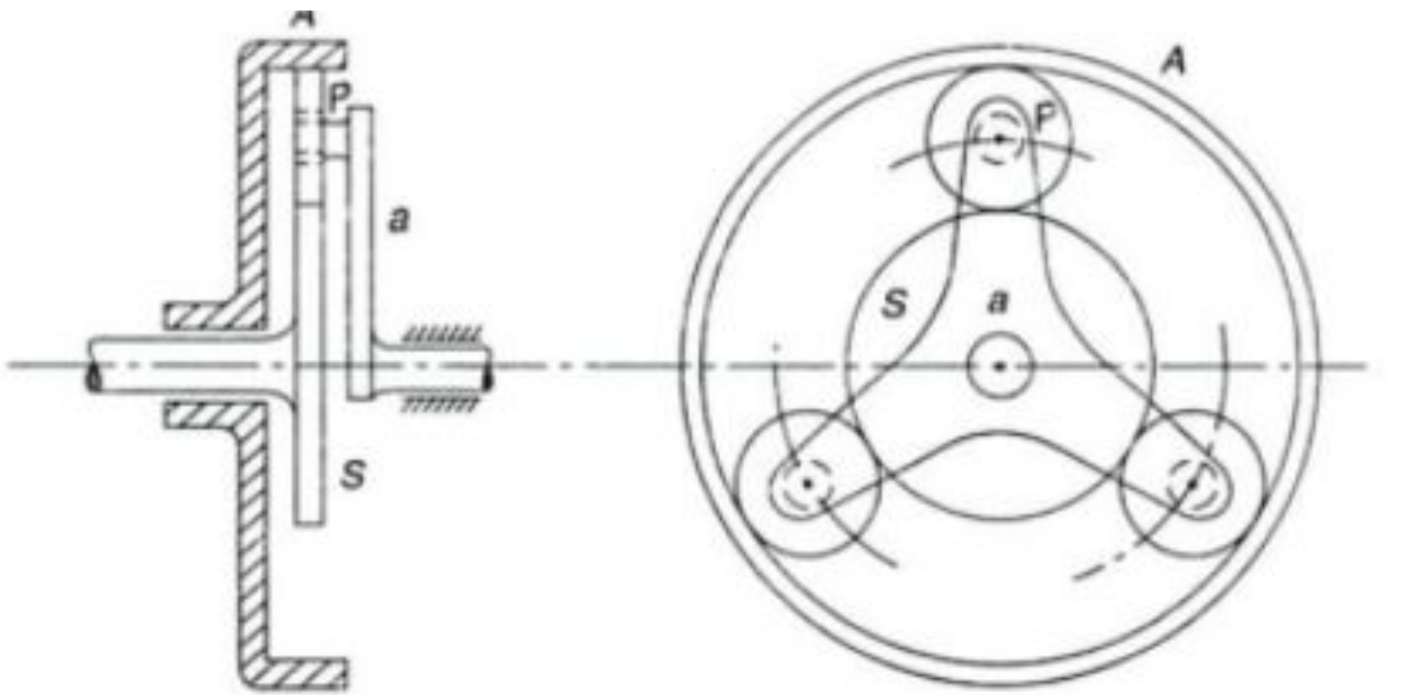
MEE-S301

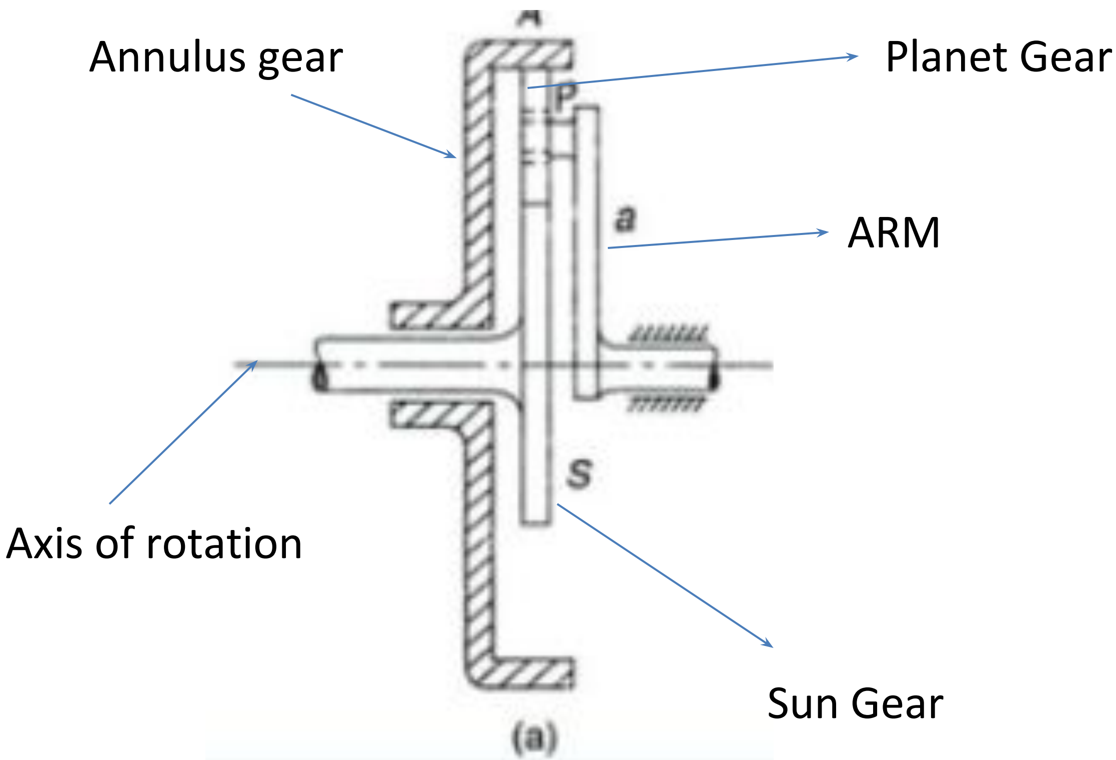
Epicyclic gear train

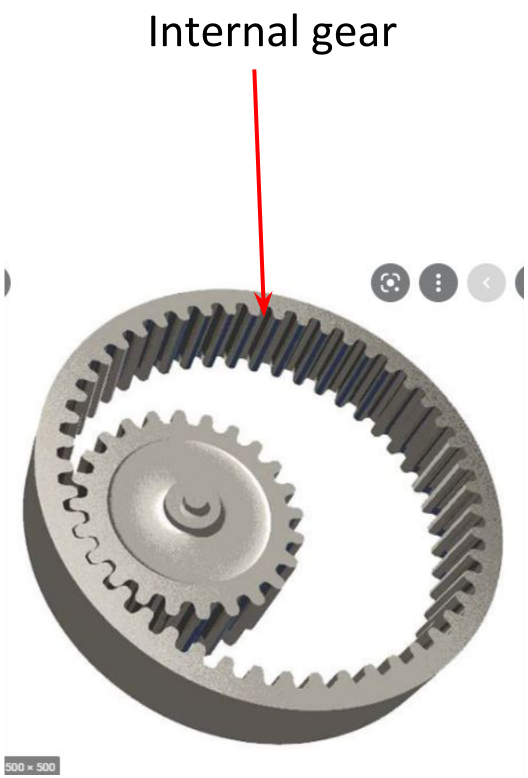
Q1. The annulus A in the gear shown in the fig 1. rotates at 300 rpm about the axis of the fixed wheel S which has 80 teeth. The