FLOW THROUGH ORIFICES AND MOUTHPIECES

INTRODUCTION

An orifice is an opening in the wall or base of a vessel through which the fluid flows. The top edge of the orifice is always below the free surface (If the free surface is below the top edge of the orifice, becomes a weir)

A mouthpiece is an attachmnent in the form of a small tube or pipe fixed to the orifice (the length of pipe extension is usually 2 to 3 times the orifice diameter) and is used to increase the amount of discharge.

- Orifices as well as mouthpieces are used to measure the discharge

CLASSIFICATION OF ORIFICES

The orifices are classified as follows

1. According to size:

(i) Small orifice (ii) Large orifice

An orifice is termed small when its dimensions are small compared to the head causing flow. The velocity does not vary appreciably from top to the bottom edge of the orifice and is assumed to be uniform. The orifice is large if the dimensions are comparable with the head causing flow. The variation in the velocity from the top to the bottom edge is considerable.

2. According to shape

(i) Circular orifice (ii) Rectangular orifice (iii) Square orifice (iv) Triangular orifice.

3. Shape of upstream edge

(i) Sharp-edged orifice (ii) Bell-mouthed orifice.

4. According to discharge conditions

(i) Free discharge orifices

(ii) Drowned or submerged orifices (a) Fully submereged (b) Partially submerged.

Note. An orifice or a mouthpiece is said to be discharging free when it discharges into atmosphere. It is said to be submerged when it discharges into another liquid.

FLOW THROUGH AN ORIFICE

Fig. 8.1 shows a small circular orifice with sharp edge in the side wall of a tank discharging free into the atmosphere. Let the orifice be at a depth H below the free surface. As the fluid flows through the orifice, it contracts and attains a parallel form (i.e., stream lines become parallel) at a distance d/2 from the plane of the orifice. The point at which the stream lines first become parallel is termed as vena-contracta (the cross-sectional area of the jet at the vena contracta is less than that of orifice). Beyond this section, the jet diverges and is attracted in the downward direction by gravity.

Considering points 1 and 2 as shown in **Fig. 8.1** and applying Bernoulli's theorem, we have:

$$\frac{P_1}{w} + \frac{V_2^1}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

But, $P_1 = P_2 = P_a$ (P_a = atmospheric pressure)

$z_1 = z_2 + \mathrm{H}$

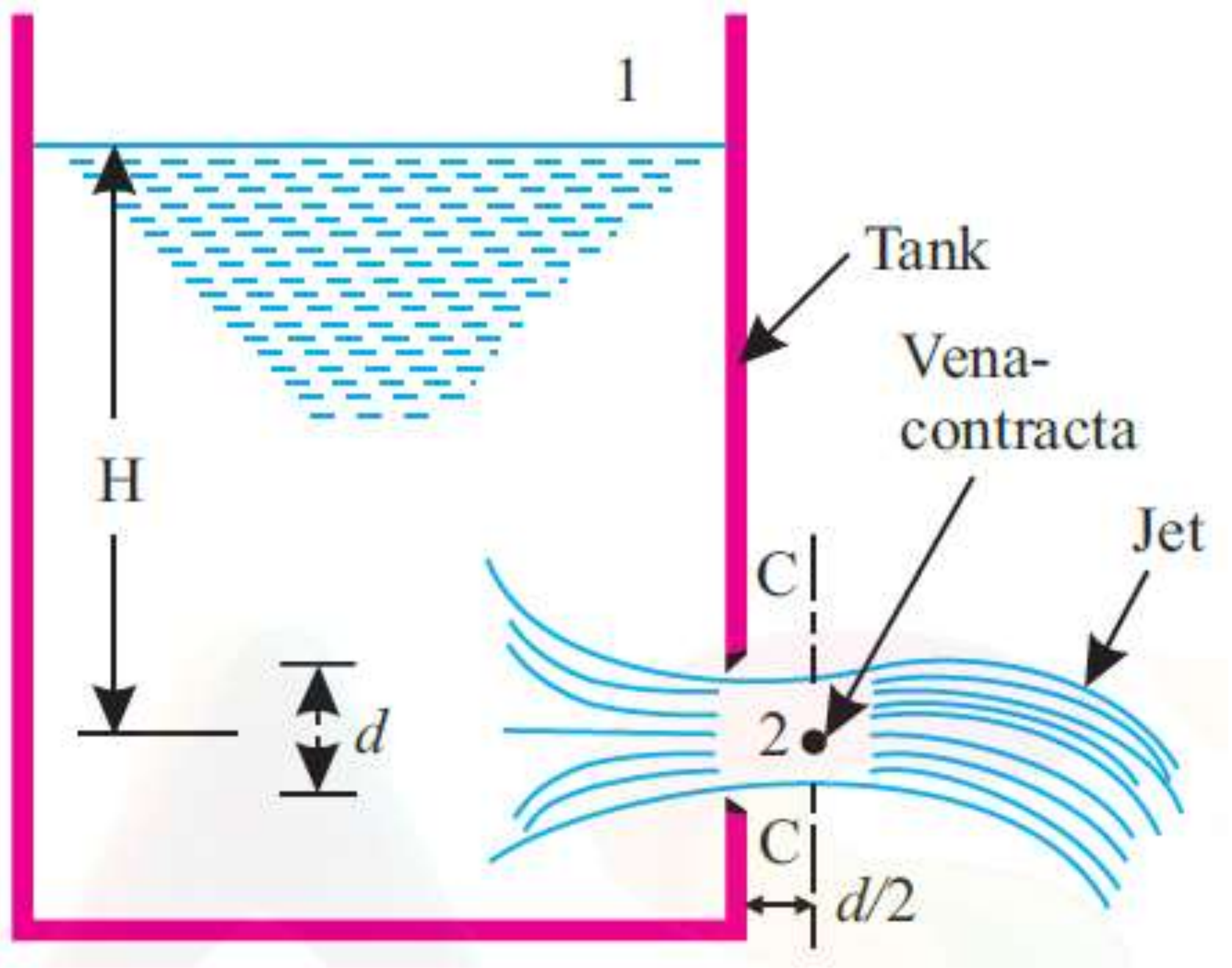
Further, if the cross-sectional area of the tank is very large, the liquid at point 1 is practically standstill and hence V1 = 0

Thus, $\frac{V_2^2}{2g} = \mathrm{H}$

$$V_2 = \sqrt{2gH}$$

Equation (8.1) is know as Torricelli's theorem.

Note. In the problems of orifices it is covenient to work in terms of gauge pressures.


1
Tank
Vena-
contracta
Jet
H
d
C
C
2
d/2

HYDRAULIC CO-EFFICIENTS

The hydraulic co-efficients (or orifice co-efficients) are enumerated and discussed below :

1. Co-efficient of contraction, C_c
2. Co-efficient of velocity, C_v
3. Co-efficient of discharge, C_d
4. Co-efficient of resistance, C_r .

8.4.1. Co-efficient of Contraction (C_c)

The ratio of the area of the jet at vena-contracta to the area of the orifice is known as Co-efficient of contraction. It is denoted by C_c.

Let, a_c = Area of jet at vena contracta, and

a = Area of orifice.

Then, $$C_c = \frac{a_c}{a} \qquad ..(8.2)$$

The value of C_c varies slightly with the available head of the liquid, size and shape of the orifice; in practice it varies from 0.613 to 0.69 but the average value is taken as 0.64.

8.4.2. Co-efficient of Velocity (C_v)

The ratio of actual velocity (V) of the jet at vena-contracta to the theoretical velocity (V_{th}) is known as Co-efficient of velocity. It is denoted by C_v and mathematically, C_v is given as:

C_v = Actual velocity of jet at vena contracta (V) /Theoretical velocity (V_{th})

i.e., C_v = V / $\sqrt{2gH}$ ... (8.3)

[where, V = Actual velocity, and

H = Head under which the fluid flows out of the orifice

The value of C_v varies from 0.95 to 0.99,depending upon the shape of orifice and the head of liquid under which the flow takes place. For sharp-edged orifices the value of C_v is taken as 0.98.

