Chapter 12

FLOW THROUGH PIPES

INTRODUCTION

A pipe is a closed conduit (generally of circular section) which is used for carrying fluids under pressure. The flow in a pipe is termed pipe flow only when the fluid completely fills the cross-section and there is no free surface of fluid. The pipe running partially full (in such a case atmospheric pressure exists inside the pipe) behaves like an open channel.

LOSS OF ENERGY (OR HEAD) IN PIPES

When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available is reduced. This loss of energy (or head) is classified as follows:

A. Major Energy Losses

This loss is due to friction.

B. Minor Energy Losses

These losses are due to:

1. Sudden enlargement of pipe,
2. Sudden contraction of pipe,
3. Bend of pipe,
4. An obstruction in pipe,
5. Pipe fitting, etc.

MAJOR ENERGY LOSSES

These losses which are due to friction are calculated by:

1. Darcy-Weisbach formula
2. Chezy's formula

Darcy-Weisbach formula

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach formula (derived in chapter 11 Art. 11·2) which is given by:

$$h_f = \frac{4fLV^2}{D \times 2g} \qquad \text{...(12·1)}$$

where, h_f = Loss of head due to friction,

f = Co-efficient of friction, (a function of Reynolds number, Re)

$h = \dfrac{0.0791}{(Re)^{1/4}}$ for Re varying from 4000 to 10^6

$= \dfrac{16}{Re}$ for $Re < 2000$ (laminar/viscous flow)

L = Length of the pipe,

V = Mean velocity of flow, and

D = Diameter of the pipe.

Chezy's Formula for Loss of Head due to Friction

Refer to Fig. 11·2. An equilibrium between the propelling force due to pressure difference and the frictional resistance gives :

$$(p_1 - p_2)\, A = f'\, PLV^2$$

or $$\frac{(p_1 - p_2)}{w} \cdot A = \frac{f'}{w} PLV^2$$ [Refer to Art. 11.2]

or $$h_f = \frac{f'}{w} \frac{P}{A} LV^2$$

$\therefore$ Mean velocity, $$V = \sqrt{\frac{w}{f'}} \times \sqrt{\frac{A}{P} \times \frac{h_f}{L}}$$

where, the factor $\sqrt{\dfrac{w}{f}}$, is called the Chezy's constant, C;

the ratio $\dfrac{A}{P}\left(= \dfrac{\text{area of flow}}{\text{wetted perimeter}}\right)$ is called the ***hydraulic mean depth*** or ***hydraulic radius*** and denoted by m (or R);

the ratio $\dfrac{h_f}{L}$ prescribes the *loss of head per unit length of pipe* and is denoted by **i or S** (slope).

$\therefore$ Mean velocity, $$V = C\sqrt{m\, i} \quad ...(12\cdot 2)$$

Eqn. (12·2) is *known as* **Chezy's formula.** This formula helps to find the head loss due to friction if the mean flow velocity through the pipe and also the value of Chezy's constant C are known.

Note : (*i*) Darcy-Weisbach formula (for loss of head) is generally used for the flow through *pipes*.

(*ii*) Chezy's formula (for loss of head) is generally used for the flow through *open channels*.

(*iii*) The values of hydraulic mean depth for a *circular pipe*,

$$m = \frac{D}{4} \left[\because m = \frac{\text{Area}}{\text{Perimeter}} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4} \right]$$

MINOR ENERGY LOSSES

Whereas the major loss of energy or head is due to friction, the minor loss of energy (or head) include the following cases:

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head due to an obstruction in the pipe,
4. Loss of head at the entrance to a pipe,
5. Loss of head at the exit of a pipe,
6. Loss of head due to bend in the pipe, and
7. Loss of head in various pipe fittings.

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT

A liquid flowing through a pipe which has sudden enlargement. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy (or head). Consider two sections 1 – 1(before enlargement) and 2 – 2 (after enlargement).

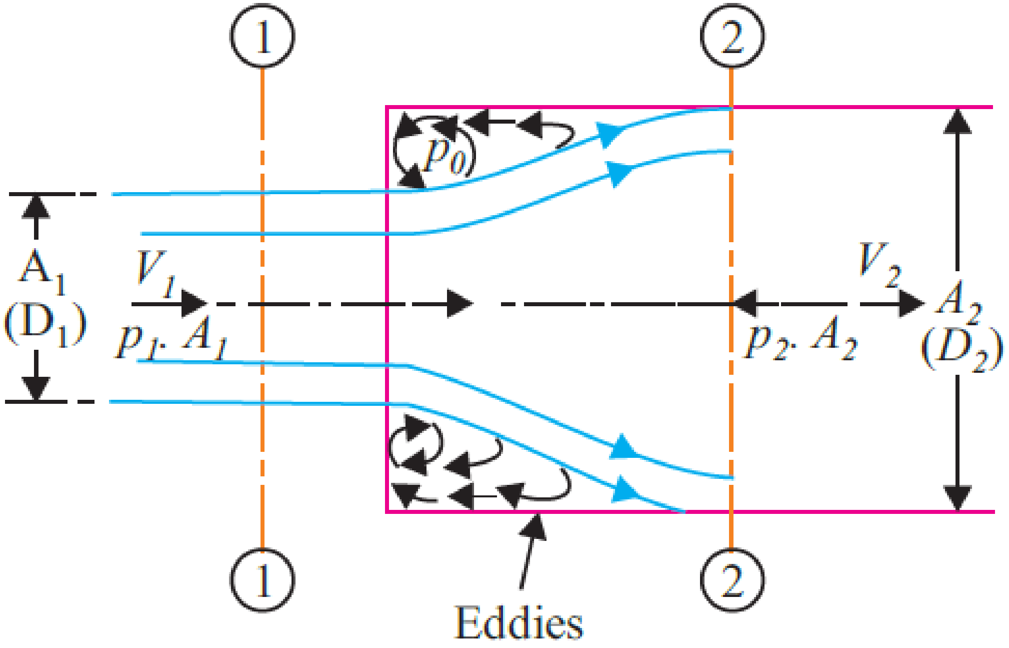
Let, A_1 = Area of pipe at section 1–1.

$= \frac{\pi}{4} D_1^2$ (where D_1 is the diameter of the pipe),

p_1 = Intensity of pressure at section 1–1,

V_1 = Velocity of flow at section 1–1,

$A_2 \left(= \frac{\pi}{4} D_2^2 \right)$, p_2 and V_2 = Corresponding values at section 2–2,



p_0 = Intensity of pressure of the liquid eddies on the area $(A_2 - A_1)$, and

h_e = Loss of head due to sudden enlargement.

