Four variable k-maps:

Four variable k-map expressions can have 2^4=16 possible combinations of input variables such as , ,------------ABCD with minterm designations m_0,m_1 -----------------m_{15} respectively in SOP form & A+B+C+D, A+B+C+ ,---------- + + + with maxterms M_0,M_1, ----------M_{15} respectively in POS form. It has 2^4=16 squares or cells.The binary number designations of rows & columns are in the gray code. Here follows 01 & 10 follows 11 called Adjacency ordering.

AB \ CD	00	01	11	10
00	0: $\bar{A}\bar{B}\bar{C}\bar{D}$	1: $\bar{A}\bar{B}\bar{C}D$	3: $\bar{A}\bar{B}CD$	2: $\bar{A}\bar{B}C\bar{D}$
01	4: $\bar{A}B\bar{C}\bar{D}$	5: $\bar{A}B\bar{C}D$	7: $\bar{A}BCD$	6: $\bar{A}BC\bar{D}$
11	12: $AB\bar{C}\bar{D}$	13: $AB\bar{C}D$	15: $ABCD$	14: $ABC\bar{D}$
10	8: $A\bar{B}\bar{C}\bar{D}$	9: $A\bar{B}\bar{C}D$	11: $A\bar{B}CD$	10: $A\bar{B}C\bar{D}$

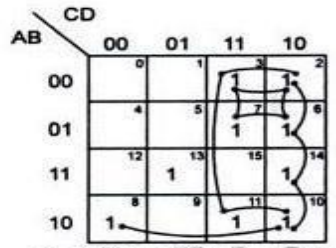
SOP form

AB \ CD	00	01	11	10
00	0: $A+B+C+D$	1: $A+B+C+\bar{D}$	3: $A+B+\bar{C}+\bar{D}$	2: $A+B+\bar{C}+D$
01	4: $A+\bar{B}+C+D$	5: $A+\bar{B}+C+\bar{D}$	7: $A+\bar{B}+\bar{C}+\bar{D}$	6: $A+\bar{B}+\bar{C}+D$
11	12: $\bar{A}+\bar{B}+C+D$	13: $\bar{A}+\bar{B}+C+\bar{D}$	15: $\bar{A}+\bar{B}+\bar{C}+\bar{D}$	14: $\bar{A}+\bar{B}+\bar{C}+D$
10	8: $\bar{A}+B+C+D$	9: $\bar{A}+B+C+\bar{D}$	11: $\bar{A}+B+\bar{C}+\bar{D}$	10: $\bar{A}+B+\bar{C}+D$

POS form

EX: Reduce using mapping the expression Σ m(2, 3, 6, 7, 8, 10, 11, 13, 14).

Start with the minterm with the least number of adjacencies. The minterm m_{13} has no adjacency. Keep it as it is. The m_8 has only one adjacency, m_{10}. Expand m_8 into a 2-square with m_{10}. The m_7 has two adjacencies, m_6 and m_3. Hence m_7 can be expanded into a 4-square with m_6, m_3 and m_2. Observe that, m_7, m_6, m_2, and m_3 form a geometric square. The m_{11} has 2 adjacencies, m_{10} and m_3. Observe that, m_{11}, m_{10}, m_3, and m_2 form a geometric square on wrapping the K-map. So expand m_{11} into a 4-square with m_{10}, m_3 and m_2. Note that, m_2 and m_3, have already become a part of the 4-square m_7, m_6, m_2, and m_3. But if m_{11} is expanded only into a 2-square with m_{10}, only one variable is eliminated. So m_2 and m_3 are used again to make another 4-square with m_{11} and m_{10} to eliminate two variables. Now only m_6 and m_{14} are left uncovered. They can form a 2-square that eliminates only one variable. Don't do that. See whether they can be expanded into a larger square. Observe that, m_2, m_6, m_{14}, and m_{10} form a rectangle. So m_6 and m_{14} can be expanded into a 4-square with m_2 and m_{10}. This eliminates two variables.