8.4.3. Co-efficient of Discharge

The ratio of actual discharge (Q) through an arifice to the theorerical discharge,(Q_{th}) is known as Co-efficient of discharge. It is dinoted by C_d.

Mathematically, C_d = Actual discharge (Q) / Theoretical discharge (Q_{th})

= Actual area × actual velocity / Theoretical area × theoretical velocity

= Actual area / Theoretical area x actual velocity x theoretical velocity

$$\therefore C_d = C_c \times C_v \qquad \dots \ (8.4)$$

The value of C_d varies from 0.62 to 0.65 depending upon size and the shape of the orifice and the head of liquid under which the flow takes place.

8.4.4. Co-efficient of Resistance (C_r)

The ratio of loss of head (or loss of kinetic energy) in the orifice to the head of water (actual kinetic energy) available at the exit of the orifice is known as Co-efficient of resistance. It is denoted by C_r.

Mathematically, C_r = Loss of head in the orifice / Head of water

The loss of head in the orifice takes place, because the walls of the orifice offer some resistance to the liquid, as it comes out. While solving numerical problems C_r is generally neglected.

8.4. EXPERIMENTAL DETERMINATION OF HYDRAULIC CO-EFFICIENTS

8.5.1. Determination of Co-efficient of Velocity (C_v).

A tank containing water at a constant level, maintained by a costant supply. Let the water flow out of the tank through an orifice, fitted in one side of the tank. Let the section C–C represents the point of vena contracta. Consider a particle of water in the jet at P.

Let, x = Horizontal distance travelled by the particle in time 't',

y = Vertical distance between C-C and P,

V = Actual velocity of the jet at vena-contracta, and

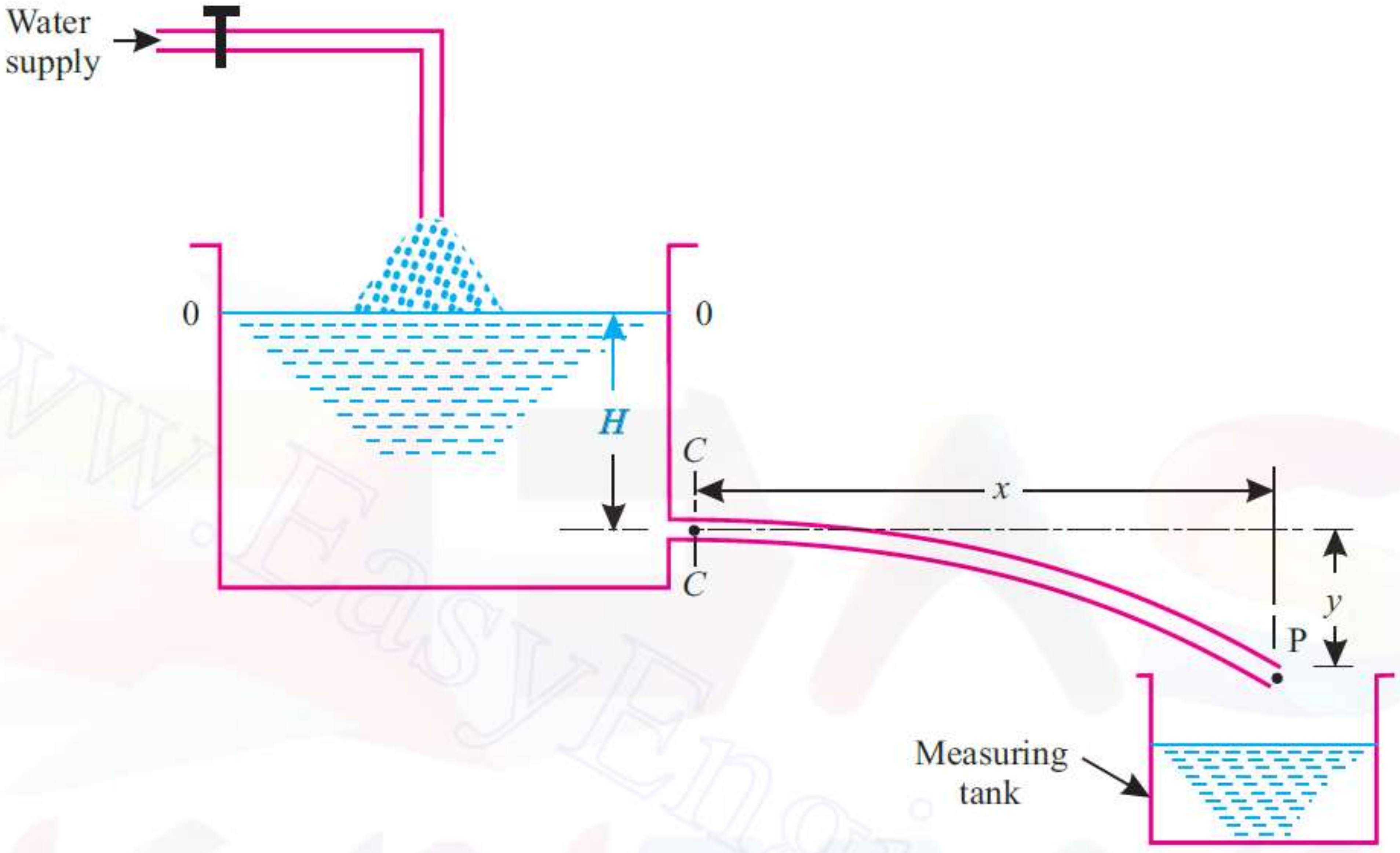
H = Constant water head.

Then, horizontal distance, x = V × t ...(i)

and, vertical distance, y = $\frac{1}{2}\, gt^2$ ...(ii)

From eqn. (i), t = $\frac{x}{V}$

Substituting this value of 't' in eqn. (ii),we get:


Water
supply
0
0
H
C
C
x
y
P
Measuring
tank

$$y = \frac{1}{2} g \times \left(\frac{x}{V}\right)^2 = \frac{gx^2}{2V^2}$$

$\therefore$

$$V^2 = \frac{gx^2}{2y} \text{ or } V = \sqrt{\frac{gx^2}{2y}}$$

But, theoretical velocity, $V_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity,

$$C_v = \frac{V}{V_{th}} = \frac{\sqrt{\frac{gx^2}{2y}}}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$$

i.e.

$$C_v = \frac{x}{\sqrt{4yH}} \qquad \text{...(8.5)}$$

8.5.2. Determination of Co-efficient of Discharge (C_d)

The water flowing through the orifice under the constant head H is collected in a measuring tank for a known time 't'. The rise of water level in the measuring tank is noted down. Then actual discharge through the orifice,

Q = Area of measuring tank x rise of water level in the measuring tank / Time (t)

Theoretical discharge, Q_{th} = Area of orifice x $\sqrt{2gH}$

Hence, $$C_d = \frac{Q}{Q_{th}} = \frac{Q}{a\,X\sqrt{2gH}}$$

8.5.3. Determination of Co-efficient of Contraction (Cc)

The co-efficient of contraction (C_c) can be found from the following relation:

$$C_d = C_c \times C_v$$

$$C_c = \frac{C_d}{C_v}$$

8.5.4. Loss of head in Orifice Flow

The loss of head through an orifice can be determined by applying the Bernoulli's equation between points *O* and *C* (Fig. 8.2).

$$\frac{p_0}{w} + \frac{V_0^2}{2g} + z_0 = \frac{p_C}{w} + \frac{V_C^2}{2g} + z_C + \text{losses}$$

Substituting the proper values, we get:

$$0 + 0 + H = 0 + \frac{V^2}{2g} + 0 + h_f$$

Where, V is the actual flow velocity through the orifice.

$$\therefore \quad h_f = H - \frac{V^2}{2g} = H\left(1 - \frac{V^2}{2gH}\right) \quad \text{...[8.8(a)]}$$

$$= H(1 - C_v^2)$$

$$\text{or} \quad h_f = \frac{V^2}{2g}\left(\frac{2gH}{V^2} - 1\right) = \frac{V^2}{2g}\left(\frac{1}{C_v^2} - 1\right) \quad \text{...[8.8(b)]}$$

DISCHARGE THROUGH A LARGE RECTANGULAR ORIFICE

When the available hetad of a liquid is less than 5 times the height of the orifice, the orifice is called a large orifice. In case of a small orifice, the velocity is considered to be constant in the entire cross- section and the discharge can be calculated by the formula Q = C_d X a X $\sqrt{2gH}$. But in case of a large orifice, the velocity of a liquid, flowing through the orifice, varies with the available head of the liquid and hence Q cannot be calculated as mentioned above (i.e. Q = C_d X a X $\sqrt{2gH}$).

