3-D- Stress transformation

Given the state of stress at a point as below

$$\sigma = \begin{bmatrix} 100 & 80 & 0 \\ 90 & -60 & 0 \\ 0 & 0 & 40 \end{bmatrix} \text{kPa}$$

Considering another set of coordinate axes x' y' z' in which z' coincides with z and x' is rotated by 30^0 anticlockwise from x –axis, determine the stress components in the new co-ordinates system.

Solution: (a) Find the direction cosine along x' y' and z' axis

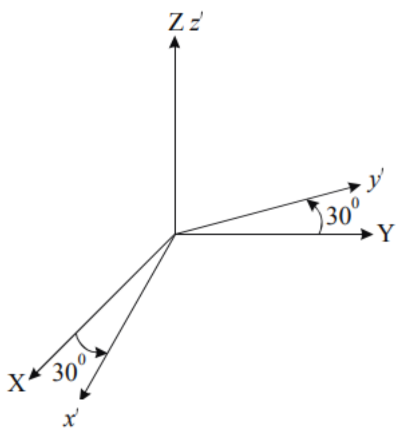

Figure 2.22 Co-ordinate system

	l	m	n
x'	Cos (30^0)	sin (30^0)	0
y'	Cos ($90^0 + 30^0$)	sin ($90^0 + 30^0$)	0
z'	0	0	1

	l	m	n
x'	0.866	0.5	0
y'	-0.5	0.866	0
z'	0	0	1

(b) Find the traction vector on a plane whose normal vector along x' axes

$$\{T\} = [\sigma]\{x'\} = \begin{bmatrix} 100 & 80 & 0 \\ 90 & -60 & 0 \\ 0 & 0 & 40 \end{bmatrix} \begin{Bmatrix} 0.866 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{bmatrix} 100 \times 0.866 + 80 \times 0.5 + 0 \times 0 \\ 90 \times 0.866 - 60 \times 0.5 + 0 \times 0 \\ 0 \times 0.866 + 0 \times 0.5 + 0 \times 40 \end{bmatrix} = \begin{Bmatrix} 126.6 \\ 47.94 \\ 0 \end{Bmatrix}$$

$$Traction\, Vector\, \vec{T} = 126.6\,\hat{\imath} + 47.94\,\hat{\jmath} + 0\,\hat{k}$$

Normal stress on a plane whose normal vector along x' axes is represents as $\boldsymbol{\sigma_{x'x'}}$.

$$\sigma_{x'x'} = \text{Component of traction vector act on plane whose normal vector along } x' - \text{axes in the direction of } x' - \text{axis (i.e. along nomal vector)}$$

$$\sigma_{x'x'} = \text{Dot product of Traction vector and } x' \text{axes}$$

$$= \vec{T} . \overrightarrow{x'}$$

$$= (126.6\,\hat{\imath} + 47.94\,\hat{\jmath} + 0\,\hat{k}).(0.866\hat{\imath} + 0.5\,\hat{\jmath} + 0\,\hat{k})$$

$$= 126.6 \times 0.866 + 47.94 \times 0.5 = 133.61 \text{ kPa}$$

Shear stress on a plane whose normal vector along x' axes and shear stress act along y' axes is represents as $\boldsymbol{\tau_{x'y'}}$

$$\tau_{x'y'} = \text{Component of traction vector act on plane whose normal vector along } x' - \text{axes in the direction of } y' - \text{axis .}$$

$$\tau_{x'y'} = \text{Dot product of Traction vector and } y' \text{axes}$$

$$= \vec{T} . \overrightarrow{y'}$$

$$= (126.6\,\hat{\imath} + 47.94\,\hat{\jmath} + 0\,\hat{k}).(-0.5\hat{\imath} + 0.866\,\hat{\jmath} + 0\,\hat{k})$$

$$= 126.6 \times -0.5 + 47.94 \times 0.866 = -21.784 \text{ kPa}$$

Shear stress on a plane whose normal vector along x' axes and shear stress act along z' axes is represents as $\boldsymbol{\tau_{x'z'}}$

$$\tau_{x'z'} = \text{Component of traction vector act on plane whose normal vector along } x' - \text{axes in the direction of } z' - \text{axis .}$$

$$\tau_{x'z'} = \text{Dot product of Traction vector and } z' \text{axes}$$

$$= \vec{T} . \overrightarrow{z'}$$

$$= (126.6\,\hat{\imath} + 47.94\,\hat{\jmath} + 0\,\hat{k}).(0\hat{\imath} + 0\,\hat{\jmath} + 1\,\hat{k})$$

$$= 126.6 \times 0 + 47.94 \times 0 + 0 \times 0 = 0 \text{ kPa}$$

$$\sigma = \begin{bmatrix} 133.61 & -21.784 & 0 \\ -- & --- & -- \\ --- & --- & --- \end{bmatrix} \text{kPa}$$

Remaining portion do yourself