three-armed spider (only one arm a is shown in figure) is driven at 180 rpm. Determine the number of teeth required on the wheel P.



(a) (b)

First identify gear name



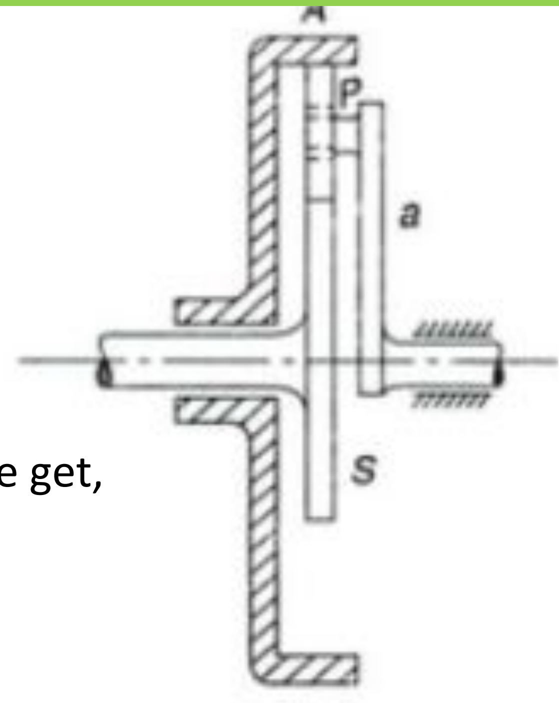


Sun Gear : Sun gear rotates about fixed axis of rotation.

