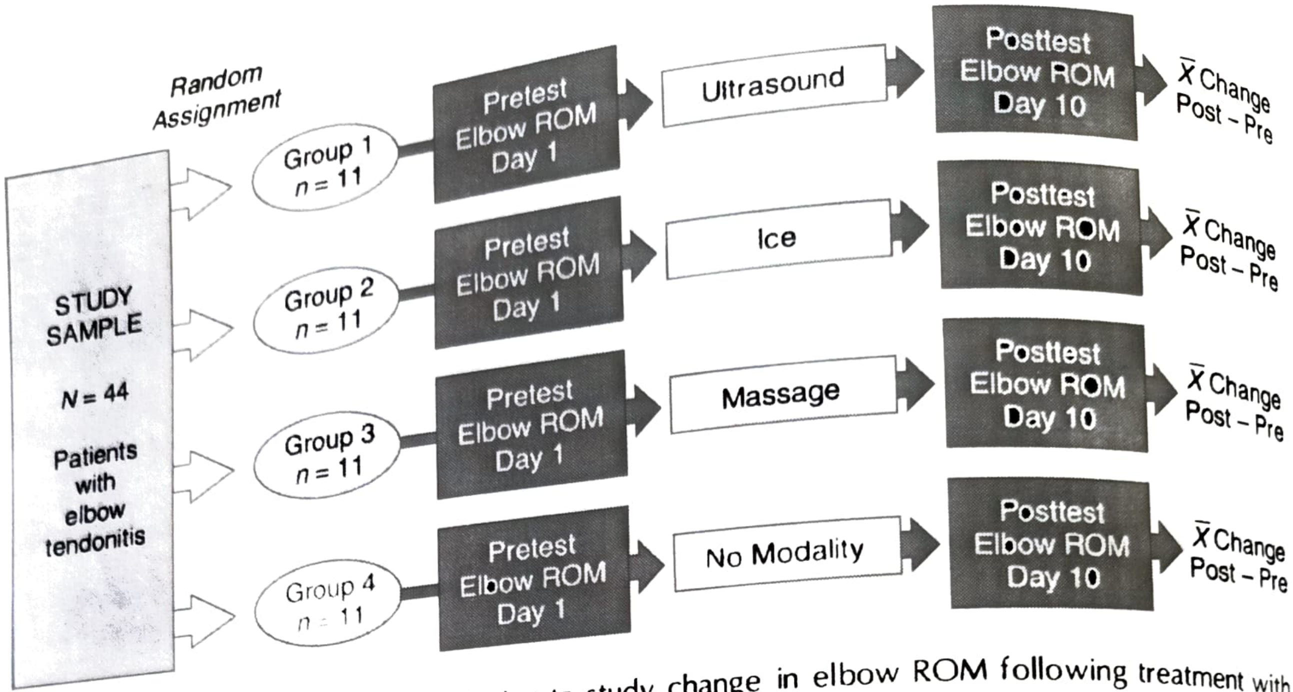

FIGURE 20.1 One-way multi-group design to study change in elbow ROM following treatment with different modalities in patients with tendonitis.

Total Sum of Squares

To estimate the total variability in these data, consider the set of 44 scores as one total sample, ignoring group assignment. We can calculate a mean for this total sample, called the **grand mean,** $\overline{X}_G$, around which all 44 scores will vary. For the data in Table 20.1, the sum of all 44 scores is 1,638, and $\overline{X}_G = 37.23$. The sum of squares for this total sample ($\Sigma(X - \overline{X}_G)^2$) represents the deviations of each individual score from the grand mean. This *total sum of squares* (SS_t) reflects the *total variability* that exists within this set of 44 scores. This variability is illustrated in Figure 20.2A, showing the entire distribution of scores above and below the grand mean.