Applying Bernoulli's equation to sections 1–1 and 2–2, we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + \text{Loss of head due to sudden enlargement } (h_e)$$

But, $\quad z_1 = z_2 \quad$ ...pipe being horizontal

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} + h_e$$

or, $$h_e = \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad ...(i)$$

Now, the force acting on liquid in the control volume (between sections 1–1 and 2–2) in the flow direction is given by :

$$F_x = p_1 \cdot A_1 + p_0 (A_2 - A_1) - p_2 \cdot A_2$$

Assuming $p_0 \simeq p_1$, we have:

$$F_x = p_1 \cdot A_1 + p_1 (A_2 - A_1) - p_2 \cdot A_2$$

$$= p_1 A_2 - p_2 A_2 = (p_1 - p_2) A_2 \quad ...(ii)$$

Consider **momentum** of liquid at the sections 1–1 and 2–2; momentum of liquid /sec at

section 1–1 = Mass × velocity.

$$= \rho A_1 V_1 \times V_1 = pA_1 V_1^2$$

Momentum of liquid/sec. at section 2–2 = $\rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$

$\therefore$ Change of momentum of liquid/sec.

$$= \rho A_2 V_2^2 - \rho A_1 V_1^2$$

But from continuity equation, we have:

$$A_1 V_1 = A_2 V_2$$

or, $$A_1 = \frac{A_2 V_2}{V_1}$$

$\therefore$ Change of momentum/sec.

$$= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2$$

$$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 (V_2^2 - V_1 V_2) \qquad ...(iii)$$

Now, Net force = Change of momentum

$$\therefore \quad (p_1 - p_2) A_2 = \rho A_2 (V_2^2 - V_1 V_2)$$

or,

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1 V_2$$

Dividing both sides by g, we get:

$$\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

or,

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{V_2^2 - V_1 V_2}{g} \qquad (\because \rho g = w)$$

Substituting the value of $\left(\frac{p_1}{w} - \frac{p_2}{w}\right)$ in eqn. (*i*), we get:

$$h_e = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g} = \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g} = \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore \quad h_e = \frac{(V_1 - V_2)^2}{2g} \qquad \text{...(12·2)}$$

Loss of Head due to Sudden Contraction

Due to sudden contraction, the stream lines converge to a minimum cross-section called the vena-contracta then expand to fill the downstream pipe.

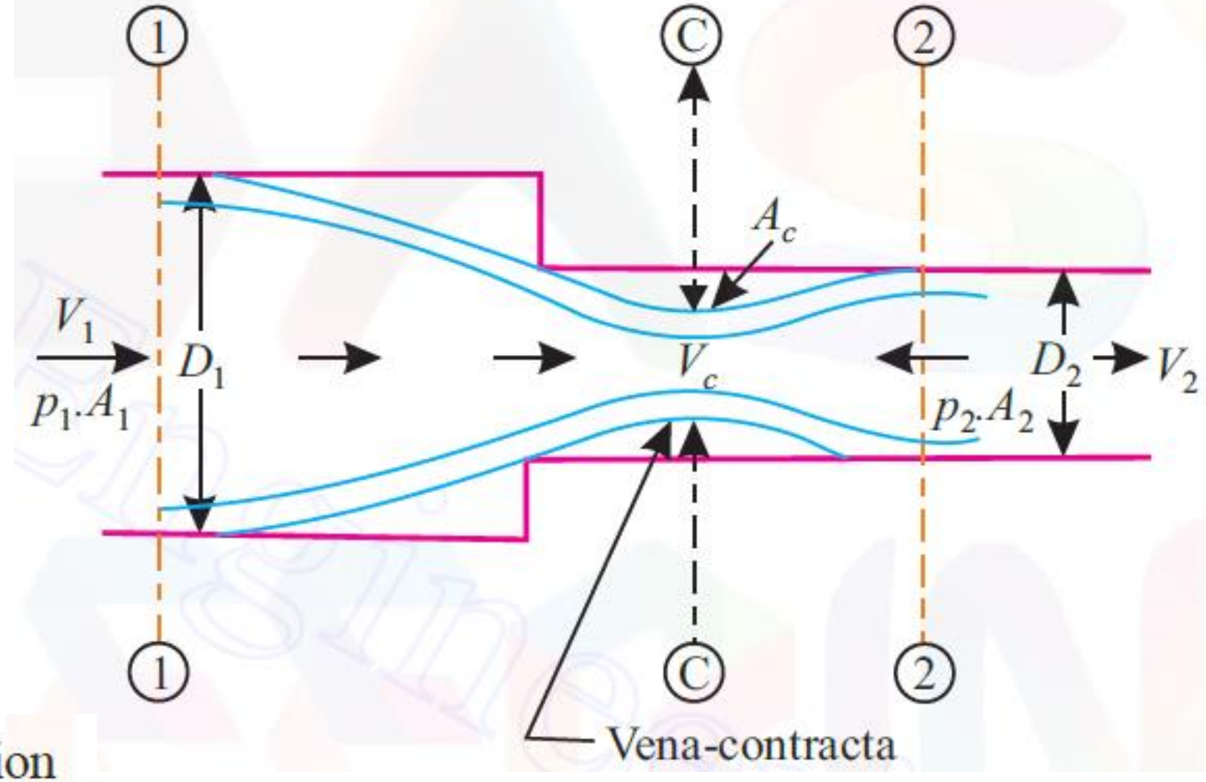
Let, A_c = Area of flow at section C–C,

V_c = Velocity of flow at section C–C,

A_2 = Area of flow at section 2–2,

V_2 = Velocity of flow at section 2–2, and

h_c = Loss of head due to sudden contraction.



Loss of head due to sudden contraction = Loss up to vena-contracta + loss due to sudden enlargement beyond vena-contracta or, $$h_c = \text{Negligibly small} + \frac{(V_c - V_2)^2}{2g} \quad \ldots (i)$$

From continuity equation, we have:

$$A_c V_c = A_2 V_2$$

or, $$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c / A_2)} = \frac{1}{C_c} \qquad \left(\because C_c = \frac{A_c}{A_2}\right)$$

or, $$V_c = \frac{V_2}{C_c}$$

Substituting the value of V_c in eqn. (i), we get:

$$h_c = \frac{\left(\frac{V_2}{C_c} - V_2\right)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

i.e., $$\boldsymbol{h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2} \quad \ldots(12{\cdot}3)$$

In general, $$h_c = k\frac{V_2^2}{2g}$$

where, $$k = \left(\frac{1}{C_c} - 1\right)^2$$

From experiments : $C_c = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$

and thus the loss co-efficient k is a function of ratio

$$\frac{A_1}{A_2} \text{ or } \frac{D_2}{D_1}$$

and, $k = 0{\cdot}375$ for $C_c = 0{\cdot}62$.

For gradual contraction (conical reducers) k is a function of cone angle and $\simeq 0{\cdot}1$.

Note : If the value of C_c is not given then loss of head due to contraction may be taken as $0{\cdot}5\,\frac{V_2^2}{2g}$

i.e., $$h_e = 0.5\frac{V_2^2}{2g} \qquad ...(12{\cdot}4)$$

Loss of Head due to Obstruction in Pipe

The loss of energy due to an obstruction in pipe takes on account of the reduction in the cross-sectional area of the pipe by the presence of obstruction which is followed by an abrupt enlargement of the stream beyond the obstruction.