Arm : Arm rotates about fixed axis of rotation.

Planet: Axis of rotation of planet is not fixed.

	Sun gear (S)	Planet gear (P)	Annulus (A)	ARM (a)
Speed (N)	$N_S = 0$	N_P = ??	N_A = 300 rpm	N_{arm} = 180 rpm
No of teeth	$Z_S = 80$	Z_P = ??	Z_A = ??	-



(a)

- Sun gear, planet gear, arm constitute external epicyclic gear train

By using relative velocity method,

$$\frac{N_S - N_{Arm}}{N_P - N_{Arm}} = -\frac{Z_P}{Z_S} \quad [N_S = 0,\ N_{ARM} = 180 \text{ rpm}]$$

$$\frac{0-180}{N_P-180} = -\frac{Z_P}{80} \text{ - - - - (1)}$$

- Internal gear A, planet gear P , arm constitute internal epicyclic gear train

By using relative velocity method,

$$\frac{N_A - N_{Arm}}{N_P - N_{Arm}} = +\frac{Z_P}{Z_A} \quad [N_A = 300 \text{ rpm},\ N_{ARM} = 180 \text{ rpm}]$$

$$\frac{300-180}{N_P-180} = +\frac{Z_P}{Z_A} \text{ - - - - (2)}$$

Divide (1) and (2) we get,

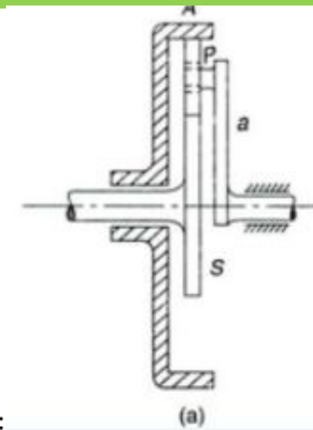
$$\frac{\left(\frac{0-180}{N_P-180}\right)}{\left(\frac{300-180}{N_P-180}\right)} = \frac{-\frac{Z_P}{80}}{\frac{Z_P}{Z_A}}$$

$$\frac{180}{120} = \frac{Z_A}{80}$$

$$Z_A = \frac{3}{2} \times 80 = 120$$

Number of teeth on gear A is equal to 120.

	Sun gear (S)	Planet gear (P)	Annulus (A)	ARM (a)
Speed (N)	$N_S = 0$	N_P = 900 rpm	N_A = 300 rpm	N_{arm} = 180 rpm
No of teeth	$Z_S = 80$	$Z_P = 20$	$Z_A = 120$	-



Since gear S is mesh with gear P. Therefore module of gear S is equal to module of gear P

Module of gear S = $\frac{2r_S}{z_S}$

Module of gear P = $\frac{2r_p}{z_P}$

Similarly gear P is mesh with internal gear A. Therefore module of gear P is equal to module of gear A.