Consider a large rectangular orifice in one side of the tank discharging water freely into the atmosphere, as shown in Fig. 8.9.

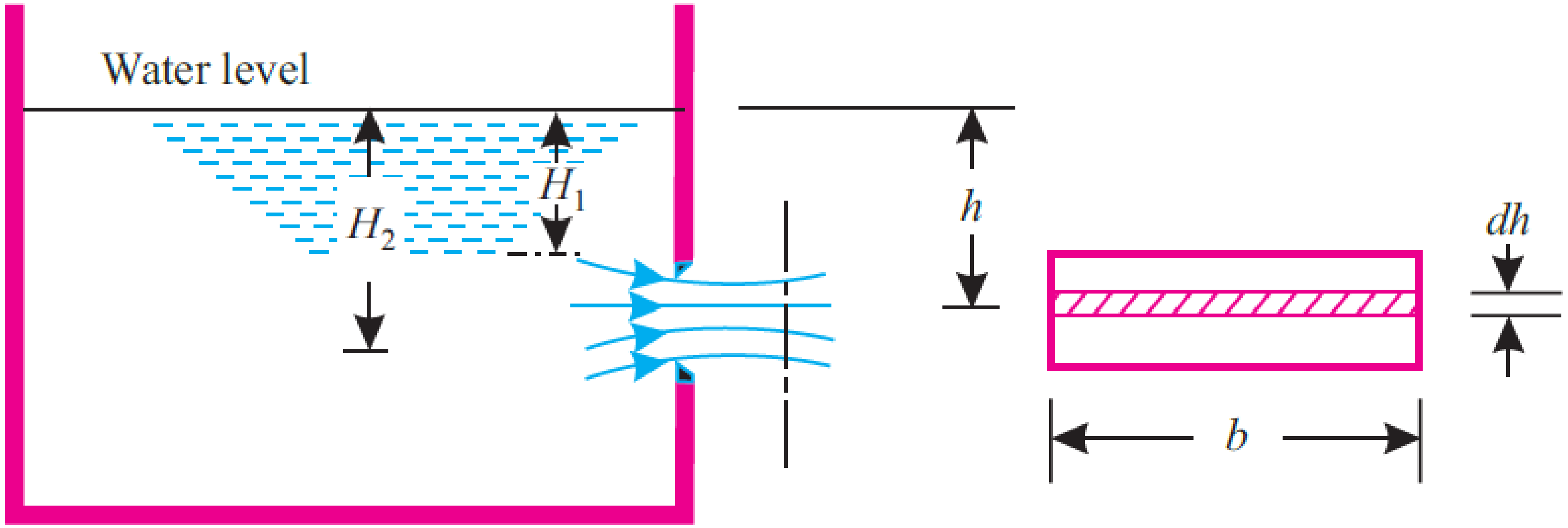
Let, H_1 = Height of liquid above the top of the orifice,

H_1 = Height of liquid above the bottom of the orifice,

b = Breadth of the orifice, and

C_d = Co-efficient of discharge.

Consider an elementary horizontal strip of depth ‘dh’ at depth of ‘h’ below the water level as shown in Fig. 8.9.



∴ Area of the strip = b . dh

Theoretical velocity of water through the strip = $\sqrt{2gH}$

∴ Discharge through the strip, dQ = C_d × area of strip × velocity

= C_d × b x dh x $\sqrt{2gH}$

= C_d . b . dh $\sqrt{2gH}$

Total discharge through the whole orifice may be found out by integrating the above equation between the limits $H_1$and H_2.

$\therefore$ $$Q = \int_{H_1}^{H_2} C_d \,.\, b \,.\, dh \ \sqrt{2gH}$$
$$= C_d \,.\, b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}$$
$$= C_d \,.\, b \times \sqrt{2g} \, [\frac{h^{3/2}}{3/2}] \, H_2/H_1$$
$$= 2/3 \, C_d \,.\, b\sqrt{2g} \, (H_2^{3/2} - H_1^{3/2})$$

8.7 DISCHARGE THROUGH FULLY SUBMEROGED ORIFICE

If an orifice has its whole of the outlet side submerged under liquid so that it discharges a jet of liquid into the liquid of the same kind then it is known as fully submerged (or drowned) orifice. Consider a fully submerged orifice as shown in

Let, H_1 = Height of water (on the upstream side) above the top of the orifice,

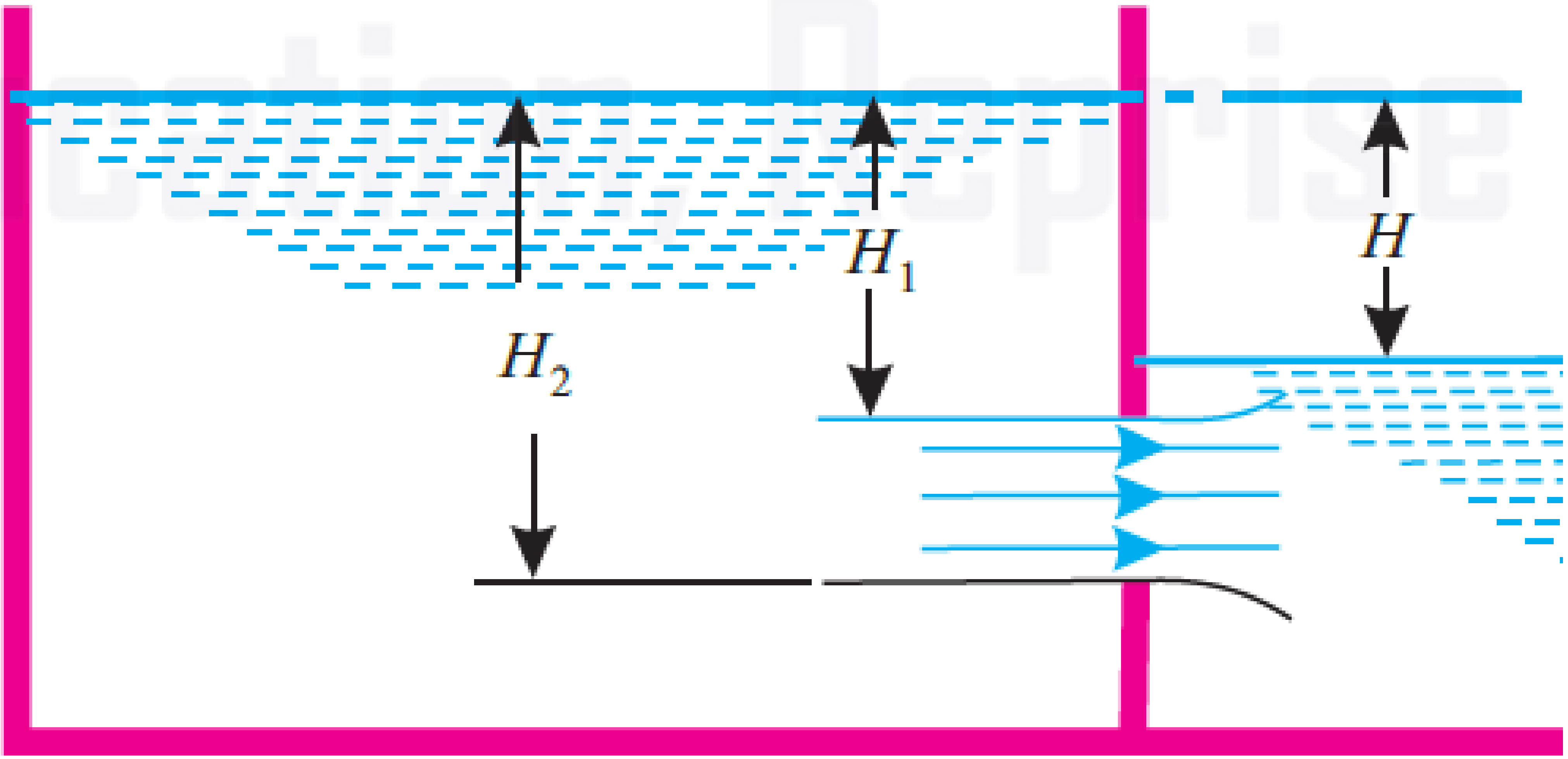
H_2 = Height of water (on the upstream side) above the bottom of the orifice,