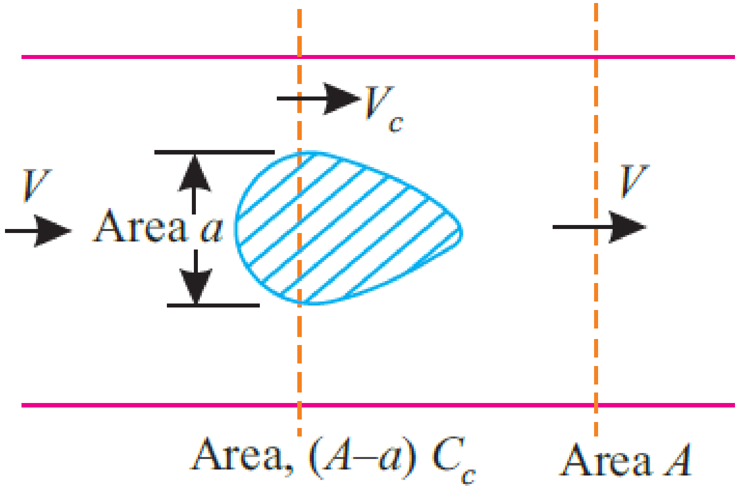
Head loss due to obstruction (h_{obs}) is given by the relation:

$$h_{obs.} = \left[\frac{A}{C_c\,(A-a)}\right]^2 \frac{V^2}{2g} \qquad \text{...(12.5)}$$

where, A = Area of the pipe,

a = Maximum area of obstruction, and

V = Velocity of liquid in pipe.



12.4.4 Loss of Head at the Entrance to Pipe

Loss of head at the entrance to pipe (h_i) is given by the relation :

$$h_i = 0.5\,\frac{V^2}{2g} \qquad \text{...(12.6)}$$

where, V = Velocity of liquid in pipe.

12·4·5 Loss of Dead at the Exit of a Pipe

Loss of head at the exit of a pipe is denoted by h_0 and is given by the relation:

$$h_0 = \frac{V^2}{2g} \qquad ...(12.7)$$

where, V = Velocity at outlet of pipe.

12·4·6 Loss of Head due to Bend in the Pipe

In general the loss of head in bends (h_b) provided in pipes may be expressed as :

$$h_0 = k\frac{V^2}{2g} \qquad ...(12.8)$$

where, V = Mean velocity of flow of fluid, and

and, k = Co-efficient of bend; it depends upon *angle of bend, radius of curvature of bend* and *diameter of pipe.*

12·4·7 Loss of Head in Various Pipe Fittings

The loss of head in the various pipe fittings (such as valves, couplings, etc.) may also be represented as :

$$h_{\text{fittings}} = k\frac{V^2}{2g}$$

where, V = Mean velocity flow in the pipe, and k = value of the co-efficient; it depends on the type of the pipe fitting.

12.5. HYDRAULIC GRADIENT AND TOTAL ENERGY LINES

The concept of hydraulic gradient line and total energy line is quite useful in the study of flow of fluid in pipes. These lines may be obtained as indicated below.

Total Energy Line (T.E.L. or E.G.L.):

It is known that the *total head* (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head, *i.e.,*

$$\text{Total head} = \frac{p}{w} + z + \frac{V^2}{2g}$$

When the fluid flows along the pipe, there is loss of head (energy) and the total energy decreases in the direction of flow. If the total energy at various points along the axis of the pipe is plotted and joined by a line, the line so obtained is called the ***'Energy gradient line' (E.G.L.).***

In literature, energy gradient line (E.G.L.) is *also known as* ***'Total energy line' (T.E.L.).***

Hydraulic Gradient Line (H.G.L.):

The sum of potential (or elevation) head and the pressure head $\left(\frac{p}{w}+z\right)$ at any point is called the *piezometric head. If a line is drawn joining the piezometric levels at various points,* the line so obtained is called the ***'Hydraulic gradient line.'***

The following points are worth noting :

1. Energy gradient line (E.G.L.) *always drops in the direction of flow because of loss of head.*
2. Hydraulic gradient line (H.G.L.) may *rise* or *fall* depending on the pressure changes.
3. Hydraulic gradient line (H.G.L.) is *always below the energy gradient line (E.G.L.)* and the vertical intercept between the two is equal to the velocity head $\left(\frac{V^2}{2g}\right)$.
4. For a pipe of *uniform cross-section the slope of the hydraulic gradient line is equal to the slope of energy gradient line.*
5. *There is no relation whatsoever between the slope of energy gradient line and the slope of the axis of the pipe.*

12.6. PIPES IN SERIES OR COMPOUND PIPES

Fig. 12·15 shows a system of pipes in series.

Let, D_1, D_2, D_3 = Diameters of pipes 1, 2 and 3 respectively,

L_1, L_2, L_3 = Lengths of pipes 1, 2 and 3 respectively,

V_1, V_2, V_3 = Velocities of flow through pipes 1, 2 and 3 respectively

f_1, f_2, f_3 = Co-efficients of friction for pipes 1, 2 and 3 respectively, and

H = Difference of water level in the two tanks.

As the rate of flow (Q) of water through each pipe is same, therefore,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

Also, The difference in liquid surface levels = Sum of the various head losses in the pipes

i.e.,
$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^3}{2g} \quad \text{...}(i)$$

where, h_i = Head loss at entrance $= \dfrac{0.5V_1^2}{2g}$

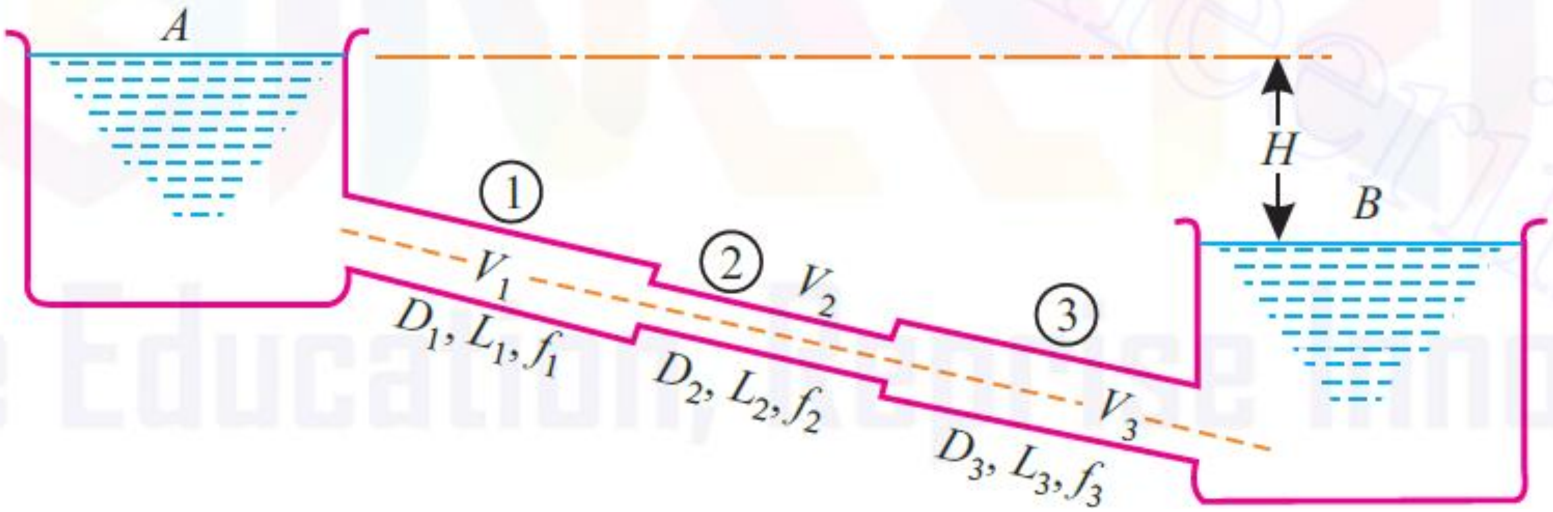
h_{f_1} = Head loss due to friction in pipe 1 $= \dfrac{4f_1L_1V_1^2}{D_1 \times 2g}$