Module of gear P = $\frac{2r_p}{z_P}$

Module of gear A= $\frac{2r_A}{z_A}$

Module of gear S = Module of gear P = Module of gear A

$$\frac{2r_S}{Z_S} = \frac{2r_p}{z_P} = \frac{2r_A}{Z_A} = m$$

Also $r_A = r_s + 2r_P$

$Z_A = Z_S + 2Z_P$

$120 = 80 + 2 \times Z_P$

$Z_P = 20$

$N_P = 900\ rpm$

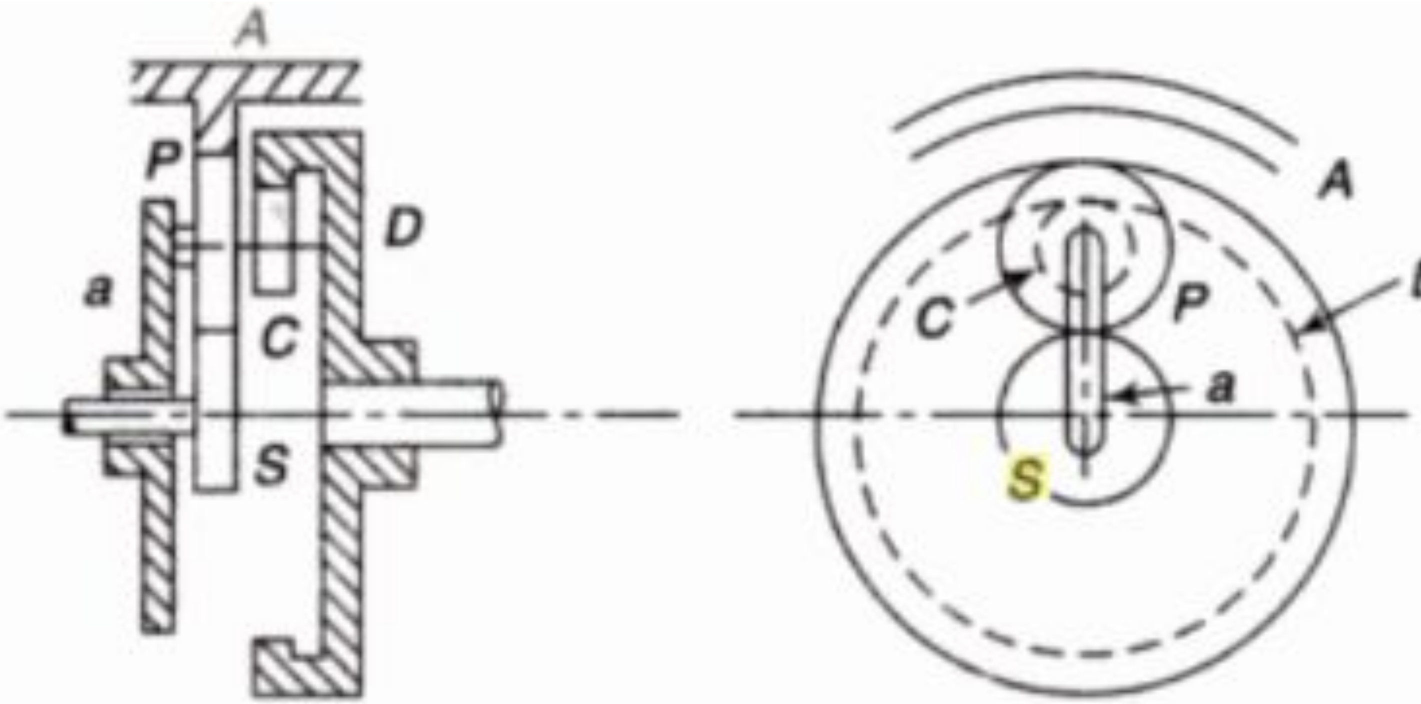
$$\frac{0 - 180}{N_P - 180} = -\frac{Z_P}{80}$$