H = Difference between the two water levels on either side of the orifice,

b = Width of orifice, and

C_d = Co-efficent of discharge.

$\therefore$ Area of the orifice = b $(H_2 - H_1)$



H
H_1
H_2

We know that theoretical velocity of water through the orifice = $\sqrt{2gH}$

∴ Actual velocity of water = $C_v \sqrt{2gH}$

Since in this case co-efficient of contraction is 1, therefore, taking C_d equal to Cv' we find that

the actual velocity of water = $C_d \times \sqrt{2gH}$

∴ Discharge through the orifice,

Q = Area of orifice × actual velocity

$= b\,(H_2 - H_1) \times C_d \sqrt{2gH}$

$= C_d \cdot b\,(H_2 - H_1) \times \sqrt{2gH}$ ... (8.10)

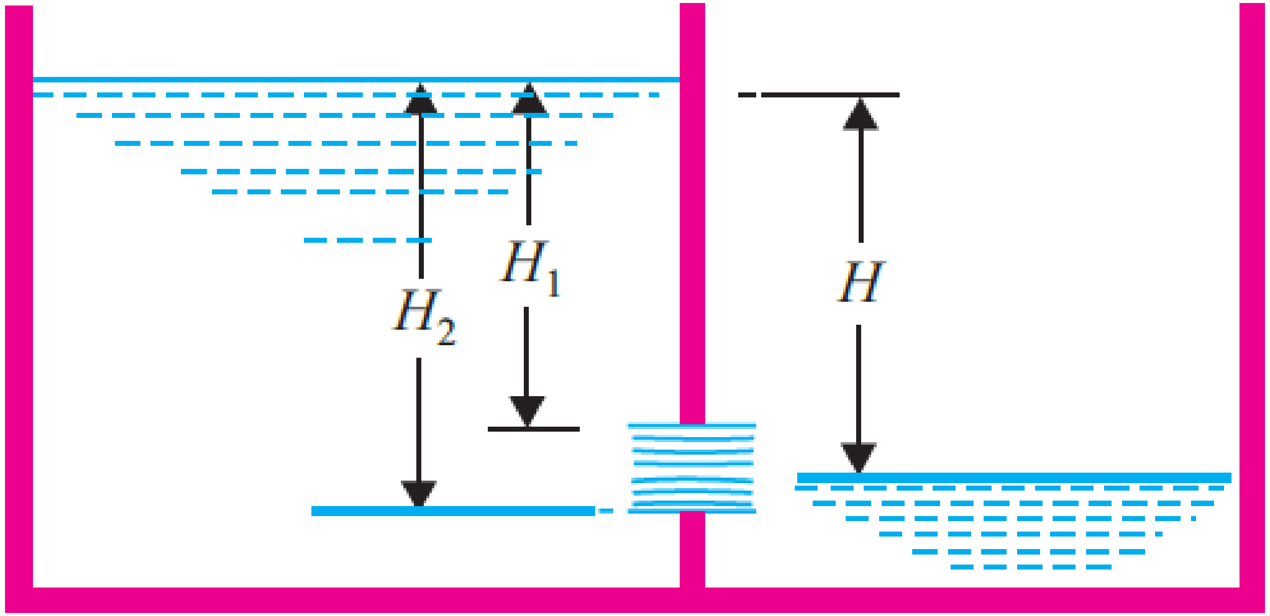
Sometimes, depth of submerged orifice (d) is given instead of H_1 and H_2. In such cases, the discharge,

$Q = C_d \cdot b \cdot d \sqrt{2gH}$ ... (8.11)

DISCHARGE THROUGH PARTIALLY SUBMERGED ORIFICE

If the outlet side of an orifice is only partly submerged (or drowned) under liquid then it is known as **partially submerged** (*or drowned*) **orifice** (Fig. 8.11). The upper portion behaves as an *orifice discharging free,* while the lower portion behaves as a submerged orifice. The total discharge is determined by computing separately the discharges through the free and the submerged portions and then adding together the two discharges thus computed.

Fig. 8.11. Partially submerged orifice.



Discharge through the submerged portion,

$$Q_1 = C_d . b.(H_2 - H) \times \sqrt{2gH}$$

and, the discharge through the free portion,

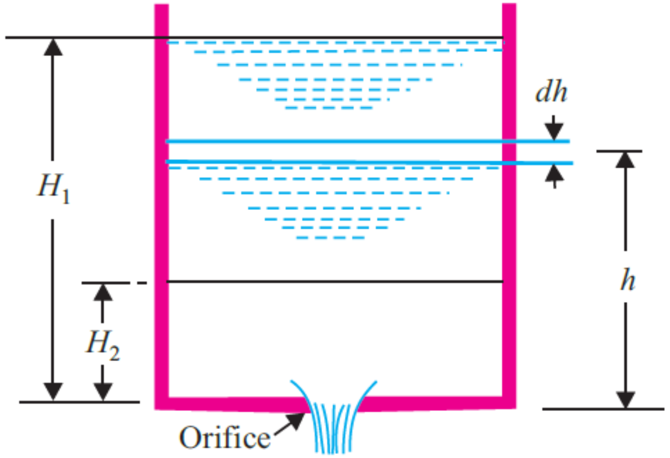
$$Q_2 = \frac{2}{3} C_d . b . \sqrt{2g} (H^{3/2} - H_1^{3/2}$$

Total discharge $Q = Q_1 + Q_2$

$$= C_d . b.(H_2 - H) \times \sqrt{2gH} + \frac{2}{3} C_d . b . \sqrt{2g} (H^{3/2} - H_1^{3/2} \quad ...(8.12)$$

TIME REQUIRED FOR EMPTYING A TANK THROUGH AN ORIFICE AT ITS BOTTOM

Consider a tank, of uniform cross-sectional area, containing some liquid, and having an orifice at its bottom as shown in Fig. 8.13.



Let, A = Cross-sectional area of the tank,

a = Area of the orifice,

H_1 = Initial height of liquid,

H_2 = Final height of liquid

T = Time in seconds, required to bring the level from H_1 to H_2

Let at some instant the height of the liquid be h above the orifice and let the liquid surface fall by an amount dh after a small interval for time dt.