Partitioning Sum of Squares

As we have described before, total variability in a set of data can be attributed to two sources: a treatment effect (*between* the groups), and unexplained sources of variance, or **error variance**, among the subjects (*within* the groups). As its name implies, the analysis of variance partitions the total variance within a set of data (SS_t) into these two components. The *between-groups sum of squares* (SS_b) reflects the spread of group means around the grand mean. The larger this effect, the greater the separation between the groups. The within-groups or *error sum of squares* (SS_e) reflects the spread of scores within each group around the group mean, or the differences among subjects. In Figure 20.2B, we can see that the means for groups 1 and 2 are close together, and both appear separated from groups 3 and 4. The spread of scores in group 4 appears to be less than in the other groups.

Because hand calculations are complex, computer programs will most often be used to obtain results for an ANOVA. For those who like to see the math, computational

TABLE 20.1 ONE-WAY ANALYSIS OF VARIANCE FOR INDEPENDENT SAMPLES: CHANGE IN ELBOW ROM (IN DEGREES) FOLLOWING TREATMENT FOR TENDONITIS ($k = 4, N = 44$)

A. DATA

	Grp	ROM		Grp	ROM
1	1	23	23	3	47
2	1	54	24	3	49
3	1	52	25	3	29
4	1	33	26	3	33
5	1	48	27	3	45
6	1	52	28	3	29
7	1	58	29	3	43
8	1	31	30	3	19
9	1	43	31	3	34
10	1	47	32	3	27
11	1	45	33	3	33
12	2	44	34	4	19
13	2	52	35	4	14
14	2	53	36	4	23
15	2	52	37	4	14
16	2	33	38	4	36
17	2	46	39	4	29
18	2	56	40	4	37
19	2	42	41	4	22
20	2	43	42	4	19
21	2	29	43	4	18
22	2	48	44	4	35

	Group 1 US	Group 2 Ice	Group 3 Massage	Group 4 Control	Total
$\sum X$	486.00	498.00	388.00	266.00	1,638.00
n	11	11	11	11	44
$\bar{X}$	44.18	45.27	35.27	24.18	37.23

B. OUTPUT

Test of Homogeneity of Variances

	Levene Statistic	df1	df2	❶ Sig.
LENGTH	.321	3	40	.810

ANOVA

LENGTH	Sum of Squares	df	Mean Square	F	❷ Sig.
Between Groups	3158.09	3	1052.70	11.89	.000
❸ Within Groups	3541.64	40	88.54		
Total	6699.73	43			

❶ As with the *t*-test, the Levene statistic indicates that there is no significant difference (p = .810) between the variances across the four groups.

❷ The probabilities associated with the *F* test do not distinguish between one and two-tailed tests. Because the probability is less than .05, we reject H_0 and conclude that there is a significant difference among groups.

❸ In different programs, the source of variance "Within Groups" may also be called "Error" or "Residual" variance.

formulae for calculating total, between-groups and error sums of squares are shown in Table 20.2.

The *F* Statistic

Degrees of Freedom

The total degrees of freedom (df_t) within a set of data will always be one less than the total number of observations, in this case $N - 1$. In our example, $N = 44$ and $df_t = 43$. The number of degrees of freedom associated with the between-groups variability (df_b)

