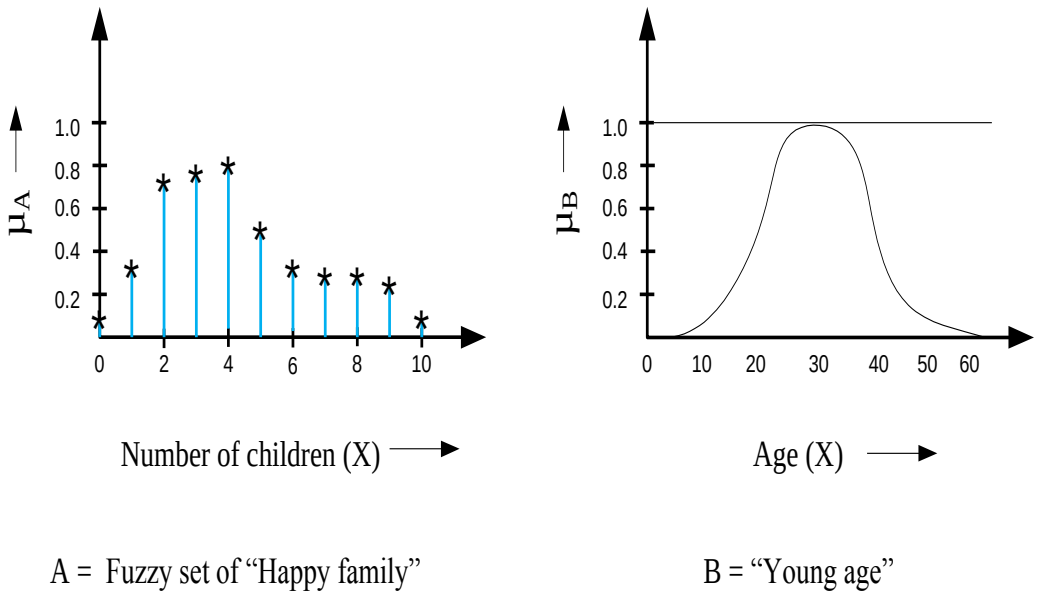
A fuzzy set is completely characterized by its membership function (sometimes abbreviated as MF and denoted as μ). So, it would be important to learn how a membership function can be expressed (mathematically or otherwise).

Note: A membership function can be on

a) a discrete universe of discourse and

b) a continuous universe of discourse.

Example:



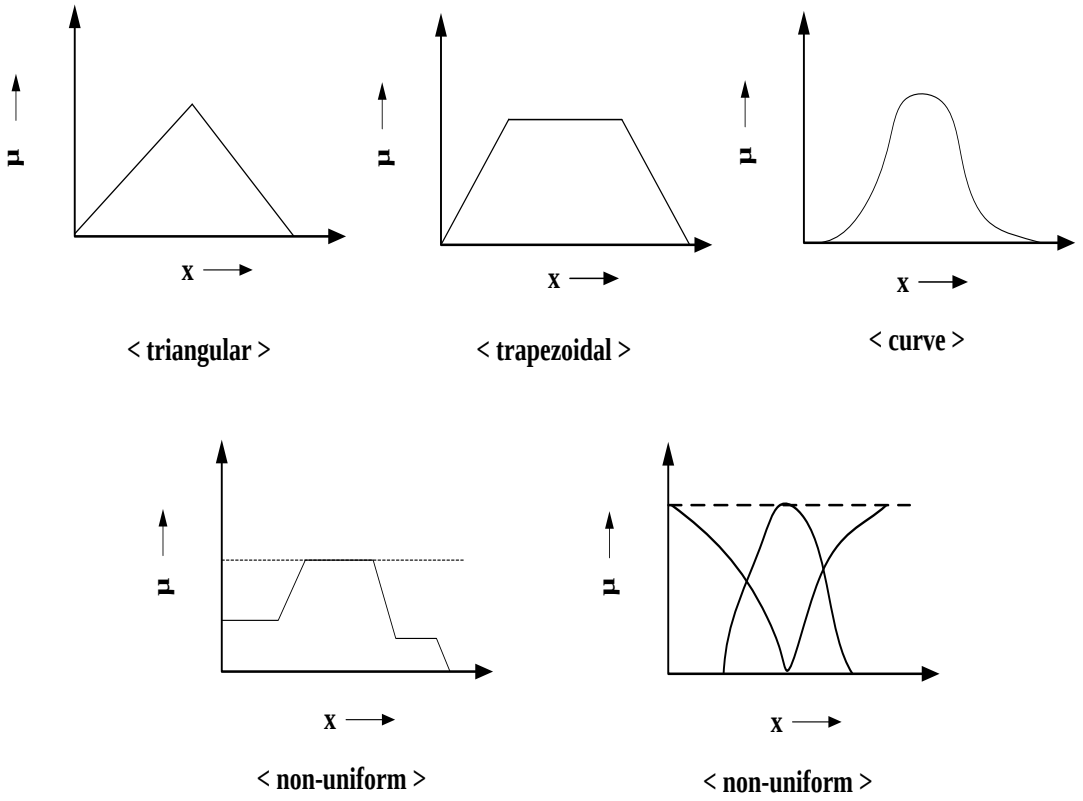
A = Fuzzy set of "Happy family"