Then, volume of the liquid that has passed the tank in time dt,

$$dq = -A \cdot dh \qquad \text{...}(i)$$

(– ve sign of dh is taken because the value of h decreases when the discharge increases). Also, theoretical velocity through the orifice, $v = \sqrt{2gH}$

∴ Discharge through the orifice in a small interval of time dt,

dq = Co-efficient of discharge × area × theoretical velocity × time.

$$= C_d \,.\, a \,.\, \sqrt{2gH} \,.\, dt \qquad \text{...}(ii)$$

Equating (i) and (ii), we get:

$$-A \cdot dh = C_d \,.\, a \,.\, \sqrt{2gH} \,.\, dt$$

$$\therefore \qquad dt = \frac{-A.dh}{C_d.a.\sqrt{2gh}} = \frac{-A\,(h^{-1/2})\,dh}{C_d.a\sqrt{2g}}$$

Time taken (T) to lower the level from H_1 to H_2 is calculated by integrating the above equation between the limits H_1 and H_2.

$$i.e. \qquad T = \int_{H_1}^{H_2} \frac{-A\,(h^{-1/2})dh}{C_d.a.\sqrt{2g}} = \frac{-A}{C_d.a\,\sqrt{2g}} \int_{H_1}^{H_2} h^{-1/2}\,dh$$

$$= \frac{-A}{C_d.a.\sqrt{2g}} \left[\frac{h^{1/2}}{1/2}\right]_{H_1}^{H_2} = \frac{-2A}{C_d.a\,\sqrt{2g}} \left[h^{1/2}\right]_{H_1}^{H_2}$$

$$= \frac{-2A}{C_d.a.\sqrt{2g}} - \left[\sqrt{H_2} - \sqrt{H_1}\right] = \frac{2A\,(\sqrt{H_1} - \sqrt{H_2})}{C_d.a\,\sqrt{2g}} \qquad ...(8.13)$$

If the tank is to be emptied completely, then $H_2 = 0$

$$\text{and,} \qquad T = \frac{2A\sqrt{H_1}}{C_d.a\sqrt{2g}} \qquad ...(8.14)$$

TIME REQUIRED FOR EMPTYING A HEMISPHERICAL TANK

Consider a hemispherical tank containing some liquid and fitted with an orifice at its bottom as shown in the Fig. 8.19.

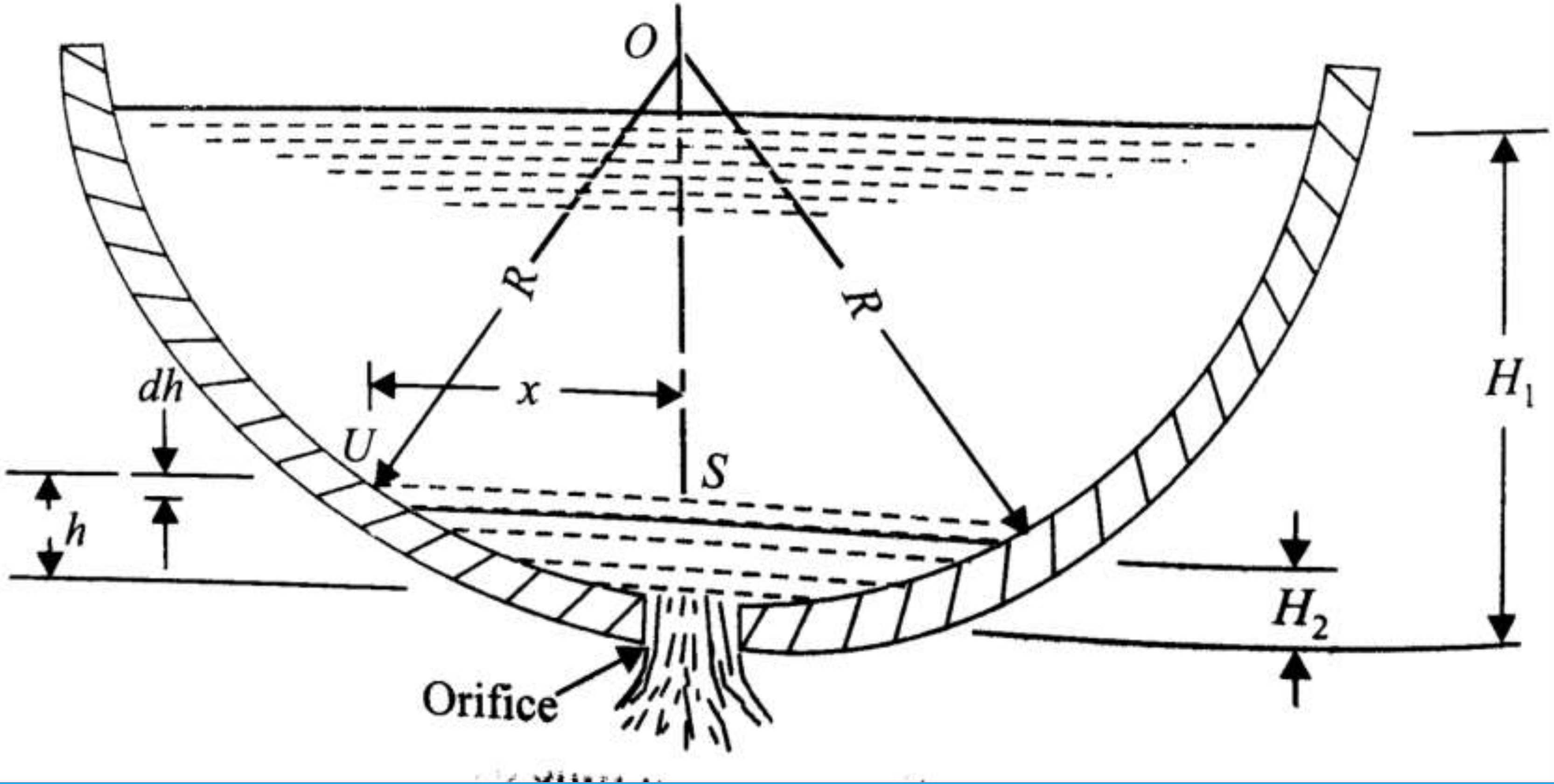
Let, R = Radius of the tank,

a = Area of the orifice,

H_1 = Initial height of the liquid,

H_2= Final height of the liquid, and

T = Time in seconds for the liquid to fall from height H_1 to H_2.



Let at any instant of time, the height of liquid over the orifice is *h* and *x* be the radius of the liquid surface.

Then, area of liquid surface, $A = \pi x^2$

Theoretical velocity of liquid = $\sqrt{2gH}$

Let the height of liquid decrease by *dh* in a small interval of time *dT.* Then,

Volume of liquid leaving the tank in time *dT*

$$= A.dh = \text{p}x^2 \times dh \qquad \text{...}(i)$$

Also, volume of liquid flowing through the orifice in time *dT*

$$= C_d \times \text{area of orifice} \times \text{velocity} \times dT$$

$$= C_d \,.\, a \,.\, \sqrt{2gH} \times dT \qquad \text{...}(ii)$$

Equating (*i*) and (*ii*), we get:

$$\pi x^2 (-dh) = C_d \,.\, a \sqrt{2gH} \times dT$$

The negative sign accounts for the *decrease in head* on the orifice with *increase in time interval.*

$$dT = \pi x^2 (-dh) \,/\, C_d \,.\, a \sqrt{2gH} \qquad \text{..}(iii)$$

From , we have:

$$OU = R \text{ and } OS = (R - h)$$

$\therefore \qquad x = US = \sqrt{OU^2 - OS^2} = \sqrt{R^2 - (R - h)^2}$

$$= \sqrt{R^2 - R^2 - h^2 + 2Rh} = \sqrt{2Rh - h^2}$$

or $\qquad x^2 = (2RH - h^2)$

Substituting this value of x^2 in eqn. (*iii*), we get:

$$dT = \frac{-\pi\left(2Rh - h^2\right)dh}{C_d \, . \, a \, . \, \sqrt{2gH}}$$

$$= \frac{-\pi}{C_d \, . \, a \, . \, \sqrt{2g}} = (2Rh - h^2)h^{-1/2}dh$$

$$= \frac{-\pi}{C_d \, . \, a \, . \, \sqrt{2g}}\left(2Rh^{1/2} - h^{3/2}\right)dh$$

The total time T required to bring the liquid level from $H_1$to H_2 is obtained by integrating the above equation between the limits $H_1$to H_2.

