## Half Adder

Half Adder- The addition of 2 bits is done using a combination circuit called a Half adder. The input variables areaugend and addend bits and output variables are sum \& carry bits. A and B are the two input bits.


## Truth Table -

| $A$ | $B$ | Sum | Carry |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

## Logical Expression:

1) For Sum:


Sum $=$ A X-OR B

## 2) For Carry:

| $A$ | 0 | 1 |
| :---: | :---: | :---: |
| $B$ | 0 | 0 |
| 0 | 0 | 1 |
| 1 | 0 |  |

## Carry $=A$. B

## Implementation:



Note: Half adder has only two inputs and there is no provision to add a carry coming from the lower order bits when multi addition is performed.

## Full Adder

Full Adder : It is the adder which adds three inputs and produces two outputs. The first two inputs are A and B and the third input is an input carry as C-IN. The output carry is designated as COUT and the normal output is designated as S which is SUM. A full adder logic is designed in such a manner that can take eight inputs together to create a byte-wide adder and cascade the carry bit from one adder to the another.


## Truth Table:

| Inputs |  |  | Outputs |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}$ | B | $\mathbf{C}-\mathbb{N}$ | Sum | C-Out |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Logical Expression for SUM:

$$
\begin{aligned}
& =\text { A' }^{\prime} \text { ' C-IN + A' B C-IN' + A B' C-IN' + A B C-IN } \\
& =\text { C-IN (A' B' + A B) + C-IN' (A' B + A B') } \\
& =\text { C-IN X-OR A X-OR B } \\
& =\sum \mathrm{m}(1,2,4,7)
\end{aligned}
$$

Logical Expression for C-OUT:

$$
\begin{aligned}
& =\mathrm{AB}+\mathrm{B} \text { C-IN + A C-IN } \\
& =\Sigma \mathrm{m}(3,5,6,7)
\end{aligned}
$$

## Implementation of Full Adder using Half Adders

2 Half Adders and a OR gate is required to implement a Full Adder.


With this logic circuit, two bits can be added together, taking a carry from the next lower order of magnitude, and sending a carry to the next higher order of magnitude.

Q1. Design and implement the following using half adder only.
$\mathrm{D}=\mathrm{A}$ XOR B XOR C
$\mathrm{E}=\mathrm{A}^{\prime} \mathrm{BC}+\mathrm{AB}^{\prime} \mathrm{C}=\left(\mathrm{A}^{\prime} \mathrm{B}+\mathrm{AB}^{\prime}\right) \mathrm{C}=(\mathrm{A}$ XOR B) C
$\mathrm{F}=\mathrm{AB}{ }^{\prime} \mathrm{C}+\left(\mathrm{A}^{\prime}+\mathrm{B}^{\prime}\right) \mathrm{C}=\mathrm{ABC} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}$
$\mathrm{G}=\mathrm{ABC}$


## BCD ADDER

## Procedure for BCD addition

1. Add two BCD numbers using ordinay binary addition.
2. If four-bit sum is equal to or less than 9 , no correction is needed. The sum is in proper BCD form.
3. If the four-bit sum is greater than 9 or if a carry is generated from the four-bit sum, the sum is invalid.
4. To correct the invalid sum, add $0110_{2}$ to the four-bit sum. If a carry results from this addition, add it to the next higher-order BCD digit.

Let $\mathrm{A}_{3}, \mathrm{~A}_{2}, \mathrm{~A}_{1}, \mathrm{~A}_{0}$ and $\mathrm{B}_{3}, \mathrm{~B}_{2}, \mathrm{~B}_{1}, \mathrm{~B}_{0}$ are two four bit BCD numbers. these two BCD no. are added along with the carry in using 4 - bit parallel binary adder as given below

$$
\begin{array}{lllll}
1^{\text {st }} \text { no. } & \mathrm{A}_{3} & \mathrm{~A}_{2} & \mathrm{~A} 1 & \mathrm{~A} 0 \\
2^{\text {nd }} \text { no. } & \mathrm{B} 3 & \mathrm{~B}_{2} & \mathrm{~B} 1 & \mathrm{~B} 0 \\
& \mathrm{~S} 3 & \mathrm{~S} 2 & \mathrm{~S} 1 & \mathrm{~S} 0 \\
& \mathrm{C} 3 & \mathrm{C} 2 & \mathrm{C} 1 & \mathrm{C} 0 \\
& & & & \\
& \mathrm{~S} 3 & \mathrm{~S} 2 & \mathrm{~S} 1 & \mathrm{~S} 0 \\
0 & \mathrm{Y} & \mathrm{Y} & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0
\end{array}
$$



The result obtained will be a binary no i.e; ( C3 S3 S2 S1 S0) The logical circuit AND / OR gate is used to check the binary result obtained.
If (S3 S2 S1 S1 ) greater than 9 than, $\mathrm{A}=1$ or $\mathrm{B}=1$, Hence $\mathrm{Y}=1$. similarly if last carry C 3 , then $\mathrm{Y}=1$.
So when $Y=1,6\left(\begin{array}{lll}0 & 1 & 1\end{array}\right)$ is added with a binary result. If $\mathrm{Y}=0$ then, $0\left(\begin{array}{lll}0 & 0 & 0\end{array}\right)$ is added with a binary result .

Using half adder zero $\left(\mathrm{HA}_{0}\right)$ full adder $\left(\mathrm{FA}_{4}\right)$ and $\left(\mathrm{HA}_{1}\right)$ the addition of two numbers is performed as given below-

| carry generated | C 2 | C 1 |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $1^{\text {st }}$ No. | S3 | S2 | S1 | S0 |
| $2^{\text {ND }}$ NO. | 0 | Y | Y | 0 |
|  | Z3 | Z2 | Z1 | Z0 |

$------->\quad$ BCD RESULT

