Job Sequencing

Introduction

In this Module, we discuss the problem of determining the sequence (order) in which a number of jobs should be performed on deferent machines in order to make effective use of available facilities and achieve greater output. If there are n jobs which are to be performed on m deferent machines then the problem is to determine the sequence of jobs, which minimizes the total elapsed time, that is, the time from the start of the first job up to the completion of the last job.

Basic Terminologies

In the following, we describe the basic terminologies which are commonly used in job sequencing:

Number of machines - It refers to the number of service facilities through which a job must pass before it is assumed to be completed.

Processing time - This is the time required by each job on each machine.

Processing order - This refers to the order (sequence) in which machines are required for completing the job.

Idle time on a machine - This is the time during which a machine does not have a job to process.

Total elapsed time - This is the time interval between starting the first job and completing the last job, including the idle time (if any), in a particular order by the given set of machines.

No passing rule - This means that the passing is not allowed, i.e., the same order of jobs is maintained over each machine. If n jobs are to be processed through two machines A and B in the order AB, then this means that each job will go to machine A first and then to B.

Assumptions

General assumptions for sequencing problems are as follows:

- The processing time on each machine is known.
- The time required to complete a job is independent of the order of jobs in which they are to be processed.
- No machine can process more than one job simultaneously.
- The time taken by each job in changing over from one machine to another is negligible.

- Each job, once started on a machine is to be performed up to completion on that machine.

- The order of completion of job has no significance, i.e., no job is to be given priority.

- A job starts on the machine as soon as the job and the machine both are idle.

Processing of *n* Jobs through 2 Machines

Suppose that n jobs are to be processed on 2 machines, say, A and B. Each job has to pass through the same sequence of operations in the same order, i.e., passing is not allowed. After a job is completely processed on machine A, it is assigned to machine
B. If machine B is not free at that moment, then the job enters in the waiting queue. Each job from the waiting queue is assigned to machine B according to FIFO (first in first out) discipline.
Let A_i = processing time for ith job on machine A B_i =
processing time for ith job on machine B
T = total elapsed time
The problem is to determine the sequence in which n jobs should be processed through machines A and B so that the total elapsed time (T) is minimum. In the following, we present a technique developed by Johnson and Bellman for determining an optimal sequence.

Johnson's Algorithm

Step 1 Select the minimum processing time out of all A_i 's and B_i 's. If it is A_r then do the rth job first. If it is B_s then do the sth job at last.

Step 2 If there is a tie in selecting the minimum of all the processing times, then such a situation is dealt with the following three ways:

(i) If the minimum of all the processing times is A_r, which is also equal to B_s, that is, $\min(A_i, B_i) = A_r = B_s$, then do the rth job first and sth job at last.

(ii) If $\min(A_i , B_i) = A_r$, but $A_r = A_k$, i.e., there is a tie for minimum among A_i 's, then select any one.

(iii) If $\min(A_i , B_i) = B_s$, but $B_s = B_t$, i.e., there is a tie for minimum among B_i 's, then select any one.

Step 3 Now, eliminate the job which has already been assigned from further consideration, and repeat steps 1 and 2 until an optimal sequence is found.

Example 1: Suppose that there are five jobs, each of which has to be processed on two machines *A* and *B* in the order *AB*. Processing times are given in the following table:

Job	Machine *A*	Machine *B*
1	6	3
2	2	7
3	10	8
4	4	9
5	11	5

Determine a sequence in which these jobs should be processed so as to minimize the total processing time.

Solution: The minimum time in the given table is 2, which corresponds to job 2 on machine *A*. So the allocation of jobs will start as $\begin{array}{|c|c|c|c|c|}\hline 2 & \ & \ & \ & \ \\ \hline\end{array}$. Now, we eliminate job 2 from further consideration. The reduced set of processing times is as follows:

Job	Machine *A*	Machine *B*
1	6	3
3	10	8
4	4	9
5	11	5

Now, the minimum time is 3 for job 1 on machine B. Therefore, this job would be done at last. The allocation of jobs till this stage would be job 1, $\begin{array}{|c|c|c|c|c|}\hline 2 & \ & \ & \ & 1 \\ \hline\end{array}$. After deletion of the reduced set of processing times is as follows:

Job	Machine *A*	Machine *B*
3	10	8
4	4	9
5	11	5

Similarly, by repeating the above steps, the optimal sequence is obtained as

2	4	3	5	1

. On the basis of this optimal sequence, the minimum elapsed time is obtained from the following table as 36 hours.