$f = AB\bar{C}D + A\bar{B}\bar{D} + \bar{A}C + \bar{B}C + C\bar{D}$

Five variable k-map:

Five variable k-map can have 2^5 =32 possible combinations of input variable as , E,--------ABCDE with minterms m_0, m_1-----m_{31} respectively in SOP & A+B+C+D+E, A+B+C+ ,---------- + + + + with maxterms M_0,M_1, ----------- M_{31} respectively in POS form. It has 2^5=32 squares or cells of the k-map are divided into 2 blocks of
16 squares each.The left block represents minterms from m_0 to m_{15} in which A is a 0, and the right block represents minterms from m_{16} to m_{31} in which A is 1.The 5-variable k-map may contain 2-squares, 4-squares , 8-squares , 16-squares or 32-squares involving these two blocks. Squares are also considered adjacent in these two blocks, if when superimposing one block on top of another, the squares coincide with one another.

Some possible 2-squares in a five-variable map are m_0, m_{16}; m_2, m_{18}; m_5, m_{21}; m_{15}, m_{31}; m_{11}, m_{27}.

Some possible 4-squares are m_0, m_2, m_{16}, m_{18}; m_0, m_1, m_{16}, m_{17}; m_0, m_4, m_{16}, m_{20}; m_{13}, m_{15}, m_{29}, m_{31}; m_5, m_{13}, m_{21}, m_{29}.

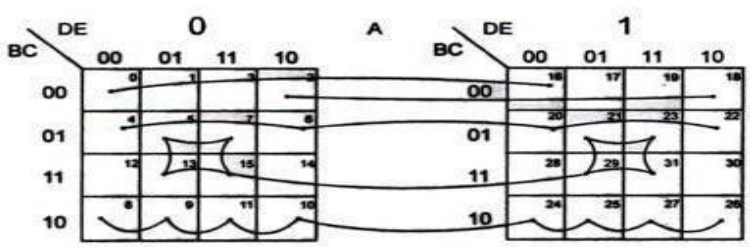
Some possible 8-squares are m_0, m_1, m_3, m_2, m_{16}, m_{17}, m_{19}, m_{18}; m_0, m_4, m_{12}, m_8, m_{16}, m_{20}, m_{28}, m_{24}; m_5, m_7, m_{13}, m_{15}, m_{21}, m_{23}, m_{29}, m_{31}.

The squares are read by dropping out the variables which change. Some possible

Grouping s is

(a) m_0, m_{16} = $\bar{B}\bar{C}\bar{D}\bar{E}$
(b) m_2, m_{18} = $\bar{B}\bar{C}D\bar{E}$
(c) m_4, m_6, m_{20}, m_{22} = $\bar{B}C\bar{E}$
(d) m_5, m_7, m_{13}, m_{15}, m_{21}, m_{23}, m_{29}, m_{31} = CE
(e) m_8, m_9, m_{10}, m_{11}, m_{24}, m_{25}, m_{26}, m_{27} = $B\bar{C}$

M_0, M_{16} = B + C + D + E
M_2, M_{18} = B + C + $\bar{D}$ + E
M_4, M_6, M_{20}, M_{22} = B + $\bar{C}$ + E
M_5, M_7, M_{13}, M_{15}, M_{21}, M_{23}, M_{29}, M_{31} = $\bar{C}$ + $\bar{E}$
M_8, M_9, M_{10}, M_{11}, M_{24}, M_{25}, M_{26}, M_{27} = $\bar{B}$ + C



Ex: F=∑m(0,1,4,5,6,13,14,15,22,24,25,28,29,30,31) is SOP

POS is F=πM(2,3,7,8,9,10,11,12,16,17,18,19,20,21,23,26,27)

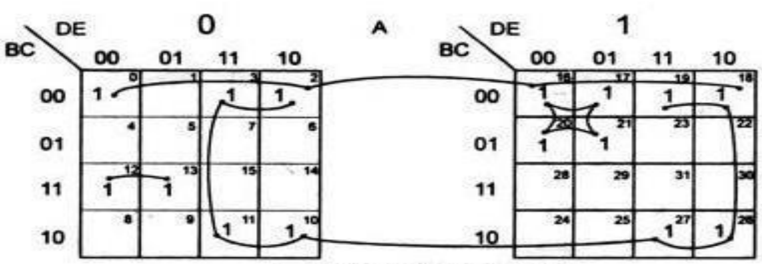
The real minimal expression is the minimal of the SOP and POS forms.