h_c = Head loss at contraction $= \dfrac{0.5V_2^2}{2g}$

h_{f_2} = Head loss due to friction in pipe 2 $= \dfrac{4f_2L_2V_2^2}{D_2 \times 2g}$

$$h_e = \text{Head loss due to enlargement} = \frac{(V_2 - V_3)^2}{2g}$$

$$h_{f_3} = \text{Head loss due to friction in pipe 3} = \frac{4 f_3 L_3 V_3^2}{D_3 \times 2g}$$



Substituting the values in (*i*), we have:

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$= \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g} \quad ...(12\cdot9)$$

If *minor losses are neglected*, then above equation becomes:

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} \quad ...(12\cdot10)$$

If, $f_1 = f_2 = f_3 = f$, then:

$$H = \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_2V_2^2}{D_2 \times 2g} + \frac{4fL_3V_3^2}{D_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{D_1} + \frac{L_2V_2^2}{D_2} + \frac{L_3V_3^2}{D_3}\right] \quad ...(12\cdot11)$$

12.7. EQUIVALENT PIPE

An **equivalent pipe** *is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.* The uniform diameter of the equivalent pipe is known as the *equivalent diameter* of the series or compound pipe.

Let, $L_1, L_2, L_3,$ etc. = Lengths of pipes 1, 2, 3, etc.

$D_1, D_2, D_3,$ etc. = Diameters of pipes 1, 2, 3, etc.,

H = Total head loss,

L = Length of the equivalent pipe, and

D = Diameter of the equivalent pipe.

Then, neglecting minor losses, total head loss,

$$h_f = h_{f_1} + h_{f_2} + h_{f_3} + \ldots$$

or,

$$H = \frac{4 f_1 L_1 V_1^2}{D_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{D_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{D_3 \times 2g} + \ldots \qquad \ldots(12{\cdot}12)$$

(where, f_1, f_2 and f_3, etc. are co-efficients of friction)

Also, from continuity considerations:

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$= \frac{\pi}{4} \times D_1^2 V_1 = \frac{\pi}{4} \times D_2^2 V_2 = \frac{\pi}{4} \times D_3^2 V_3$$

$$\therefore \quad V_1 = \frac{4Q}{\pi D_1^2}, V_2 = \frac{4Q}{\pi D_2^2}, V_3 = \frac{4Q}{\pi D_3^2}$$

Substituting these values in eqn. (12·12), assuming $f_1 = f_2 = f_3$, etc. $= f$, we get:

$$H = \frac{4 f L_1 \times \left(\frac{4Q}{\pi D_1^2}\right)^2}{D_1 \times 2g} + \frac{4 f L_2 \times \left(\frac{4Q}{\pi D_2^2}\right)^2}{D_2 \times 2g} + \frac{4 f L_3 \times \left(\frac{4Q}{\pi D_3^2}\right)^2}{D_3 \times 2g} + \ldots$$

$$= \frac{4 \times 16 f Q^2}{\pi^2 \times 2g} \left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\right) \qquad \ldots(12\cdot13)$$

Head loss in the equivalent pipe,

$$H = \frac{4 f L V^2}{D \times 2g} \text{ (assuming the same value of } f \text{ as in compound pipe)}$$

where, $$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} \times D^2} = \frac{4Q}{\pi D^2}$$

$$\therefore \qquad H = \frac{4fL\left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} = \frac{4 \times 16 fQ^2 f}{\pi^2 \times 2g}\left[\frac{L}{D^5}\right] \qquad \text{...(12·14)}$$

From eqns. (12·13) and (12·14), we have:

$$\frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + ...\right) = \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left(\frac{L}{D^5}\right)$$

or,
$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + ... \qquad \text{...(12.15)}$$

Eqn. 12·15 is known as **Dupit's equation.** If the length of the equivalent pipe is equal to the length of the compound pipe *i.e.,* $L = (L_1 + L_2 + L_3 + ...)$, the diameter D of the equivalent pipe may be determined by using this equation. Sometimes a pipe of a given diameter D which is available may be required to be used as equivalent pipe to replace a compound pipe; in this case the length of the equivalent pipe may be required to be determined and the same may also be determined by using eqn. (12·15).

12.8. PIPES IN PARALLEL

The pipes are said to be in *parallel* (Fig. 12·19) when a main line divides into two or more parallel pipes which again join together downstream and continues as a main line.

It may be seen from Fig. 12·19 that the rate of discharge in the main line is equal to the pipes.

Thus, $$Q = Q_1 + Q_2 \qquad ...(12·16)$$

When the pipes are arranged in parallel, the *loss of head in each pipe* (*branch*) *is same*.

∴ Loss of head in pipe 1 = Loss of head in pipe 2.

or, $$h_f = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4f_2L_2V_2^2}{D_2 \times 2g} \qquad ...(12·17)$$

When, $f_1 = f_2$, then:

$$\frac{L_1V_1^2}{D_1 \times 2g} = \frac{L_2V_2^2}{D_2 \times 2g} \qquad ...(12·18)$$