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-\pi}{C_d \,.a\,\sqrt{2g}} \left(2Rh^{1/2} - h^{3/2}\right) dh$$

$$T = \frac{-\pi}{C_d \,.a\,\sqrt{2g}} \int_{H_1}^{H_2} \left(2Rh^{1/2} - h^{3/2}\right) dh$$

$$= \frac{-\pi}{C_d \,.a\,\sqrt{2g}} \left[2R \times \frac{h^{1/2+1}}{\frac{1}{2}+1} - \frac{h^{3/2+1}}{\frac{3}{2}+1}\right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \,.a\,\sqrt{2g}} \left[\frac{2}{3} \times 2R\, h^{3/2} - \frac{2}{5}\, h^{5/2}\right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \,.a\,\sqrt{2g}} \left[\frac{4}{3} R \left(H_2^{3/2} - H_1^{3/2}\right) - \frac{2}{5} \left(H_2^{5/2} - H_1^{5/2}\right)\right]$$

or

$$T = \frac{\pi}{C_d \,.a.\sqrt{2g}} \left[\frac{4}{3} R \left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5} \left(H_1^{5/2} - H_2^{5/2}\right)\right] \quad ...(8.15)$$

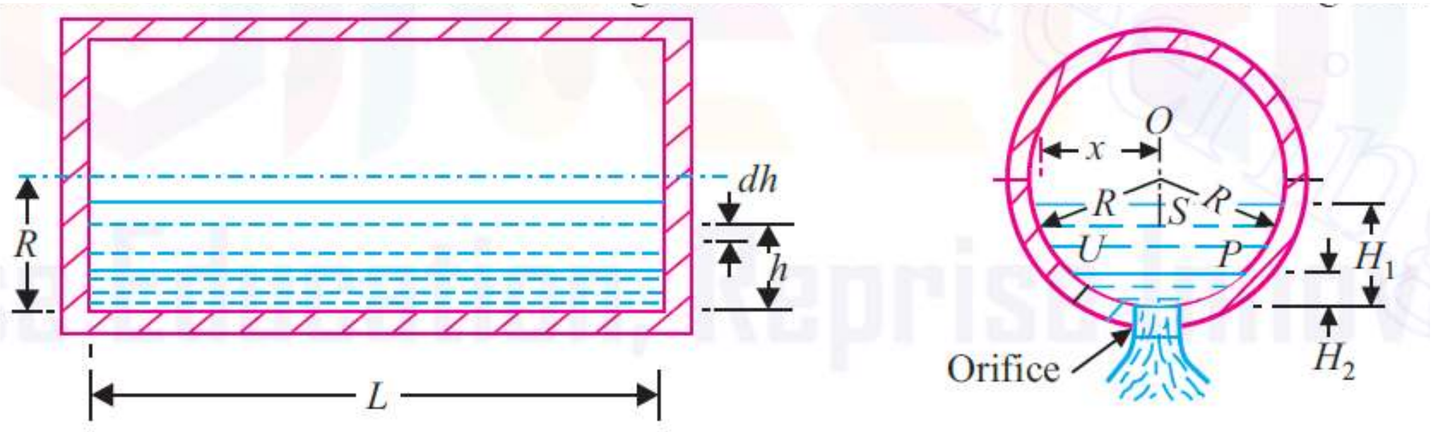
For emptying the tank completely, $H_2 = 0$ and hence,

$$T = \frac{\pi}{C_d \,.a.\sqrt{2g}} \left[\frac{4}{3} RH_1^{3/2} - \frac{2}{5} H_1^{5/2}\right] \quad ...(8.16)$$

TIME REQUIRED FOR EMPTYING A CIRCULAR HORIZONTAL TANK

Consider a circular horizontal tank having an orifice at its bottom and containing some liquid.

Fig. 8.21



Let, R = Radius of the tank,

L = Length of the tank,

H_1 = Initial height of the liquid,

H_2 = Final height of the liquid, and

T = Time in seconds for the liquid to fall from height H_1 to H_2.

Let at any time, the height of liquid over the orifice is h and it decreases dh in a small interval of time dT. Further, let x be the radius of liquid surface at this instant. Then,

Volume of liquid leaving the tank in time $dT = A.dh$ ...(*i*)

$$= A.dh = UP \times L \times dh = 2xL.dh \;(\because UP = 2x)$$

(where, A = surface area)

Velocity of liquid through the orifice = $\sqrt{2gH}$

Volume of liquid flowing through the orifice in time $dT = C_d \times a \times \sqrt{2gh} \times dT$ ...(*ii*)

Volume of liquid leaving the tank equals the volume of liquid flowing through the orifice.

i.e. $-2x\,L.dh = C_d \times a \times \sqrt{2gh} \times dT$

The negative sign accounts for the decrease in head on the orifice with increase in time interval.

$$\therefore \quad dT = \frac{-2xL\,.\,dh}{C_d\,.\,a\,.\,\sqrt{2gh}} \qquad ...(iii)$$

From Fig. 8.21, we have:

$OU = R$ and $OS = (R - h)$

$\therefore \qquad x = US = \sqrt{OU^2 - OS^2} = \sqrt{R^2 - (R - h)^2} = \sqrt{2Rh - h^2}$

Substituting this value of *x* in eqn. (*iii*), we get:

$$dT = \frac{-2\sqrt{2Rh - h^2} \; X \; L \; X \; dh}{C_d \, . \, a \, . \, \sqrt{2gh}} = \frac{-2L\sqrt{2R - h} \, .dh}{C_d \, . \, a \, . \, \sqrt{2gh}} \qquad \text{(Taking } \sqrt{h} \text{ common)}$$

The total time T required to bring the liquid level from height H_1 to H_2 can be found out by integrating the above equation within the limits H_1 and H_2.

$$\therefore \qquad \int_0^T dT = \int_{H_1}^{H_2} \frac{-2L\sqrt{(2R-h)}\cdot dh}{C_d \,.a.\sqrt{2g}}$$

$$\text{or,} \qquad T = \frac{-2L}{C_d\,.a.\sqrt{2g}} \int_{H_1}^{H_2} (2R-h)^{1/2}\,dh$$

$$= \frac{-2L}{C_d\,.a.\sqrt{2g}} \times \frac{2}{3} \times \left[(2R-h)^{3/2} \times (-1\right]_{H_1}^{H_2}$$

$$\text{or,} \qquad T = \frac{4L}{3C_d\;.a\sqrt{2g}}\left[(2R-H_2)^{3/2} - (2R-H_1)^{3/2}\right] \qquad \text{...(8.17)}$$

For emptying the tank completely, $H_2 = 0$ and hence,

$$T = \frac{4L}{3C_d\;.a\;.\sqrt{2g}}\left[(2R)^{3/2} - (2R-H_1)^{3/2}\right] \qquad \text{...(8.18)}$$

8.12. CLASSIFICATION OF MOUTHPIECES

The mouthpieces may be classified as follows :

1. *According to the position of the mouthpiece* :

(*i*) Internal mouthpiece. (*ii*) External mouthpiece.

2. *According to the shape of the mouthpiece*:

(*i*) Cylindrical mouthpiece. (*ii*) Convergent mouthpiece.

(*iii*) Convergent - divergent mouthpiece.

3. *According to nature of discharge*:

(i) Mouthpiece running full. (*ii*) Mouthpiece running free.

- A mouthpiece is said to be *running free* if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as *running full*.

8.13. DISCHARGE THROUGH AN EXTERNAL MOUTHPIECE

A **mouthpiece** *is a small tube (two or three times its diameter in length) attached to an orifice*. An external mouthpiece is attached to the vessel such that it projects outside. Fig.8.23 shows a tank to which is attached an external cylindrical mouthpiece.