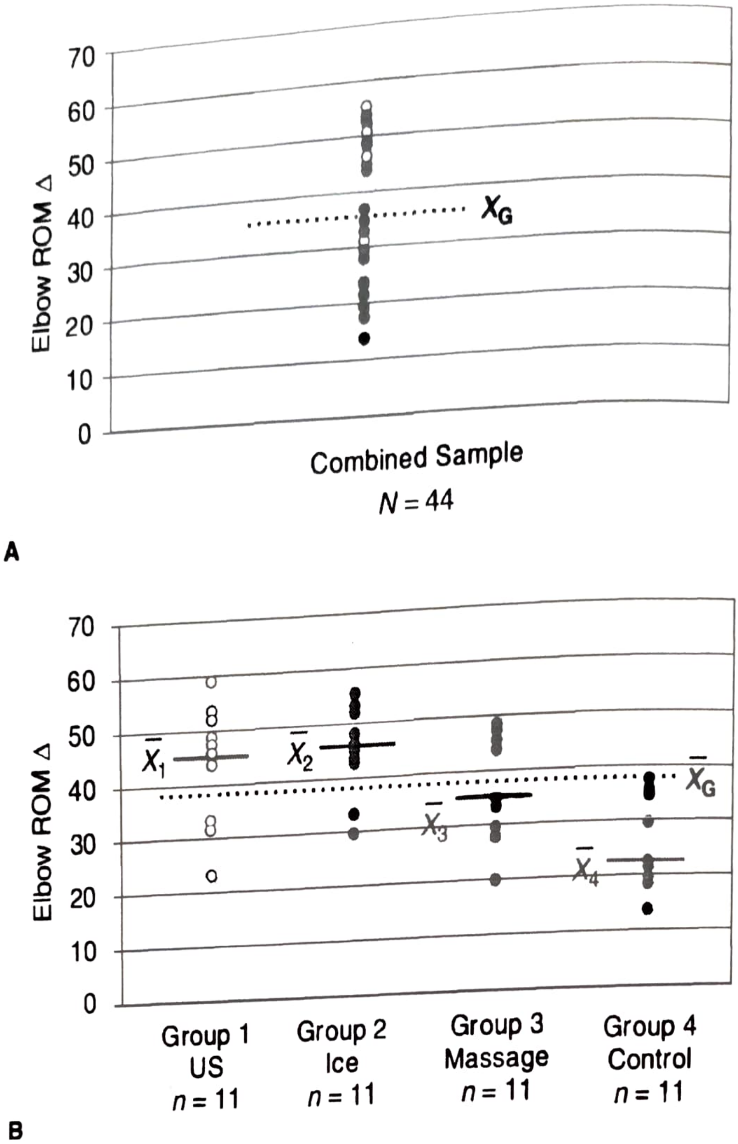

FIGURE 20.2 Scores from tendonitis study (Table 20.1). **A.** The total variance in the sample is reflected in the distribution of scores from all four groups around the grand mean ($\bar{X}_G$). **B.** The between-groups variance is determined by the distribution of the four group means. The error variance reflects the variability of scores within each of the groups around the group mean.

is one less than the number of groups ($k - 1$), in this case $df_b = 3$. There are $n - 1$ degrees of freedom within each group, so that the number of degrees of freedom for the within-groups error variance (df_e) for all groups combined will be $(n_1 - 1) + (n_2 - 1) + \ldots + (n_k - 1)$, or $N - k$. For the data in Table 20.1, $df_e = 44 - 4 = 40$. The degrees of freedom for the separate variance components are additive, so that $(k - 1) + (N - k) = (N - 1)$.

Mean Squares

The concepts of between-groups and within-groups variability are once again used to define a statistical ratio. These sources of variability are defined as between-groups and error sums of squares. We convert the sums of squares to a variance estimate, or

TABLE 20.2 **CALCULATION OF SUMS OF SQUARES AND F STATISTIC FOR ONE-WAY ANALYSIS OF VARIANCE (DATA FROM TABLE 20.1)**

A. DATA

	Group 1 US $N = 11$	Group 2 Ice $N = 11$	Group 3 Massage $N = 11$	Group 4 Control $N = 11$
ΣX	486	498	388	266
ΣX^2	22,654	23,252	14,590	7,182
$\bar{X}$	44.18	45.27	35.27	24.18
s	10.87	8.40	9.50	8.66