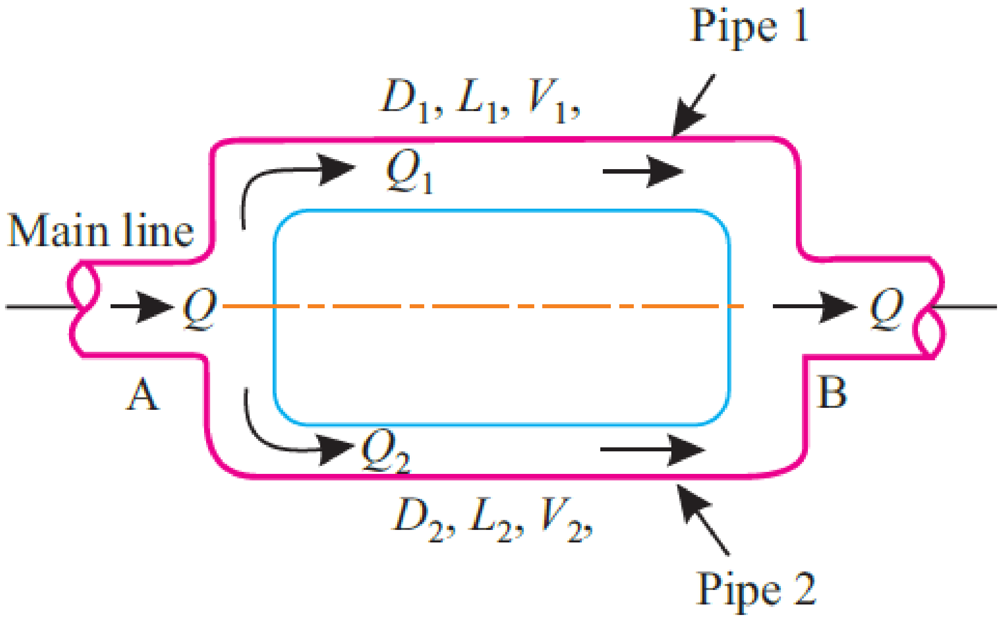
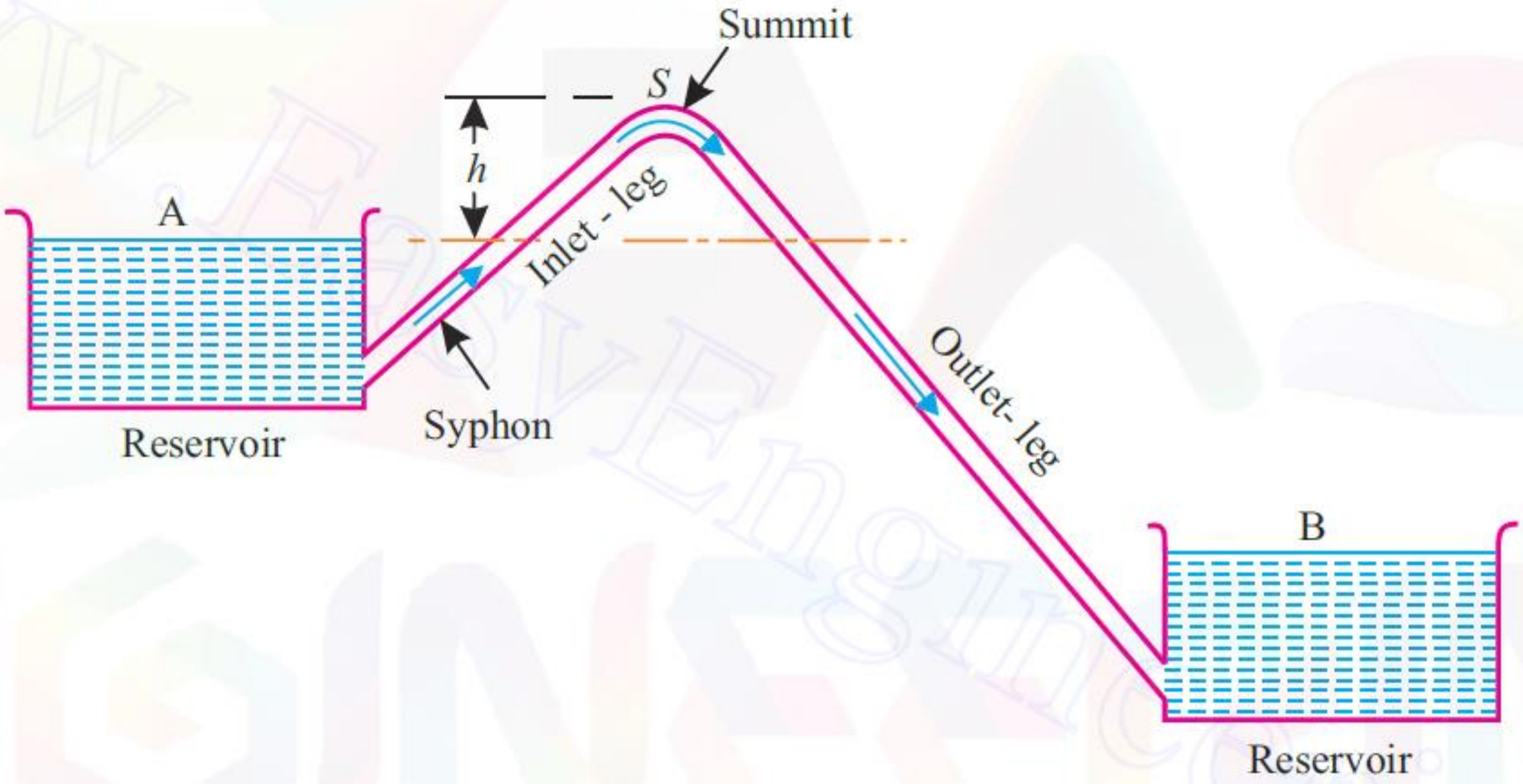


Fig. 12.19

12.9. SYPHON

A **syphon** *is a long bent pipe employed for carrying water from a reservoir at a higher elevation to another reservoir at a lower elevation when the two reservoirs are separated by a hill or high level ground in between* as shown in Fig. 12·39.



The highest point (*S*) of the syphon is called the **summit.** The pressure at the point *S* is **less** *than atmospheric pressure* (since *S* lies above the free water surface in the tank *A*). The pressure at *S* can be reduced theoretically to – 10·3 m of water but in actual practice this pressure is only – 7·6 m of water (or 10·3 – 7·6 = 2·7 m of water *absolute*). When the pressure at *S* becomes less than 2·7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit. *Therefore syphon should be so laid that no section of the pipe will be more than 7·6 m above the hydraulic gradient at that section.* Moreover, in order to limit the reduction of the pressure at the summit the length of the *inlet-leg* (rising portion of the syphon) of the syphon is also required to be limited (this is so because, if the *inlet leg* is very long a considerable loss of head due to friction is caused, resulting in further reduction of the pressure at the summit).

12.10. POWER TRANSMISSION THROUGH PIPES

The transmission of power through pipes carrying water or other liquids is commonly used for working of several hydraulic machines. The hydruaulic power transmitted by a pipe however depends on (*i*) the *discharge* passing through the pipe and (*ii*) the *total head of water* (or liquid).

