Conversion between various number systems

Decimal numbers

- In the decimal number systems each of the ten digits, 0 through 9, represents a certain quantity. The position of each digit in a decimal number indicates the magnitude of the quantity represented and can be assigned a weight. The weights for whole numbers are positive powers of ten that increases from right to left, beginning with $10^0 = 1$
- ……………10^5 10^4 10^3 10^2 10^1 10^0
- For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1}.
- 10^2 10^1 10^0 . 10^{-1} 10^{-2} 10^{-3} ……..
- The value of a decimal number is the sum of digits after each digit has been multiplied by its weights as in following examples.

Q.1 Express the decimal number 87 as a sum of the values of each digit.

Sol: The digit 8 has a weight of 10, which is 10□, as indicated by its position. The digit 7 has a weight of 1, which is 10^o, as indicated by its position.

- $87 = (8 \times 10^1) + (7 \times 10^o)$
- $= (8 \times 10) + (7 \times 1)$
- $= 87$

Q.2.Express the decimal number 725.45 as a sum of the values of each digit.

- $725.45 = (7 \times 10^2) + (2 \times 10^1) + (5 \times 10^o) + (4 \times 10^{-1}) + (5 \times 10^{-2})$
- $= 700 + 20 + 5 + 0.4 + 0.05$
- $= 725.45$

BINARY NUMBERS

- The binary system is less complicated than the decimal system because it has only two digits, it is a base-two system. The two binary digits (bits) are 1 and 0. The position of a 1 or 0 in a binary number indicates its weight, or value within the number, just as the position of a decimal digit determines the value of that digit. The weights in a binary number are based on power of two as:
- ….. 2^4 2^3 2^2 2^1 2^0 . 2^{-1} 2^{-2}……….
- With 4 digits position we can count from zero to 15.In general, with n bits we can count up to a number equal to $2^n - 1$.
- Largest decimal number = $2^n - 1$

- A binary number is a weighted number. The right-most bit is the least significant bit (LSB) in a binary whole number and has a weight of $2^0 = 1$. The weights increases from right to left by a power of two for each bit. The left-most bit is the most significant bit (MSB); its weight depends on the size of the binary number.

Binary to Decimal Conversion

- The decimal value of any binary number can be found by adding the weights of all bits that are 1 and discarding the weights of all bits that are 0.

Example

- Let’s convert the binary whole number 101101 to decimal.
- Weight: 2^5 2^4 2^3 2^2 2^1 2^o
- Binary no: 1 0 1 1 0 1
- $101101 = 2^5 + 2^3 + 2^2 + 2^o = 32+8+4+1$
- $= (45)_{10}$

Decimal-to-Binary Conversion

- One way to find the binary number that is equivalent to a given decimal number is to determine the set of binary weights whose sum is equal to the decimal number. For example, decimal number 9, can be expressed as the sum of binary weights as follows:
- $9 = 8 + 1$ or $9 = 2^3 + 2^0$
- Placing 1s in the appropriate weight positions, 2^3 and 2^0 and 0s in the 2^2 and 2^1 positions determines the binary number for decimal 9.
- 2^3 2^2 2^1 2^0
- 1 0 0 1 Binary number for nine

Hexadecimal numbers

- The hexadecimal number system has sixteen digits and is used primarily as a compact way of displaying or writing binary numbers because it is very easy to convert between binary and hexadecimal. Long binary numbers are difficult to read and write because it is easy to drop or transpose a bit. Hexadecimal is widely used in computer and microprocessor applications. The hexadecimal system has a base of sixteen; it is composed of 16 digits and alphabetic characters.
- The maximum 3-digits hexadecimal number is FFF or decimal 4095 and maximum 4-digit hexadecimal number is FFFF or decimal 65535

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F