Job	Machine *A*		Machine *B*	
	Time in	Time out	Time in	Time out
2	0	2	2	9
4	2	6	9	18
3	6	16	18	26
5	16	27	27	32
1	27	33	33	36

Further, idle time for machine A = total elapsed time − time when the last job is out of machine $A = 36 - 33 = 3$ hours. Idle time for machine B = time at which the first job in a sequence finishes on machine A + (time when the ith job starts on machine B) − (time when the (i -1)th job finishes on machine B). Therefore, idle time for machine $B = 2 + (9 - 9) + (18 - 18) + (27 - 26) + (33 - 32) = 4$ hours

Example 2: A book binder company has one printing machine and one binding ma- chine. There are manuscripts of a number of deferent books. Processing times for printing and binding are given in the following table:

Book	Time (in hours)	
	Printing	Binding
A	5	2
B	1	6
C	9	7
D	3	8
E	10	4

Determine the sequence in which books should be processed on the machines so that the total time required is minimized.

Solution: The minimum time in the given table is 1 which corresponds to the book B on printing machine. Therefore, the allocation of jobs will start as

B				

Now, book B is eliminated. The reduced set of processing times is given in the following table:

Book	Time (in hours)	
	Printing	Binding
A	5	2
C	9	7
D	3	8
E	10	4

Now, the minimum time is 2 for book A on binding machine. Therefore, this job should be done at last. The allocation of jobs till this stage is reduced

B				*A*

The set of processing times is as follows:

Book	Time (in hours)	
	Printing	Binding
C	9	7
D	3	8
E	10	4

Similarly, by repeating the above steps, the optimal sequence is obtained as

B	*D*	*C*	*E*	*A*

Now, on the basis of this optimal sequence, we construct the following table:

Book	Printing		Binding	
	Time in	Time out	Time in	Time out
B	0	1	1	7
D	1	4	7	15
C	4	13	15	22
E	13	23	23	27
A	23	28	28	30

From the last column, we find that the minimum elapsed time is 30 hours. Idle time for printing process = total elapsed time − time when the last job is out of machine A $= 30 - 28 = 2$ hours. Idle time for binding process $= 1 + (7 - 7) + (15 - 15) + (23 - 22) + (28 - 27) = 3$ hours.

Example 3: There are seven jobs, each of which has to be processed on two machines A and B in the order AB. Processing times are given in the following table:

Job	Machine A	Machine B
1	3	8
2	12	10
3	15	10
4	6	6
5	10	12
6	11	1
7	9	3

Determine a sequence of these jobs that will minimize the total elapsed time. Also find total elapsed time and idle machines A and B.

Solution: The smallest processing time is 1 hour for job 6 on machine B. Thus job 6 will be processed last on machine A as shown below:

						6

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The reduced set of processing times becomes

Job	Machine A	Machine B
1	3	8
2	12	10
3	15	10
4	6	6
5	10	12
7	9	3

There are two equal minimal values: processing time of 3 hours for job 1 on machine A and processing time of 3 hours for job 7 on machine B. According to Johnson's rules, job 1 is scheduled first and job 7 next to 6 as shown below:

1					7	6

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The reduced set of processing times becomes

Job	Machine A	Machine B
2	12	10
3	15	10
4	6	6
5	10	12

Again there are two equal minimal values; processing time of 6 hours for job 4 on machine A as well as on machine B. We may choose arbitrarily to process job 4 next to 1 or next to job 7 as shown below:

1	4				7	6

or

1				4	7	6

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The reduced set of processing times becomes

Job	Machine A	Machine B
2	12	10
3	15	10
5	10	12

There are three equal minimal values: processing time of 10 hours for job 5 on ma- chine A and for jobs 2 and 3 on machine B. According to rules: job 5 is scheduled next to job 4 in the first or next to job 1 in the second schedule. Job 2 then is scheduled next to job 7 in the first schedule or next to job 4 in the second schedule. The optimal sequences are shown below:

or 1	4	5	3	2	7	6

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1	5	3	2	4	7	6

The calculation of both sequencing for total elapsed time for machines *A* and *B* are shown in the following tables:

Job	Machine *A*		Machine *B*	
	Time in	Time out	Time in	Time out
1	0	3	3	11
4	3	9	11	17
5	9	19	19	31
3	19	34	34	44
2	34	46	46	56
7	46	55	56	59
6	55	66	66	67

Job	Machine *A*		Machine *B*	
	Time in	Time out	Time in	Time out
1	0	3	3	11
5	3	13	13	25
3	13	28	28	38
2	28	40	40	50
4	40	46	50	56
7	46	55	56	59
6	55	66	66	67

From the above tables, we see that the total elapsed time in both sequencing is 67 hours and idle time for machine *A* is 1 hour; idle time for machine *B* is 17 hours.