Consider a pipe AB *connected to a high level storage tank as shown in Fig. 12·43.*

Let, H = Head of water available at the inlet of pipe, m,

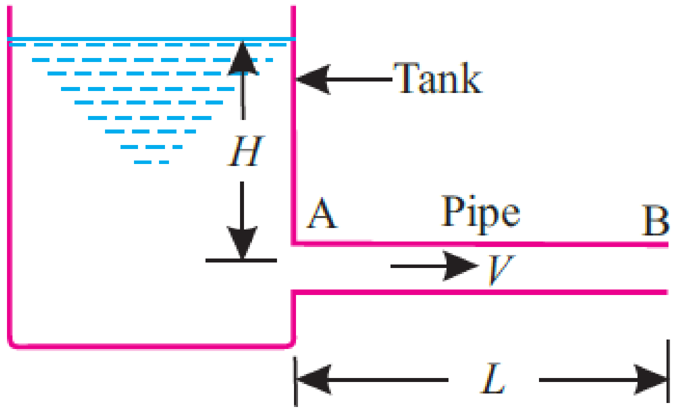
L = Length of the pipe, m,

D = Diameter of the pipe, m,

V = Velocity of water in the pipe m/s,

f = Co-efficient of friction, and

h_f = Loss of head in the pipe AB, due to friction, m.



Weight of water flowing through the pipe per second

$$= wQ = wAV \qquad ...(i)$$

(where, Q = discharge of water through the pipe, m^3/s)

and, net head of water available at B (neglecting minor losses)

$$= H - h_f = H - \frac{4fLV^2}{D \times 2g}$$

Also, The **efficiency of transmission,**

$$\eta = \frac{H - h_f}{H}$$

And, **Power, *P*** $= \dfrac{\left\{\begin{array}{c}\text{Weight of water flowing/sec.} \\ \times \text{ head of water}\end{array}\right\}}{1000}$ kW

$$= wQ\,(H - h_f)\ \text{kW}$$

(where, $w = 9{\cdot}81\ \text{kN/m}^3$ for water)

$$= wAV\left(H - \frac{4fLV^2}{D \times 2g}\right)\text{kW}$$

$$= wA\left(HV - \frac{4fLV^3}{D \times 2g}\right)\text{kW} \qquad \ldots(iii)$$

It is evident from eqn. (*iii*) that power transmitted depends upon the velocity of water (V), as the other things are constant.

$\therefore$ Power transmitted will be *maximum*, when:

$$\frac{d(P)}{dV} = 0$$

or, $$\frac{d}{dV}\left[wA\left(HV - \frac{4fLV^3}{D \times 2g}\right)\right] = 0$$

or, $$wA\left(H - \frac{4 \times 3fLV^2}{D \times 2g}\right) = 0$$

or, $$H - 3 \times \frac{4fLV^2}{D \times 2g} = 0$$

or, $$H - 3h_f = 0 \qquad \left[\because h_f = \frac{4fLV^2}{D \times 2g}\right]$$

or, $$H = 3h_f$$

or, $$\boldsymbol{h_f = \frac{H}{3}}$$

It means that *power transmitted through the pipe is maximum, when head lost due to friction in the pipe is equal to* $\frac{1}{3}$ *of the total supply head.*

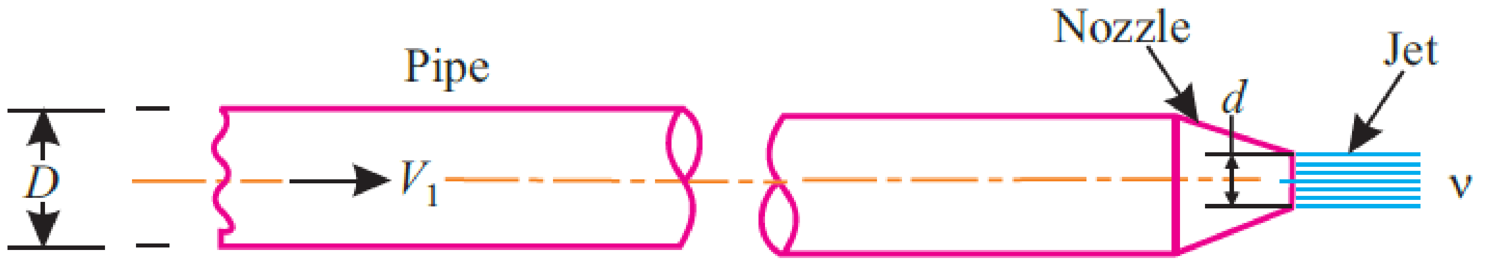
The **maximum efficiency** would correspond to the maximum power transmitted and hence maximum efficiency,

$$\eta = \frac{H - \frac{H}{3}}{H} = \frac{\frac{2}{3}H}{H} = \frac{2}{3} \quad \text{or} \quad \mathbf{66{\cdot}7\%}$$

12.11. FLOW THROUGH NOZZLE AT THE END OF A PIPE

Refer to Fig. 12·44. A **nozzle** *is a tapering mouthpiece, which is fitted to the outlet end of a pipe*. The total energy at the end of the pipe consists of pressure energy and kinetic energy. By fitting the nozzle at the end of a pipe, the total energy is converted into *kinetic energy*. A high velocity is required in the fields of power development, fire fighting, mining, etc.

Fig. 12·44 shows a nozzle fitted at the end of a pipe connected to a reservoir.



Let, D = Diameter of the pipe,
L = Length of the pipe,
d = Diameter of the nozzle,
V = Velocity of flow in pipe,
v = Velocity of flow at the outlet of the nozzle,
f = Co-efficient of friction for the pipe, and
H = Height of water level in the reservoir above the centre-line of the nozzle.

Head lost due to friction in pipe,

$$h_f = \frac{4fLV^2}{D \times 2g}$$

$\therefore$ Head available at the base of the nozzle

$$= H - h_f = H - \frac{4fLV^2}{D \times 2g}$$

Assuming the minor losses and losses in the nozzle to be negligible, we have:

$$\text{Total head at the nozzle outlet} = \frac{v^2}{2g}$$

$\therefore$

$$H = h_f + \frac{v^2}{2g} = \frac{4fLV^2}{D \times 2g} + \frac{v^2}{2g} \qquad \text{...}(i)$$

From continuity consideration, we have:

$$AV = av$$

(where A and a are the areas of the pipe and area of the nozzle at outlet respectively)

or,
$$V = \frac{av}{A}$$

Substituting the value of V in eqn. (i), we get:

$$H = \frac{4fLa^2v^2}{D \times 2g \times A^2} + \frac{v^2}{2g}$$

$$= \frac{v^2}{2g}\left(\frac{1 + 4fLa^2}{D \times A^2}\right)$$

$\therefore$
$$v = \sqrt{\frac{2gH}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}} \qquad \text{...(12.20)}$$

$\therefore$ Discharge through the nozzle $= a \times v$

12·11·1 Power Transmitted through the Nozzle

Mass of liquid flowing per second at the outlet ofthe nozzle, $m = \rho a v$

The K.E. of the jet at outlet of the nozzle

$$= \frac{1}{2} m v^2 = \frac{1}{2} \times \rho a v \times v^2 = \frac{1}{2} \rho a v^3$$

$\therefore$ Power available at the outlet of nozzle $= \frac{1}{2} \rho a v^3$ watts