From this equation we can find speed of planet gear

Problem-2

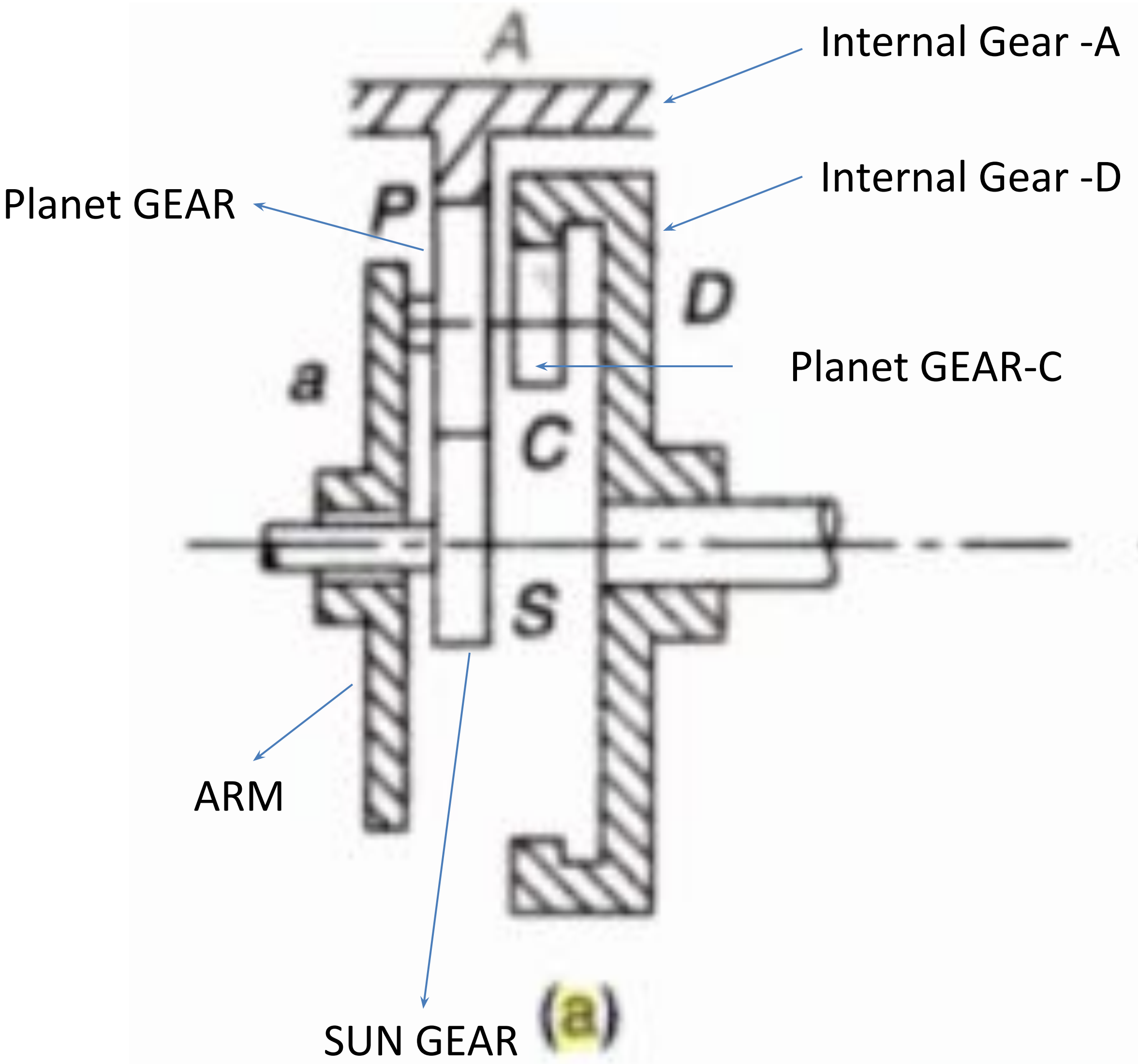
Epicyclic gear train

Q2. In a reduction gear shown in Fig -2 , the input S has 24 teeth. P and C constitute a compound planet a compound planet having 30 and 18 teeth respectively. If all the gears are of the same pitch, find the ratio of the reduction of gear. Assume A to be fixed



(a) (b)

First identify the sun gear, planet gear and arm



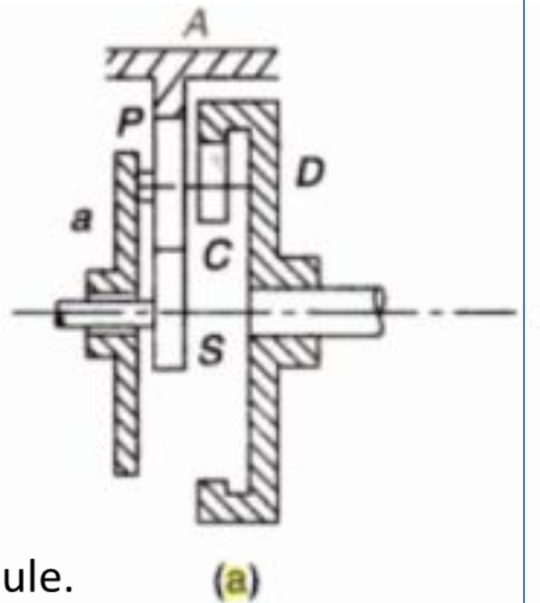
First identify the sun gear, planet gear and arm

	Sun gear (S)	Planet gear (P)	Arm (a)	Ring gear -A	Planet gear -C	Internal gear D
Speed (N) rpm				0		
No of teeth (Z)	24	30	-	84	18	

- Gear A is fixed , $N_A = 0$ rpm
- Gear S , Gear P and arm constitute epicyclic gear train.

$$\frac{N_S - N_{Arm}}{N_P - N_{Arm}} = -\frac{Z_P}{Z_S}$$

$$\frac{N_S - N_{Arm}}{N_P - N_{Arm}} = -\frac{30}{24} \quad ----(1)$$



From figure,

$r_A = r_s + 2r_p$

Since all gear are have same pitch it means it have also same module.