B = "Young age"

Fuzzy membership functions

So, membership function on a discrete universe of course is trivial. However, a membership function on a continuous universe of discourse needs a special attention.

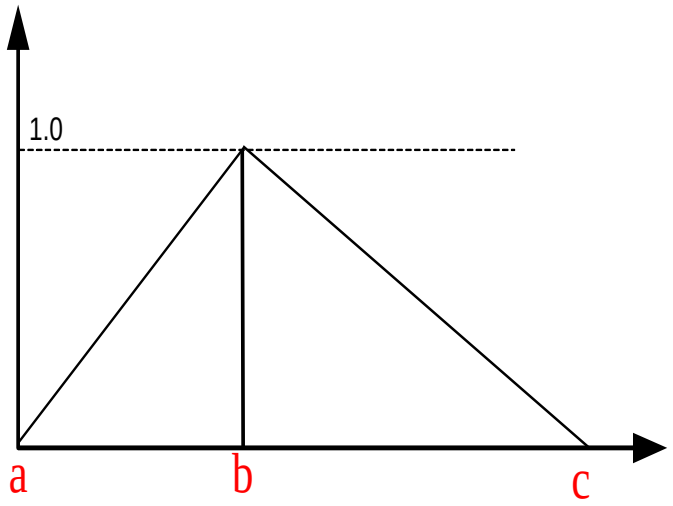
Following figures shows typical examples of membership functions.



Fuzzy MFs : Formulation and parameterization

In the following, we try to parameterize the different MFs on a continuous universe of discourse.

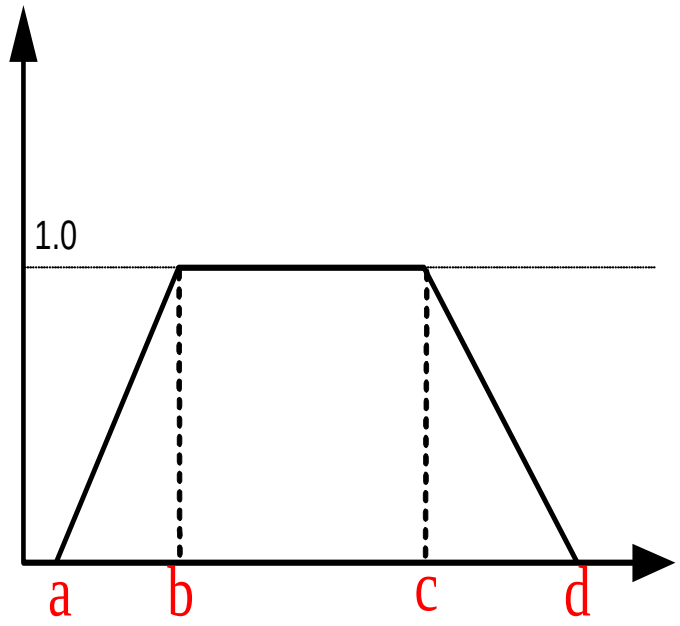
Triangular MFs : A triangular MF is specified by three parameters $\{a, b, c\}$ and can be formulated as follows.



$$\text{triangle}(x; a, b, c) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{a-b} & if\ a \le x \le b \\ \dfrac{c-x}{c-b} & if\ b \le x \le c \\ 0 & if\ c \le x \end{cases}$$

Fuzzy MFs: Trapezoidal

A **trapezoidal MF** is specified by four parameters $\{a, b, c, d\}$ and can be defined as follows:

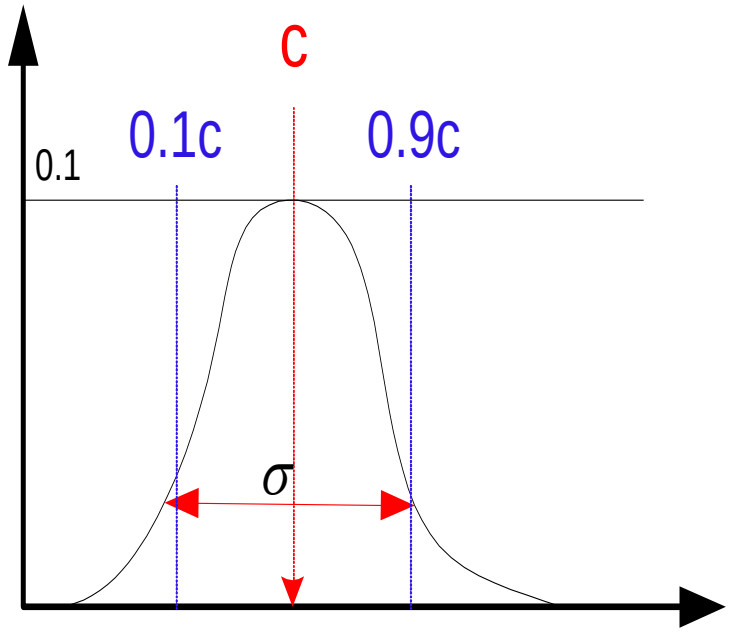


$$\text{trapozoid}\,(x; a, b, c, d) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a} & if\ a \le x \le b \\ 1 & if\ b \le x \le c \\ \dfrac{d-x}{d-c} & if\ c \le x \le d \\ 0 & if\ d \le x \end{cases}$$

Fuzzy MFs: Gaussian

A **Gaussian MF** is specified by two parameters $\{c, \sigma\}$ and can be defined as below:

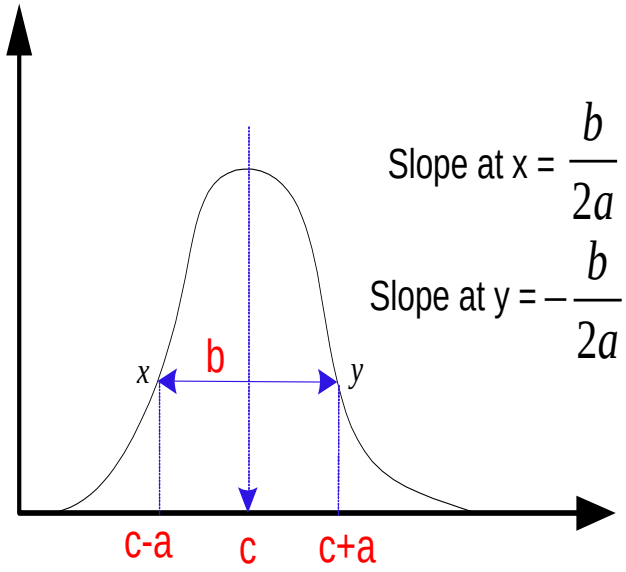
$$gaussian\,(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$



Fuzzy MFs: Generalized bell

It is also called Cauchy MF. A generalized bell MF is specified by three parameters $\{a, b, c\}$ and is defined as:

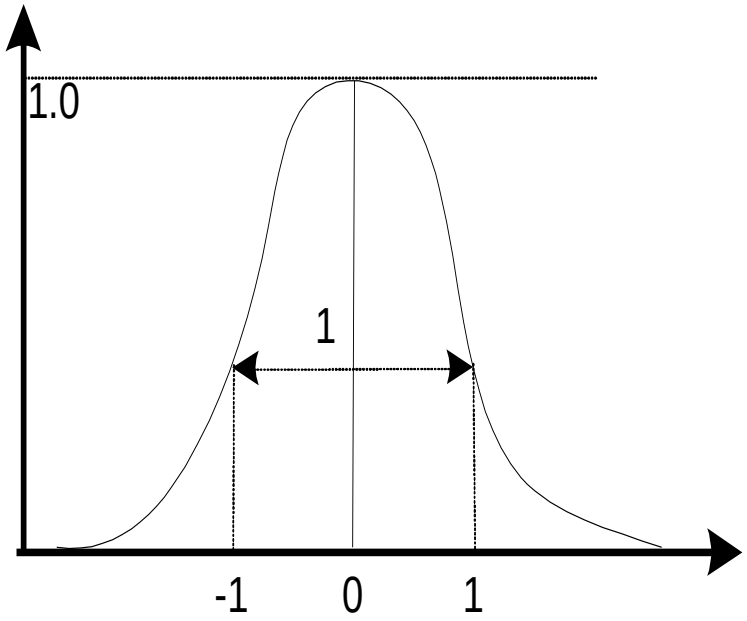
$$bell(x;\ a, b, c) = \frac{1}{1 + \left|\frac{x - c}{a}\right|^{2b}}$$