Let, a_1 = Area of mouthpiece at outlet,

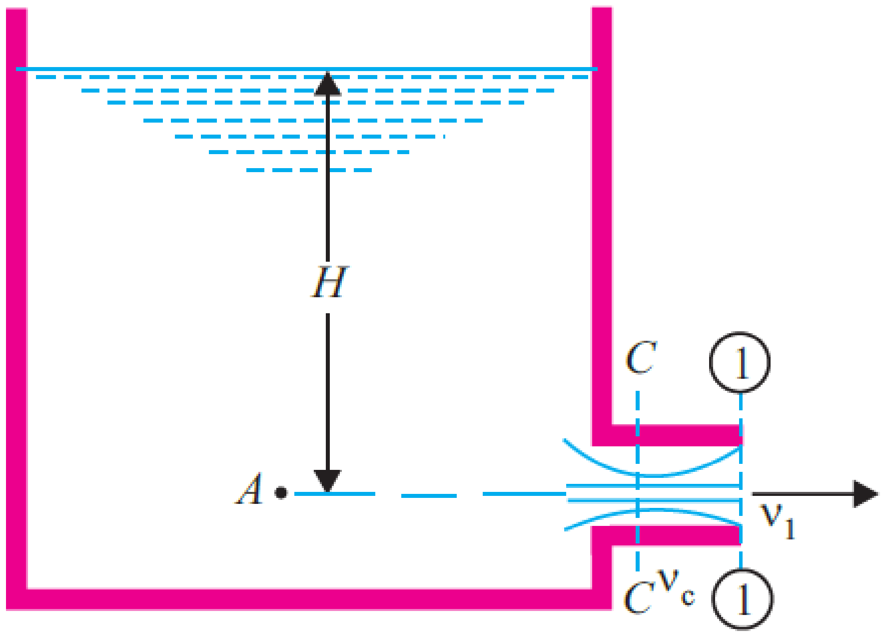
v_1 = Velocity of liquid at outlet,

a_c = Area of flow at vena-contracta,

v_c = Velocity of liquid at *C*-*C* section,

H = Height of liquid above the centre of the mouthpiece, and

C_c = Co-efficient of contraction.



Applying continuity equation at *C*-*C* and 1-1, we get:

$$a_c v_c = a_1 v_1$$

$$\therefore \qquad V_c = \frac{a_1 v_1}{a_c} = \frac{v_1}{a_c / a} = \frac{v_1}{C_c}$$

(where $a_c / a = C_c$ = co-efficient of contraction)

Taking $C_c = 0.62$, we get: $v_c = \dfrac{v_1}{0.62}$

From section C-C the jet of liquid suddenly enlarges at section 1-1; the loss of head due to sudden enlargement is given by:

$$h_L = \frac{(v_c - v_1)^2}{2g}$$

$$= \frac{\left(\frac{v_1}{0.62} - v_1\right)^2}{2g} \qquad \left(\because v_c = \frac{v_1}{0.62}\right)$$

$$= \frac{v_1^2}{2g}\left(\frac{1}{0.62} - 1\right)^2$$

$$= \frac{0.375\, v_1^2}{2g}$$

(Please refer to Art. 12.4.1 for loss of head due to sudden enlargement)

Applying Bernoulli's equation to point A and 1 -1, we get:

$$\frac{P_A}{w} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 + h_L$$

But $z_A = z_1$, $\frac{p_1}{w}$ = atmospheric pressure = 0, and v_A is negligible.

$$\therefore \qquad H + 0 = 0 + \frac{v_1^2}{2g} + \frac{0.375 v_1^2}{2g}$$

or,
$$H = \frac{1.375 v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\sqrt{2gH}$$

Theoretical velocity of liquid at outlet, $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity for mouthpiece,

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855$$

For a mouthpiece, since the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet, therefore, $C_c = 1$.

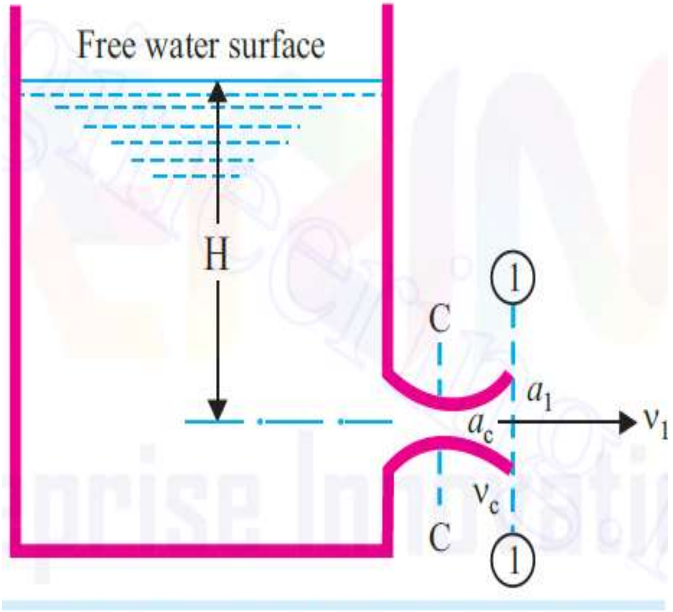
Hence $C_d = C_c \times C_v = 1 \times 0.855 = 0.855$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice, and so *discharge through mouthpiece will be more.*

Note: In actual practice $C_v = C_d \approx 0.82$.

DISCHARGE THROUGH A CONVERGENT-DIVERGENT MOUTHPIECE

shows a convergent-divergent mouthpiece (which converges upto vena-contracta and then diverges). In this mouthpiece since there is no sudden enlargement of the jet, therefore, the loss of energy due to sudden enlargement is eliminated.



For this mouthpiece. $C_d = 1$.

Let, H = Head of liquid over the mouthpiece,

H_a = Atmospheric pressure head, and

H_c = Absolute pressure head at vena-contracta.

Applying Bernoulli's equation at the free water surface and section *C-C*, we get:

$$\frac{P}{w} + \frac{v^2}{2g} + z = \frac{P_c}{w} + \frac{v_c^2}{2g} + z_c$$

Assuming that datum passes through the centre of the mouthpiece, we have:

$$\frac{P}{w} = H_a,\ v = 0,\ \frac{P_c}{w} = H,\ z_c = 0$$

$$H_a + 0 + H = H_c + \frac{v_c^2}{2g} + 0 \qquad \text{...... (i)}$$

or, $$\frac{v_c^2}{2g} = H_a + H - H_c \qquad \text{...... (ii)}$$

or, $$v_c = \sqrt{2g\,(H_a + H - H_c)}$$

Now applying Bernoulli's equation at sections *C*-*C* and 1-1, we get:

$$\frac{p_c}{w} + \frac{v_c^2}{2g} + z_c = \frac{p_1}{w} + \frac{v_1^2}{2g} + z_1$$

But, $$z_c = z_1 \text{ and } \frac{p_1}{w} = H_a$$

$\therefore$ $$H_c + \frac{v_c^2}{2g} = H_a + \frac{v_1^2}{2g}$$

Also from eqn. (*i*), we have:

$$H_c + \frac{v_c^2}{2g} = H_a + H$$

$$\therefore \quad H_a + \frac{v_1^2}{2g} = H_a + H$$

$$\therefore \quad v_1 = \sqrt{2gH}$$

By continuity equation, we have: $a_c v_c = a_1 v_1$

or, $$\frac{a_1}{a_c} = \frac{v_c}{v_1} = \frac{\sqrt{2g\,(H_a + H - H_c)}}{\sqrt{2gH}} = \sqrt{\frac{H_a}{H} + 1 - \frac{H_c}{H}}$$

i.e. $$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} \quad \text{...(8.19)}$$

The discharge, $Q = a_c \times \sqrt{2gH}$ ...(8.20)

8.15. DISCHARGE THROUGH AN INTERNAL MOUTHPIECE (OR RE-ENTRANT OR BORDA'S MOUTHPIECE)

An **internal mouthpiece** is *short cylindrical tube attached to an orifice in such a way that it (tube) projects inwardly to a tank.* If the length of the tube is *equal to diameter,* the jet of liquid comes out from mouthpiece without touching the sides of the tube (Fig. 8.26); the mouthpiece is known as ***running free***. But *if the length of the tube is about 3 times its diameter*, the *jet comes out with its diameter equal to the diameter of mouthpiece at the outlet* (Fig. 8.27); *the mouthpiece is said to be* ***running full****.*

8.15.1. Mouthpiece Running Free

Consider a mouthpiece *running free* as shown in Fig. 8.26.