Also, power available at the inlet of pipe $= wQH$

$\therefore$ Efficiency of power transmission through the nozzle,

$$\eta = \frac{\text{Power available at the outlet of nozzle}}{\text{Power available at the inlet of pipe}} = \frac{\frac{1}{2} \rho a v^3}{wQH}$$

But, $w = \rho g$ and $Q = av$

$$\therefore \quad \eta = \frac{\frac{1}{2} \rho a v^3}{\rho g \times a v \times H} = \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}} \right] \qquad \ldots(12\cdot21)$$

$$\left[\because v = \sqrt{\frac{2gH}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}} \qquad \ldots\text{eqn. } (12\cdot20) \right]$$

12·11·2 Condition for Transmission of Maximum Power Through Nozzle

We know that, $$H = h_f + \frac{v^2}{2g} = \frac{4fLV^2}{D \times 2g} + \frac{v^2}{2g}$$

or, $$\frac{v^2}{2g} = H - \frac{4fLV^2}{D \times 2g}$$

But power transmitted through the nozzle,

$$P = \frac{1}{2}\rho a v^3 = \frac{1}{2}\rho a v \times v^2$$

$$= \frac{1}{2}\rho a v \left[2g\left(H - \frac{4fLV^2}{D \times 2g}\right)\right]$$

$$= wav\left(H - \frac{4fLV^2}{D \times 2g}\right) \qquad ...(12 \cdot 22)$$

From continuity consideration, we have:

$$AV = av \quad \text{or} \quad V = \frac{av}{A}$$

Substituting the value of V in eqn. (12·22), we get:

Power transmitted through nozzle, $$P = wav\left(H - \frac{4fL \times a^2v^2}{D \times 2g \times A^2}\right) \qquad ...[12 \cdot 22(a)]$$

Power transmitted will be *maximum*, when $\dfrac{dP}{dv} = 0$

$$\frac{d}{dv}\left[wav\left(H - \frac{4fL}{D \times 2g} \times \frac{a^2 v^2}{A^2}\right)\right] = 0$$

or, $$\frac{d}{dv}\left[wa\left(Hv - \frac{4fL}{D \times 2g} \times \frac{a^2 v^3}{A^2}\right)\right] = 0$$

or, $$H - 3 \times \frac{4fL}{D \times 2g} \times V^2 = 0 \qquad \left(\because \frac{a^2 v^2}{A^2} = V^2\right)$$

or, $$H - 3h_f = 0 \qquad \left(\because h_f = \frac{4fLV^2}{D \times 2g}\right)$$

or, $$\boldsymbol{h_f = \frac{H}{3}} \qquad \text{...(12·23)}$$

The eqn. (12·23) indicates that the *power transmitted by a nozzle is maximum when the head lost due to friction in pipe is equal to one-third the total head supplied at the inlet of pipe.*

12·11·3 Diameter of the Nozzle for Transmitting Maximum Power

We know that, $$H = h_f + \frac{v^2}{2g}$$

But, $$H = 3h_f \qquad \text{[From eqn. (12·22)]}$$

$\therefore$ $$3h_f = h_f + \frac{v^2}{2g} \quad \text{or} \quad 2h_f = \frac{v^2}{2g}$$

$$\frac{2 \times 4fLV^2}{D \times 2g} = \frac{v^2}{2g}$$

For *continuity* considerations, we have:

$$AV = av \quad \text{or} \quad V = \frac{av}{A}$$

$\therefore$ $$\frac{2 \times 4fL \times a^2v^2}{D \times 2g \times A^2} = \frac{v^2}{2g}$$

or, $$\frac{A^2}{a^2} = \frac{8fL}{D} \quad \text{or} \quad \frac{A}{a} = \sqrt{\frac{8fL}{D}} \qquad \text{...(12·24)}$$

Eqn. (12·24) gives the *ratio* between the areas of the supply pipe and the nozzle for maximum power transmission.

Substituting the values of A and a in eqn. (12·24) and squaring both sides, we have:

$$\left(\frac{\frac{\pi}{4} \times D^2}{\frac{\pi}{4} \times d^2}\right)^2 = \frac{8\,fL}{D}$$

or, $$\frac{D^4}{d^4} = \frac{8\,fL}{D} \quad \text{or} \quad D^5 = 8fLd^4$$

$\therefore$ $$\boldsymbol{d = \left(\frac{D^5}{8\,fL}\right)^{1/4}} \qquad \text{...(12·25)}$$

12.12. WATER HAMMER IN PIPES

In a long pipe, when the *flowing water is suddenly brought to rest by closing the valve or by any similar cause*, there will be a *sudden rise in pressure* due to the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden rise in pressure has the effect of hammering action on the walls of the pipe. This *phenomenon of sudden rise in presssure is known as* **water hammer** or **hammer blow.** The magnitude of pressure rise depends on :

(*i*) *The speed at which valve is closed,*

(*ii*) *The velocity of flow,*

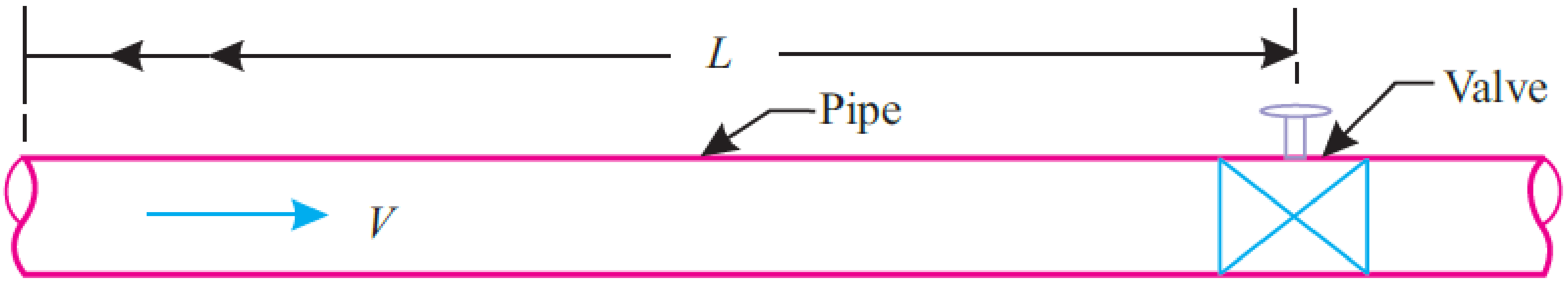
(*iii*) *The length of pipe, and*

(*iv*) *The elastic properties of the pipe material as well as that of the flowing fluid.*

The rise in pressure in some cases may be so large that the pipe may even burst and therefore it is essential to take into account this pressure rise in the design of the pipes.

12·12·1 Gradual Closure of Valve

Consider a long pipe carrying liquid (Fig. (12·45)) and provided with a valve which is closed gradually.