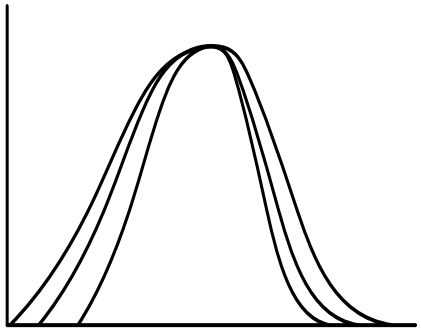
Example: Generalized bell MFs

Example: $\mu(x) = \frac{1}{1+|x|^{2}}$;

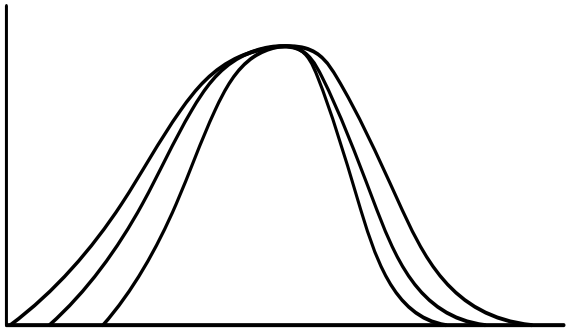
a = b = 1 and c = 0;



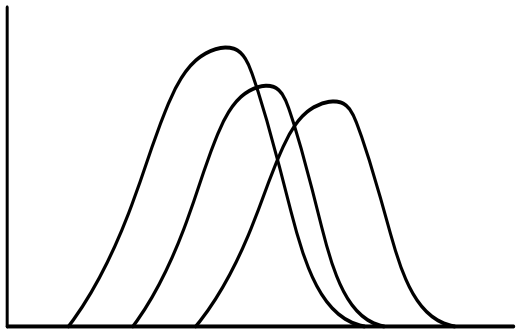
Generalized bell MFs: Different shapes



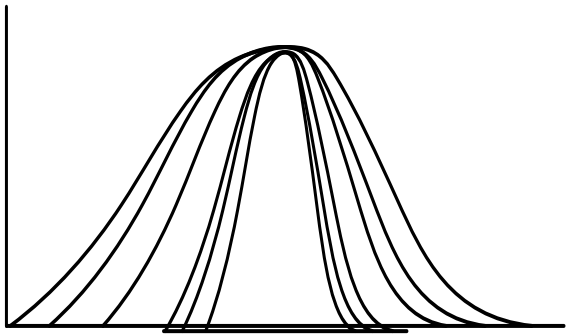
Changing a



Changing b



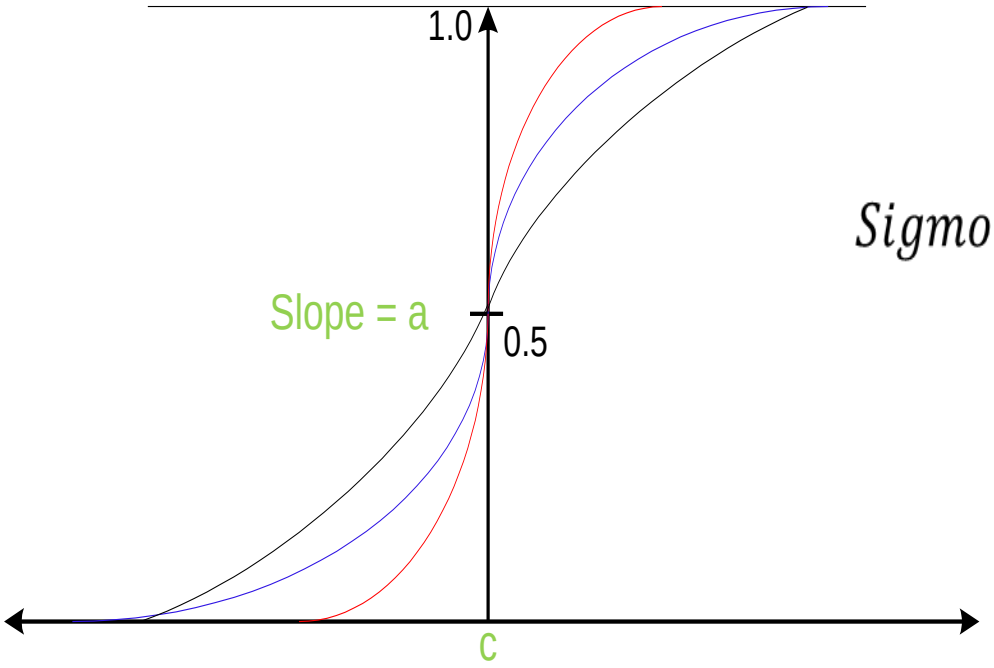
Changing a



Changing a and b

Fuzzy MFs: Sigmoidal MFs

Parameters: $\{a, c\}$; where c = crossover point and a = slope at c;