$Z_A = Z_s + 2Z_p$

$Z_A = 24 + 2 \times 30 = 84$

First identify the sun gear, planet gear and arm

	Sun gear (S)	Planet gear (P)	Arm (a)	Ring gear -A	Planet gear -C	Internal gear D
Speed (N) rpm				0		
No of teeth (Z)	24	30	-	84	18	72

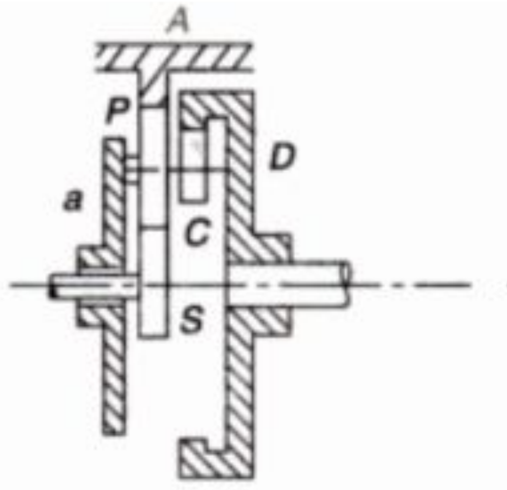
From figure,

$r_D = r_s + r_p + r_c$

Since all gear are have same pitch it means it have also same module.

$Z_D = Z_s + Z_p + Z_c$

$Z_D = 24 + 30 + 18 = 72$



(a)

- Gear P, Gear – A , arm constitute internal epicyclic gear train.

$$\frac{N_A - N_{Arm}}{N_P - N_{Arm}} = +\frac{Z_P}{Z_A}$$

$$\frac{0 - N_{Arm}}{N_P - N_{Arm}} = \frac{30}{84}$$

$$-84N_{Arm} = 30N_P - 30N_{Arm}$$

$$-54N_{Arm} = 30N_P$$

$$N_P = -\frac{54}{30}N_{Arm}$$

First identify the sun gear, planet gear and arm

	Sun gear (S)	Planet gear (P)	Arm (a)	Ring gear -A	Planet gear -C	Internal gear D
Speed (N) rpm				0		
No of teeth (Z)	24	30	-	84	18	72

By equation (1)

$$\frac{N_S - N_{Arm}}{N_P - N_{Arm}} = -\frac{30}{24} \; ----(1)$$

$$\frac{N_S - N_{Arm}}{-\frac{54}{30}N_{Arm} - N_{Arm}} = -\frac{30}{24}$$

$$N_s = N_{arm} - \frac{30}{24} \times -\frac{84}{30} N_{Arm}$$

$$N_s = N_{Arm} + \frac{84}{24} N_{Arm} = \frac{108}{24} N_{Arm}$$

$$\boldsymbol{N_s = \frac{108}{24} N_{Arm}}$$

Since planet gear P and gear C mount on same shaft. Therefore both have same speed.

$$\mathbf{N_P = N_C}$$

Gear D, gear C and arm constitute internal epicyclic gear train

$$\frac{N_C - N_{Arm}}{N_D - N_{Arm}} = +\frac{Z_D}{Z_C} = \frac{72}{18} = 4$$

$$N_D = \frac{N_C - N_{Arm}}{4} + N_{arm} = \frac{N_C}{4} + \frac{3}{4} N_{arm}$$

First identify the sun gear, planet gear and arm

	Sun gear (S)	Planet gear (P)	Arm (a)	Ring gear -A	Planet gear -C	Internal gear D
Speed (N) rpm				0		
No of teeth (Z)	24	30	-	84	18	72

$$N_D = \frac{N_C}{4} + \frac{3}{4} N_{arm}$$

$$N_D = -\frac{54}{120} N_{arm} + \frac{3}{4} N_{arm}$$

$$N_D = \frac{-54 + 3 \times 30}{120} N_{arm}$$

$$N_D = \frac{36}{120} N_{arm}$$

Since planet gear P and gear C mount on same shaft. Therefore both have same speed.

$$N_P = N_C$$

$$\text{Speed reduction} = \frac{Speed\ of\ input\ shaft}{Speed\ of\ output\ shaft}$$

$$= \frac{N_S}{N_D} = \frac{\frac{108}{24} N_{arm}}{\frac{36}{120} N_{arm}} = 15$$

Ans: 15 (it means speed of output shaft reduced

By a factor of 15)