Let, A = Area of cross-section of the pipe,

L = Length of the pipe,

V = Velocity of flow of water in the pipe,

t = Time required to close the valve (in seconds), and

p = Intensity of pressure wave produced.

The mass of liquid contained in the pipe is $= \rho AL$

Assuming that the rate of closure of the valve is so adjusted that the liquid column in the pipe is brought to rest with a uniform retardation; from an intial velocity V to zero in time t seconds, we have:

$$\text{Retardation of water} = \frac{V - 0}{t} = \frac{V}{t}$$

$\therefore$ The axial force available for producing retardation

$$= \text{Mass} \times \text{retardation}$$

$$= \rho AL \times \frac{V}{t} \qquad \ldots(i)$$

Also, force due to pressure wave is $= p.A$ ...(*ii*)

Equating the two forces given by eqns. (*i*) and (*ii*), we have:

$$\rho AL \times \frac{V}{t} = p \times A$$

or,
$$p = \frac{\rho LV}{t} \quad \text{...(12·26)}$$

$\therefore$ Head of pressure,
$$H = \frac{p}{w} = \frac{\rho LV}{w \times t} = \frac{\rho LV}{\rho \cdot g \cdot t} = \frac{LV}{gt}$$

i.e.,
$$H = \frac{LV}{gt} \quad \text{...(12·27)}$$

(*i*) The closure of valve is said to be *gradual* when $t > \frac{2L}{C}$ ...(12·28)

(*ii*) The closure of valve is said to be *instantaneous* when $t < \frac{2L}{C}$ ...(12·29)

where, C = velocity of the pressure wave.

12·12·2 Instantaneous Closure of Valve in Rigid Pipes

Eqn. (11·26) indicates that when the valve is closed instantaneously (*i.e.,* $t = 0$), the inertia head should rise to infinity. However, in practice, it is not possible to close the valve instantaneously, as it always takes some time. Thus, even for a very rapid closure of the valve, as observed during experimentation, the pressure rise is quite finite and measurable. Moreover, eqn. (12·26) has been derived on the *assumption that the liquid is incompressible.* This assumption is *incorrect,* because at very high pressures even liquids get compressed to *some extent* and *behave like cmpressible fluids*.

Consider a pipe of length L and area of cross-section A (Fig. 12·45) carrying water which is flowing through it at a velocity V. When the valve is closed instantaneously the K.E. of the flowing water is converted into strain energy of water (neglecting effect of friction and assuming the pipe wall to be perfectly rigid).

$$\text{Loss of } K.E. = \frac{1}{2} mV^2 = \frac{1}{2} \rho AL \times V^2 \qquad (\because m = \rho \times A \times L)$$

$$\text{Gain of strain energy} = \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2} \frac{p^2}{K} \times AL$$

[where, k = Bulk modulus of elasticity of water, and
p = Intensity of pressure wave produced.]

Equating the loss of $K.E.$ to the gain of strain energy, we get:

$$\frac{1}{2}\rho AL \times V^2 = \frac{1}{2} \frac{p^2}{K} \times AL$$

or,
$$p^2 = \frac{1}{2} \rho ALV^2 \times \frac{2K}{AL} = \rho KV^2$$

$$\therefore \qquad p = \sqrt{\rho K V^2} = V\sqrt{\rho K} = V\sqrt{\frac{K\rho^2}{\rho}}$$

$$\text{or,} \qquad p = V\rho C \qquad \text{...(12·30)}$$

$$\left(\text{where, } C = \sqrt{\frac{K}{\rho}}, C \text{ being the } \textit{velocity of pressure wave.}\right)$$

12·12·3 Instantaneous Closure of Valve in Elastic Pipes

As shown in Fig. 12·45, consider a pipe of length L, diameter D, thickness t (small compared to diameter).

Let, p = Increase of pressure due to water hammer,

E = Modulus of elasticity of pipe material, and

$\frac{1}{m}$ = Poisson's ratio for pipe material.

When the valve is closed intantaneously, rise of pressure takes place due to which circumferential and longitudinal stresses are produced in the pipe wall; these stresses are given as (from knowledge of strength of materials):

$$\sigma_c = \frac{pD}{2t} \quad \text{and} \quad \sigma_l = \frac{pD}{4t}$$

where, σ_c = Circumferential stress, and

σ_l = Longitudinal stress.

Also, strain energy stored in the pipe material per unit volume is

$$= \frac{1}{2E}\left(\sigma_c^2 + \sigma_l^2 - \frac{2\sigma_c\,\sigma_l}{m}\right)$$

$$= \frac{1}{2E}\left[\left(\frac{pD}{2t}\right)^2 + \left(\frac{pD}{4t}\right)^2 - \frac{2 \times \frac{pD}{2t} \times \frac{pD}{4t}}{m}\right]$$

$$= \frac{1}{2E}\left[\frac{p^2D^2}{4t^2} + \frac{p^2D^2}{16t^2} - \frac{p^2D^2}{4mt^2}\right]$$

Assuming, $\frac{1}{m} = 1/4$, we have:

Strain energy per unit volume $= \frac{1}{2E}\left[\frac{p^2D^2}{4t^2} + \frac{p^2D^2}{16t^2} - \frac{p^2D^2}{16t^2}\right] = \frac{p^2D^2}{8Et^2}$

Total strain energy stored in pipe material

$$= \frac{p^2D^2}{8Et^2} \times \text{total volume of pipe material}$$

$$= \frac{p^2D^2}{8Et^2} \times \pi\, Dt \times L = \frac{p^2 \times D^3L}{8Et}$$

$$= \frac{p^2 \times \pi D^2 \times DL}{8Et} = \frac{p^2ADL}{2Et} \quad \left[\because A \text{ (area of the pipe)} = \frac{\pi}{4} \times D^2\right]$$

$$\text{Loss of K.E. of water} = \frac{1}{2}mV^2 = \frac{1}{2}\rho AL \times V^2$$

$$\text{Gain of strain energy in water} = \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$$

Also, The loss of K.E. of water = Gain of strain energy in water + strain energy stored in material.

$$\therefore \qquad \frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL + \frac{p^2 ADL}{2Et}$$

Dividing both sides by $\frac{AL}{2}$, we get:

$$\rho V^2 = \frac{p^2}{K} + \frac{p^2 D}{Et} = p^2\left(\frac{1}{K} + \frac{D}{Et}\right)$$

$$\therefore \qquad p^2 = \frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}$$

$$\text{or,} \qquad p = \sqrt{\frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} = V \times \sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} \qquad \text{...(12·31)}$$

12·12·4 Time required by Pressure Wave to travel from the Valve to the Tank and from Tank to Valve

$$\text{Time taken, } t = \frac{\text{Distance travelled from valve to tank and back}}{\text{Velocity of pressure wave}}$$

$$= \frac{L + L}{C} = \frac{2L}{C} \quad \textit{i.e.,} \quad t = \frac{2L}{C} \qquad \text{...(12·32)}$$

where, L = Length of the pipe, and

C = Velocity of pressure wave.

THANKS