$$Sigmoid(x;\, a, c) = \frac{1}{1 + e^{-\left[\frac{x-c}{a}\right]}}$$

Fuzzy MFs : Example

Example : Consider the following grading system for a course.

Excellent = Marks ≤ 90

Very good = $75 \leq$ Marks ≤ 90

Good = $60 \leq$ Marks ≤ 75

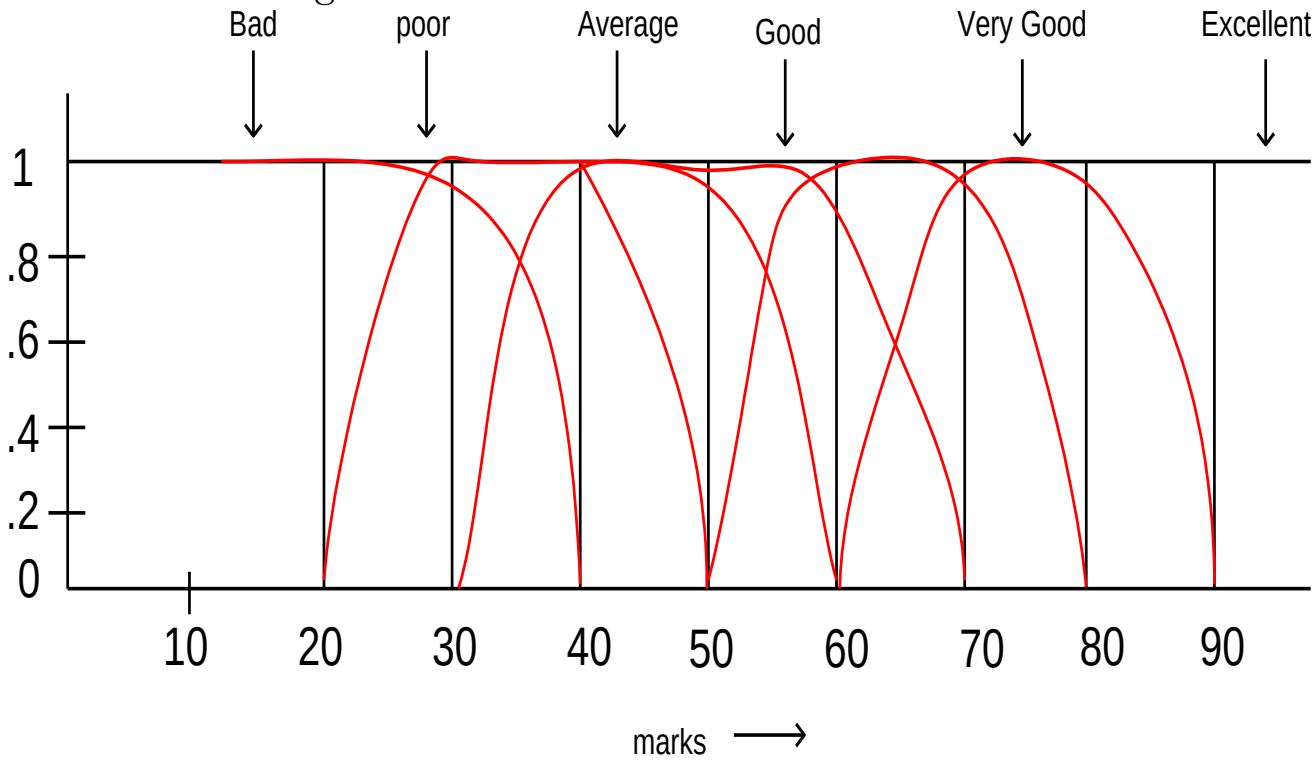
Average = $50 \leq$ Marks ≤ 60

Poor = $35 \leq$ Marks ≤ 50

Bad= Marks ≤ 35

Grading System

A fuzzy implementation will look like the following.



You can decide a standard fuzzy MF for each of the fuzzy grade.