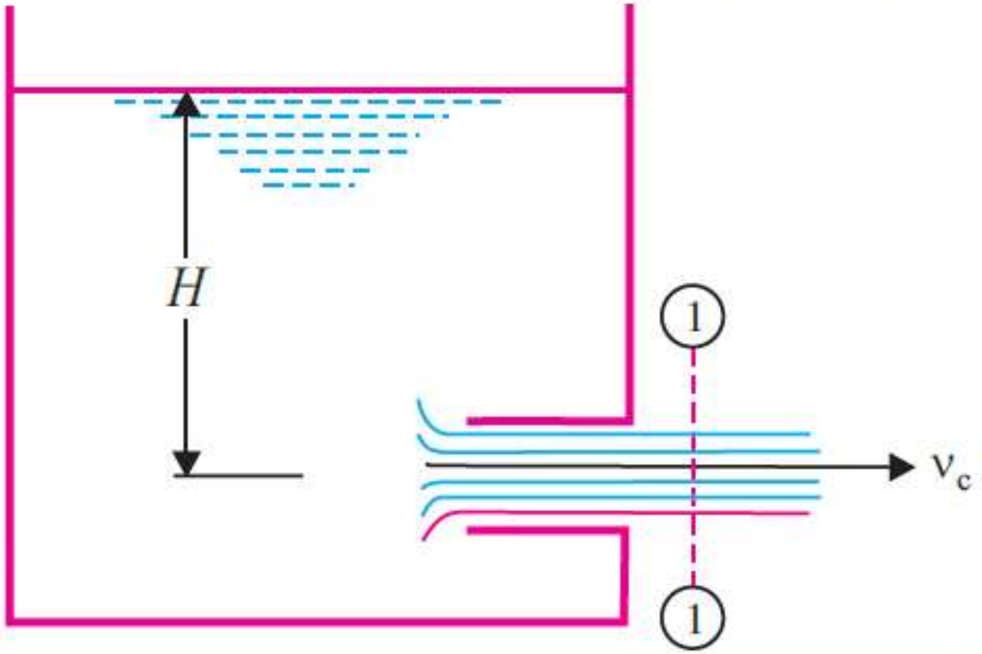


Fig. 8.26. Mouthpiece running free.

Let, H = Height of the liquid above the mouthpiece,

a = Area of orifice or mouthpiece,

a_c = Area of contracted jet, and

v_c = Velocity through mouthpiece.

$\therefore$ Pressure of the liquid on the mouthpiece, $p = wH$

and, force acting on the mouthpiece

$$= \text{Pressure} \times \text{area}$$

$$= wH \times a \quad \text{...}(i)$$

Mass of liquid flowing per second $= \dfrac{wa_c v_c}{g}$

Momentum of flowing liquid/sec.

$$= \text{Mass} \times \text{velocity} = \frac{wa_c v_c \times v_c}{g}$$

$$= \frac{wa_c v_c^2}{g} \quad \text{...}(ii)$$

Since the water is initially at rest, therefore initial momentum = 0

$\therefore$ Change of momentum $= \dfrac{wa_c v_c^2}{g}$

As per Newton's second law of motion, the force is equal to the rate of change of momentum. Therefore equating (*i*) and (*ii*), we get:

$$wH \times a = \frac{w a_c v_c^2}{g}$$

$$H \times a = \frac{a_c v_c^2}{g}$$

$$\frac{v_c^2}{2g} \times a = \frac{a_c v_c^2}{g} \qquad \left(\because H = \frac{v_c^2}{2g}\right)$$

$$\therefore \quad a = 2a_c \text{ or } \frac{a_c}{a} = \frac{1}{2} = 0.5$$

$$\therefore \quad \text{Co-efficient of contraction, } C_c = \frac{a_c}{a} = 0.5$$

Since there is no loss of head, co-efficient of velocity, $C_v = 1.0$

$$\therefore \quad \text{Co-efficient of discharge, } C_d = C_c \times C_v = 0.5 \times 1 = 0.5$$

$$\therefore \quad \text{Discharge, } Q = C_d \times a \times \sqrt{2gH}$$

$$= 0.5 \times a \times \sqrt{2gH} \qquad \text{...(8.21)}$$

8.15.2 Mouthpiece Running Full

Consider a mouthpiece *running full* as shown in Fig. 8.27.

Let, a_c = Area at vena-contracta,

a = Area of orifice or mouthpiece,

v_c = Velocity of the liquid at *C-C* (vena-contracta),

v_1 = Velocity of the liquid at 1-1 (or outlet), and

H = Height of liquid above the mouthpiece.

Since the liquid is flowing continuosly, therefore from the continuity equation, we have:

$$a_c v_c = a_1 v_1 \qquad (a_1 = a)$$

$$v_c = \frac{a_1 v}{a_c} \qquad \ldots(i)$$

We know that the co-efficient of contraction for an internal mouthpiece is 0.5. Substituting this value of $C_c \left(= \frac{a_c}{a_1} \right) = 0.5$ in (*i*), we get:

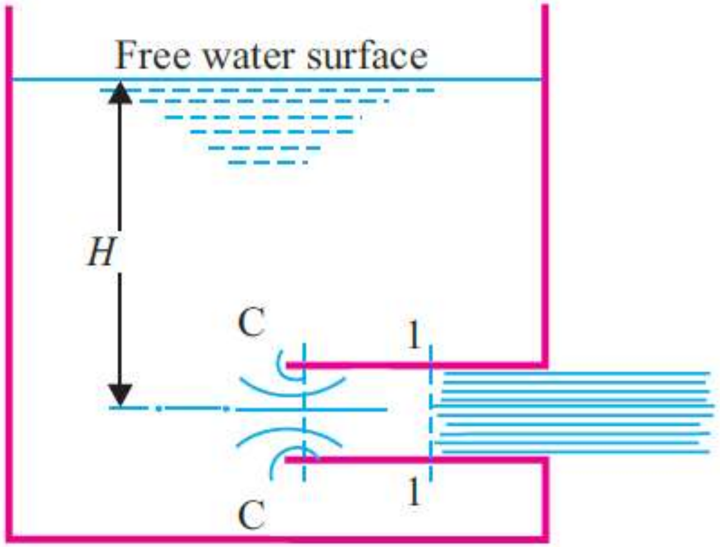


Fig. 8.27. Mouthpiece running full.

$$v_c = 2v_1 \qquad \ldots(ii)$$

The jet of liquid after passing through C–C, suddenly enlarges at section 1–1. Therefore, there will be loss of head due to sudden enlargement,

$$h_L = \frac{(v_c - v_1)^2}{2g} = \frac{(2v_1 - v_1)^2}{2g} \qquad (\because v_c = 2v_1)$$

$$= \frac{v_1^2}{2g}$$

Applying Bernoulli's equation to free water surface in tank and section 1–1 (or outlet),we get:

$$\frac{p}{w} + \frac{v^2}{2g} + z = \frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 + h_L$$

Assuming datum line passing through the centre line of mouthpiece

$$0 + 0 + H = 0 + \frac{v_1^2}{2g} + 0 + \frac{v_1^2}{2g}$$

$$\therefore \quad H = \frac{v_1^2}{2g} + \frac{v_1^2}{2g} = \frac{v_1^2}{g} \quad \text{or} \quad v_1 = \sqrt{gH}$$

Here v_1 is the actual velocity as losses have been taken into account.

But throretical velocity,

$$v_{th} = \sqrt{2gH}$$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{v_1}{v_{th}} = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}}$$

As the area of the jet at outlet is equal to the area of the mouthpiece, hence co-efficient of contraction = 1

$$\therefore \quad C_d = C_c \times C_v = 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\therefore \quad \text{Discharge, } Q = C_d \times a \times \sqrt{2gH} = 0.707 \times a \times \sqrt{2gH} \qquad \text{...(8.22)}$$

THANKS