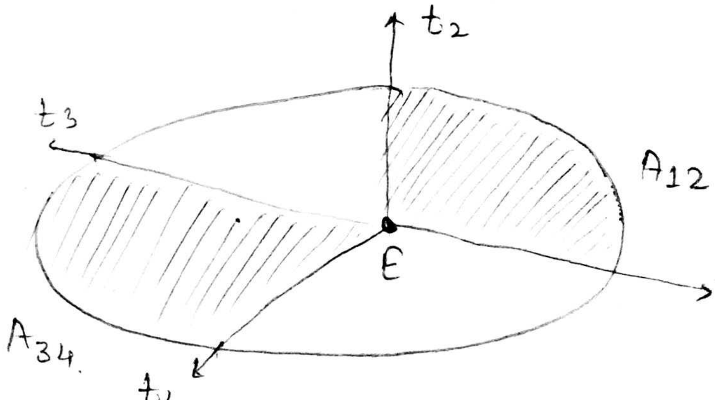
*. Kepler's three Law of Planetary Motion:-

Kepler give three law of planetary motion, they are given as bellow-

1. The orbit of any smaller body about a larger body is always an elipse, with the centre of mass of larger body as one of two foci.

2. The orbit of smaller body sweeps out equal areas in equal time as shown in the figure.



$$\boxed{A_{12} = A_{34}}$$

3. The square of the period of revolution of smaller body about the larger body is proportional to third power of semimajor axis of the of the orbital ellipse. Means that.

$$T^2 \propto a^3$$

$$T^2 = k a^3$$

$$\text{where } k = \text{Constant} = \frac{4\pi^2}{\mu}$$

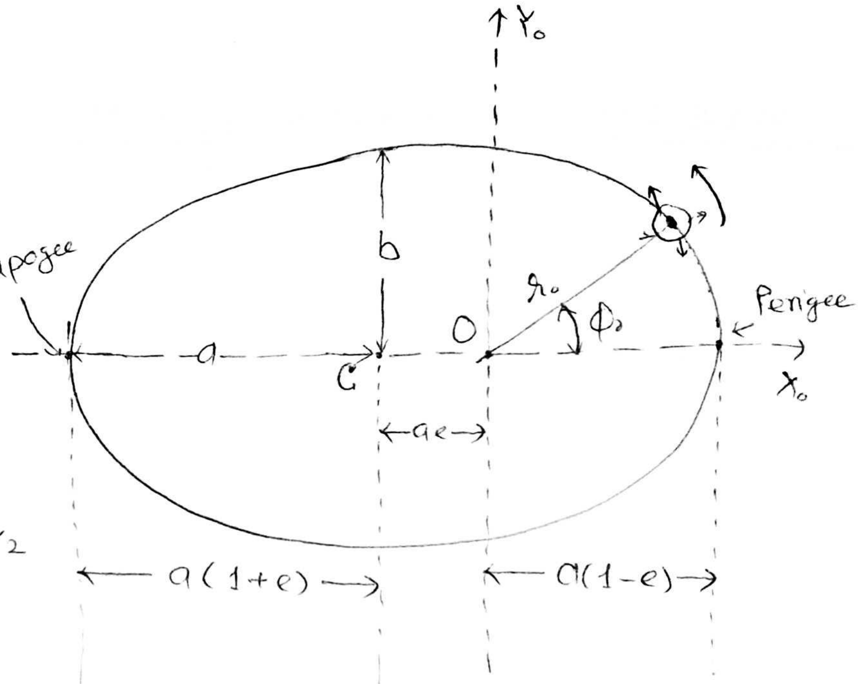
Describing the orbit of a satellite:- The equation of satellite orbit is given as bellow-

$$r_0 = \frac{p}{1 + e\,(\cos(\phi_0 - \theta_0))} \quad \text{------ (i)}$$

As we know that orbit of satellite is an ~~orb~~ ellipse, we can always choose, x_0, & y_0 so that, θ_0 is zero. Hence equation (i) becomes-

$$r_0 = \frac{p}{1 + e\cos\phi_0} \quad \text{------------ (ii)}$$

The ~~pl~~ path of satellite is shown in the figure. the length a & b of semimajor and semiminor axes are given bellow-



$a = p/(1-e^2)$

$b = a\,(1-e^2)^{1/2}$

$\Rightarrow a = \frac{p}{(1-e^2)}$

$\Rightarrow b = a\sqrt{(1-e^2)}$

$\therefore b^2 = a^2(1-e^2)$

O = Centre of earth.
C = Centre of ellipse.
These two centres never coincide unless e=0. When e=0, the ellipse becomes circle. a=b.

Perigee:- The point in the orbit where the satellite is closest to the earth is called perigee.

Apogee:- The point in the orbit where the satellite is farthest to the earth is called apogee.

The perigee and apogee are always exactly opposite to ...

other. To make θ_0 is equal to zero we have chosen x_0 such that the apogee and perigee both lie along it and therefore. x_0 axis is major axis of the ellipse.

The differential area swept by the vector r_0 from origin to satellite in time dt is given by-

$$dA = 0.5 r_0^2 \left(\frac{d\phi_0}{dt}\right) dt$$
$$= 0.5\, h\, dt$$

$\Rightarrow$ $\boxed{\text{Area swept in orbital revolution} = 0.5 h T}$

where h = ~~an mom~~
= magnitude of orbital angular momentum,

→ according to kepler's Second Law;-

" Radius vector of satellite swept out equal areas in equal times."

mean's that,

Area of ellipse = Area swept in one orbit - revolution.

$$\pi ab = 0.5 h T$$
$$= \frac{hT}{2}$$

$$T = \frac{2\pi ab}{h} = \frac{2\pi ab}{\sqrt{p\mu}}$$

$\Rightarrow p = b^2/a$

$$\therefore T = \frac{2\pi a\cancel{b}}{\cancel{b}\sqrt{\frac{\mu}{a}}}$$
$$= \frac{2\pi a^{3/2}}{\sqrt{\mu}}$$

$$\therefore \quad T^2 = \frac{4\pi^2 a^3}{\mu}$$

$$\boxed{T^2 = \frac{4\pi^2 a^3}{\mu}}$$

$$\Rightarrow \quad T^2 \propto a^3 \quad \text{------- (*)}$$

A

a (*) Keplers third Law of planet Motion.

b

The orbital period of a GEO satellite is exactly equal to the period of the earth that is 24 hour = 23 hour, 56 min, & 4.1 Sec. But, to an observer on the ground, the satellite appears to have an infinite orbital period. It always stays in the same place in the sky.

Locating the Satellite in the Orbit :- The equation of the orbit of satellite can be written as-

$$r_0 = \frac{p}{1 + e\cos\phi_0} = \frac{a(1-e^2)}{1 + e\cos\phi_0} \quad \text{------ (i)}$$

The angle ϕ_0 is measured from x_0 axis and called anomaly. x_0 such that passes through perigee. The angle ϕ_0 measured from perigee to the instaneous position of the satellite. The rectangular co-ordinates of the satellite are given by-

$$x_0 = r_0 \cos\phi_0$$

$$y_0 = r_0 \sin\phi_0$$

The angular velocity η of satellite can be gi