BOOLEAN THEOREMS

Boolean algebraic theorems are the theorems that are used to change the form of a Boolean expression.

- Identity Law : $X+0=X$ and $X.1=X$
- Complement Law : $X+X'=1$ and $X.X'=0$
- Idempotent Law : $X+X=X$ and $X.X=X$
- Dominant Law : $X+1=1$ and $X.0=0$
- Involution Law : $(X')'=X$

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- Commutative Law : X+Y=Y+X and X.Y=Y.X
- Associative Law : X+(Y+Z)=(X+Y)+Z and X.(Y.Z)=(X.Y).Z
- Absorption Law :X+(X.Y)=X and X.(X+Y) =X
- Distributive Law : X.(Y+Z)=X.Y+X.Z and X+Y.Z=(X+Y).(X+Z)

De-morgan's Theorem

(i) (X+Y)'=X'.Y'

(ii) (X.Y)' =X'+Y'

Example

(i) $ABC' + AB'C + A'BC + ABC$

$= ABC' + ABC + AB'C + ABC + A'BC + ABC$

$= AB(C' + C) + AC(B + B') + BC(A + A')$

$= AB + BC + AC$

(ii) $((XYZ + X'Y')' + YZ)'$

$= (XYZ + X'Y').(YZ)'$

$= ((XYZ + X'Y')Y') + ((XYZ + X'Y').Z')$

$= XYZ.Y' + X'Y'Y' + XYZ.Z' + X'Y'Z'$

$= 0 + X'Y' + 0 + X'Y'Z'$

$= X'Y'(1 + Z')$

$= X'Y'.1$

$= X'Y'$

Q. Design & Implement XOR Gate using NAND Gate

Sol. F= x XOR y

= x'y+xy'

= x'y+xy'+xx'+yy'

= (x+y) (x'+y')

Now we need to implement this circuit using NAND gates

F= (x+y)(xy)'

= x. (xy)' + y. (xy)'

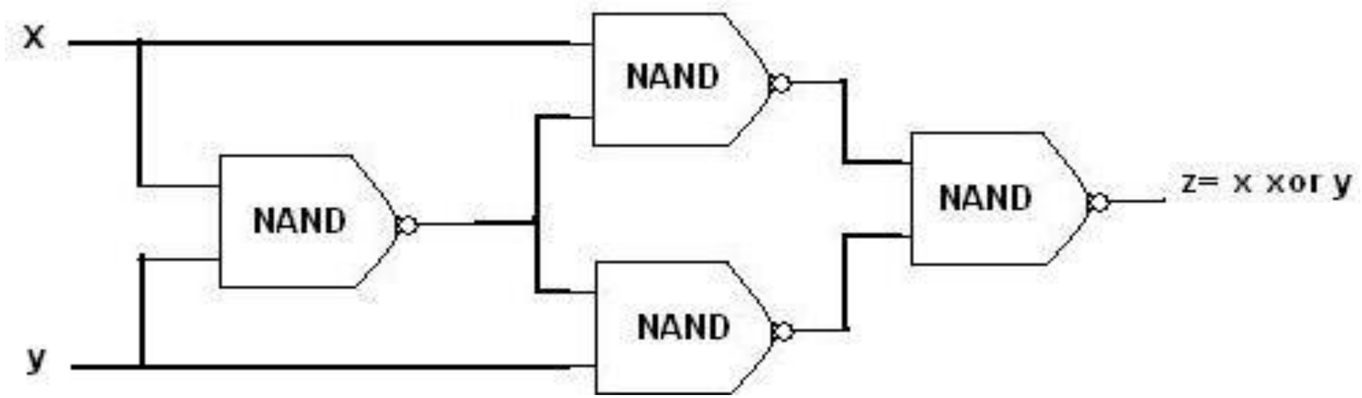
Take compliment

F'= (x. (xy)' + y. (xy)')'

= (x. (xy)')'. (y. (xy)')

Take compliment again

F=((x. (xy)')'. (y. (xy)'))'



Now we can implement this using NAND gates

So, we get that we need minimum of 4 NAND gates to implement XOR gate.