The reduction is done as

1. There is no isolated 1s
2. M_{12} can go only with m_{13}. Form a 2-square which is read as A‘BCD‘
3. M_0 can go with m_2,m_{16} and m_{18} . so form a 4-square which is read as B‘C‘E‘
4. M_{20},m_{21},m_{17} and m_{16} form a 4-square which is read as AB‘D‘
5. M2,m3,m18,m19,m10,m11,m26 and m27 form an 8-square which is read as C‘d
6. Write all the product terms in SOP form.

So the minimal expression is

F_{min}= A‘BCD‘+B‘C‘E‘+AB‘D‘+C‘D(16 inputs)



$f = \bar{A}BC\bar{D} + \bar{B}\bar{C}\bar{E} + A\bar{B}\bar{D} + \bar{C}D$

In the POS k-map ,the reduction is done as:

1. There are no isolated 0s

M_1 can go only with M_5. So, make a 2-square, which is read as $(A + B + D + \bar{E})$.

3. M_4 can go with M_5, M_7, and M_6 to form a 4-square, which is read as $(A + B + \bar{C})$.

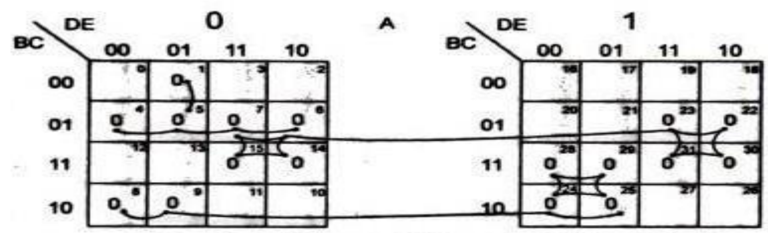
4.M_8

5. M_{28}

6.M_{30}

7. Sum terms in POS form. So the minimal expression in POS is

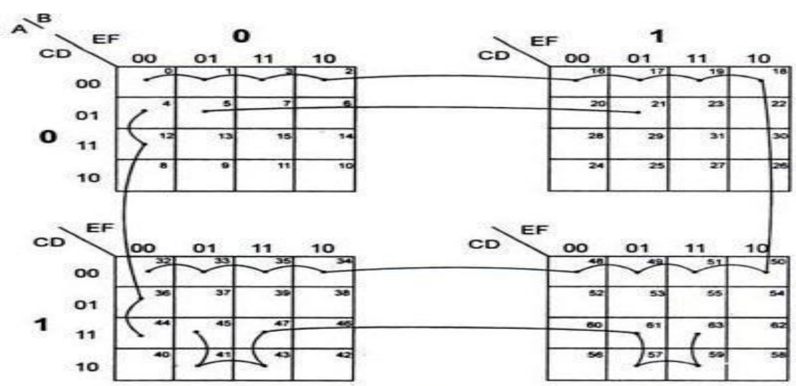
F_{min}= A‘BcD‘+B‘C‘E‘+AB‘D‘+C‘D



$$f = (A + B + D + \bar{E})(A + B + \bar{C})(\bar{B} + C + D)(\bar{A} + \bar{B} + D)(\bar{C} + \bar{D})$$

Six variable k-map:

Six variable k-map can have 2^6 =64 combinations as , ,---------
---ABCDEF with minterms m_0, m_1-----m_{63} respectively in SOP & (A+B+C+D+E+F), ---------- (
+ + + + +) with maxterms M_0,M_1, -----------M_{63} respectively in POS form. It has 2^6=64 squares or cells of the k-map are divided into 4 blocks of 16 squares each.



Some possible groupings in a six variable k-map

Don't care combinations: For certain input combinations, the value of the output is unspecified either because the input combinations are invalid or because the precise value of the output is of no consequence. The combinations for which the value of experiments are not specified are called don‘t care combinations are invalid or because the precise value of the output is of no consequence. The combinations for which the value of expressions is not specified are called don‘t care combinations or Optional Combinations, such expressions stand incompletely specified. The output is a don‘t care for these invalid combinations.

Ex:In XS-3 code system, the binary states 0000, 0001, 0010,1101,1110,1111 are unspecified. & never occur called don‘t cares.

A standard SOP expression with don‘t cares can be converted into a standard POS form by keeping the don‘t cares as they are & writing the missing minterms of the SOP form as the maxterms of the POS form viceversa.