B. COMPUTATION OF SUMS OF SQUARES

$\Sigma X = 486 + 498 + 388 + 266 = 1{,}638$ $\quad \Sigma X^2 = 22{,}654 + 23{,}252 + 14{,}590 + 7{,}182 = 67{,}678$

$$SS_t = \sum X^2 - \frac{(\sum X)^2}{N} = 67{,}678 - \frac{(1638)^2}{44} = 6{,}699.73 \qquad (20.1)$$

$$SS_b = \sum \frac{(\sum X_i)^2}{n} - \frac{(\sum X)^2}{N} = \left[\frac{(486)^2}{11} + \frac{(498)^2}{11} + \frac{(388)^2}{11} + \frac{(266)^2}{11}\right] - \frac{(1{,}638)^2}{44} = 3{,}158.09 \qquad (20.2)^a$$

$$SS_e = \sum X^2 - \sum \frac{(\sum X_i)^2}{n} = 67{,}678 - \left[\frac{(486)^2}{11} + \frac{(498)^2}{11} + \frac{(388)^2}{11} + \frac{(266)^2}{11}\right] = 3{,}541.64 \qquad (20.3)$$

C. COMPUTATION OF F STATISTIC

$$df_b = k - 1 = 4 - 1 = 3 \qquad MS_b = \frac{SS_b}{df_b} = \frac{3158.09}{3} = 1{,}052.70$$

$$F = \frac{MS_b}{MS_e} = \frac{1052.70}{88.54} = 11.89$$

$$df_e = N - k = 44 - 4 = 40 \qquad MS_e = \frac{SS_e}{df_e} = \frac{3541.64}{40} = 88.54$$

D. HYPOTHESIS TEST

$H_0: \mu_1 = \mu_2 = \mu_3 \quad H_1: \mu_1 \neq \mu_2 \neq \mu_3 \qquad _{(\alpha=.05)}F_{(3,40)} = 2.84$

Reject H_0

[a]The term $\frac{(\Sigma X_i)^2}{n}$ in equation 20.2 represents the square of the sum of scores within each individual group divided by the number of subjects in that group.

mean square (MS), by dividing each sum of squares by its respective degrees of freedom. A mean square can be calculated for the between- and error-variance components as follows:

$$MS_b = \frac{SS_b}{df_b} \tag{20.4a}$$

$$MS_e = \frac{SS_e}{df_e} \tag{20.4b}$$

The F-Ratio

Mean square values are used to calculate the ***F* statistic** as a ratio of the between-groups variance to the error variance:

$$F = \frac{MS_b}{MS_e} \tag{20.5}$$

When H_0 is true and no treatment effect exists, the total variance in a sample is due to error, and MS_e is equal to or larger than MS_b, yielding an *F*-ratio of approximately 1.0 or less. When H_0 is false and the treatment effect is significant, the between-groups variance is large, yielding an *F*-ratio greater than 1.0. The larger the *F*-ratio, the greater the difference between the group means relative to the variability within the groups. In our example, $F = 11.89$, as shown in Table 20.2C.

Critical Values of F

Like *t*, the calculated *F*-ratio is compared to a critical value to determine its significance. Table A.3 in the Appendix contains critical values of *F* at $\alpha = .05$. Because mean squares are based on squared values, the *F*-ratio cannot be a negative number, and therefore, we do not distinguish tails for an *F* test.

The critical value of *F* for the desired α is located in the table by the degrees of freedom associated with the between-groups and error variances, with df_b across the top of the table and df_e along the side. For our example, $df_b = 3$ and $df_e = 40$ (always given in that order). Therefore, from Table A.3,

$$_{(.05)}F_{(3,40)} = 2.84$$

We compare this critical value with our calculated value, $F = 11.89$. The calculated value must be *greater than or equal to* the critical value to achieve statistical significance. In this case, we can reject H_0.

A significant *F*-ratio does not indicate that each group is different from all other groups. Actually, it only tells us that there is a significant difference between at least two of the means (largest versus smallest). At this point, a separate test must be done to determine exactly where the significant differences lie. Various **multiple comparison tests** are described for this analysis in the next chapter. When the *F*-ratio is smaller than the critical value, H_0 is not rejected and no further analyses are appropriate.