Don‘t cares denoted by _X‘ or _φ‘

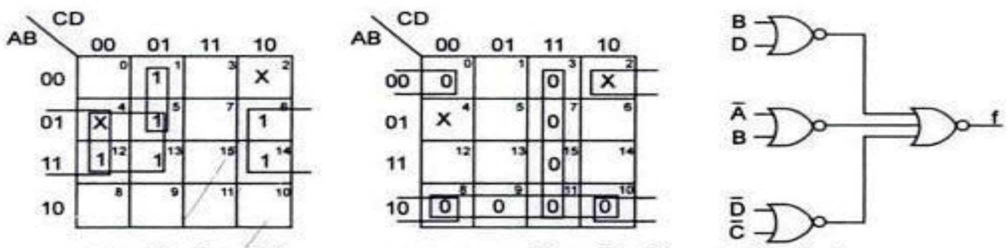
Ex:f=∑m(1,5,6,12,13,14)+d(2,4)

Or f=π M(0,3,7,9,10,11,15).πd(2,4)

SOP minimal form f_{min}= +B +

POS minimal form f_{min}=(B+D)(+B)(+D)

= + + + + (+



(a) $f = B\bar{C} + \bar{B}D + \bar{A}\bar{C}D$ (b) $f = (B + D)(\bar{A} + B)(\bar{C} + \bar{D})$ (c) NOR logic

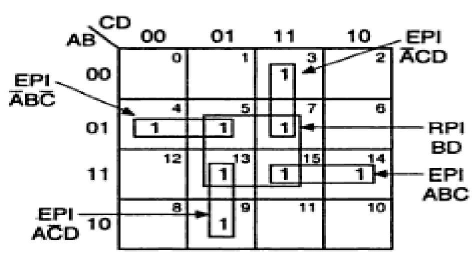
Prime implicants, Essential Prime implicants, Redundant prime implicants:

Each square or rectangle made up of the bunch of adjacent minterms is called a subcube. Each of these subcubes is called a Prime implicant (PI). The PI which contains at leastone which cannot be covered by any other prime implicants is called as Essential Prime implicant (EPI).The PI whose each 1 is covered at least by one EPI is called a Redundant Prime implicant (RPI). A PI which is neither an EPI nor a RPI is called a Selective Prime implicant (SPI).

The function has unique MSP comprising EPI is

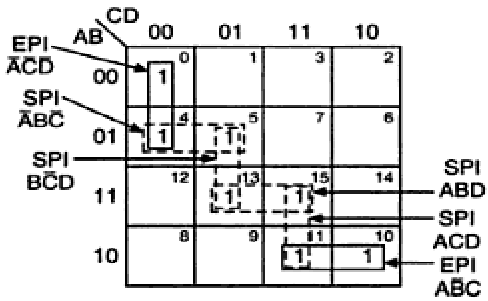
F(A,B,C,D)= CD+ABC+A D + B

The RPI ‗BD‘ may be included without changing the function but the resulting expression would not be in minimal SOP(MSP) form.



Essential and Redundant Prime Implicants

F(A,B,C,D)=∑m(0,4,5,10,11,13,15) SPI are marked by dotted squares, shows MSP form of a function need not be unique.



Essential and Selective Prime Implicants

Here, the MSP form is obtained by including two EPI's & selecting a set of SPI's to cover remaining uncovered minterms 5,13,15. & these can be covered as

(A) (4,5) &(13,15) ----------B +ABD
(B) (5,13) & (13,15) --------B D+ABD
(C) (5,13) & (15,11) -------B D+ACD

F(A,B,C,D)= +A C---------EPI's + B +ABD

(OR) F(A,B,C,D)= +A C---------EPI's + B D+ABD

(OR) F(A,B,C,D)= +A C---------EPI's + B D+ACD

False PI's Essential False PI's, Redundant False PI's & Selective False PI's:

The maxterms are called falseminterms. The PI's is obtained by using the maxterms are called False PI's (FPI). The FPI which contains at least one _0' which can't be covered by only other FPI is called an Essential False Prime implicant (ESPI)

F(A,B,C,D)= ∑m(0,1,2,3,4,8,12)

=π M(5,6,7,9,10,11,13,14,15)

F_{min}= (+)(+)(+)(+)

All the FPI, EFPI's as each of them contain atleast one _0' which can't be covered by any other FPI