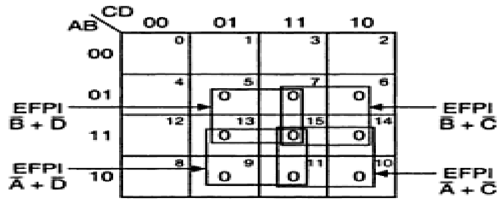
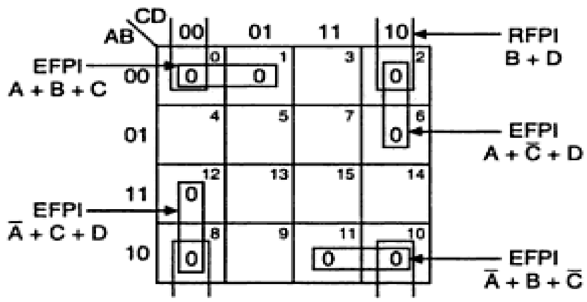


Essential False Prime implicants

Consider Function F(A,B,C,D)= π M(0,1,2,6,8,10,11,12)



Essential and Redundant False Prime Implicants

Mapping when the function is not expressed in minterms (maxterms):

An expression in k-map must be available as a sum (product) of minterms (maxterms). However if not so expressed, it is not necessary to expand the expression algebraically into its minterms (maxterms). Instead, expansion into minterms (maxterms) can be accomplished in the process of entering the terms of the expression on the k-map.

Limitations of Karnaugh maps:

- Convenient as long as the number of variables does not exceed six.
- Manual technique, simplification process is heavily dependent on the human abilities.

Quine-Mccluskey Method:

It also known as *Tabular method.* It is more systematic method of minimizing expressions of even larger number of variables. It is suitable for hand computation as well as computation by machines i.e., programmable. . The procedure is based on repeated application of the combining theorem.

PA+P =P (P is set of literals) on all adjacent pairs of terms, yields the set of all PI‘s from which a minimal sum may be selected.

Consider expression

∑m(0,1,4,5)= + C+A +A C

First, second terms & third, fourth terms can be combined

(+)+ (C+)= +A

Reduced to

(+)=

The same result can be obtained by combining m_0& m_4 & m_1&m_5 in first step & resulting terms in the second step .

Procedure:

- Decimal Representation
- Don‘t cares
- PI chart
- EPI
- Dominating Rows & Columns
- Determination of Minimal expressions in comples cases.

Branching Method:

EXAMPLE 3.29 Obtain the set of prime implicants for the Boolean expression $f = \Sigma m(0, 1, 6, 7, 8, 9, 13, 14, 15)$ using the tabular method.

Solution

Group the minterms in terms of the number of 1s present in them and write their binary designations. The procedure to obtain the prime implicants is shown in Table 3.3.

Table 3.3 Example 3.29

	Column 1		Column 2		Column 3
	Minterm	**Binary designation**		**A B C D**	**A B C D**
Index 0	0	0 0 0 0 ✓	0, 1 (1)	0 0 0 – ✓	0, 1, 8, 9 (1, 8) – 0 0 – Q
Index 1	1	0 0 0 1 ✓	0, 8 (8)	– 0 0 0 ✓	... ...
	8	1 0 0 0 ✓	1, 9 (8)	– 0 0 1 ✓	... ...
Index 2	6	0 1 1 0 ✓	8, 9 (1)	1 0 0 – ✓	6, 7, 14, 15 (1, 8) – 1 1 – P
	9	1 0 0 1 ✓	6, 7 (1)	0 1 1 – ✓	
Index 3	7	0 1 1 1 ✓	6, 14 (8)	– 1 1 0 ✓	
	13	1 1 0 1 ✓	9, 13 (4)	1 – 0 1 S	
	14	1 1 1 0 ✓	7, 15 (8)	– 1 1 1 ✓	
Index 4	15	1 1 1 1 ✓	13, 15 (2)	1 1 – 1 R	
			14, 15 (1)	1 1 1 – ✓	

Comparing the terms of index 0 with the terms of index 1 of column 1, $m_0(0000)$ is combined with $m_1(0001)$ to yield 0, 1 (1), i.e. 000 –. This is recorded in column 2 and 0000 and 0001 are checked off in column 1. $m_0(0000)$ is combined with $m_8(1000)$ to yield 0, 8 (8), i.e. – 000. This is recorded in column 2 and 1000 is checked off in column 1. Note that 0000 of column 1 has already been checked off. No more combinations of terms of index 0 and index 1 are possible. So, draw a line below the last combination of these groups, i.e. below 0, 8 (8), – 000 in column 2. Now 0, 1 (1), i.e. 000 – and 0, 8 (8), i.e. – 000 are the terms in the first group of column 2.

Comparing the terms of index 1 with the terms of index 2 in column 1, $m_1(0001)$ is combined with $m_9(1001)$ to yield 1, 9 (8), i.e. – 001. This is recorded in column 2 and 1001 is checked off in column 1 because 0001 has already been checked off. $m_8(1000)$ is combined with $m_9(1001)$ to yield 8, 9 (1), i.e. 100 –. This is recorded in column 2. 1000 and 1001 of column 1 have already been checked off. So, no need to check them off again. No more combinations of terms of index 1 and index 2 are possible. So, draw a line below the last combination of these groups, i.e. 8, 9 (1),

-- 001 in column 2. Now 1, 9 (8), i.e. – 001 and 8, 9 (1), i.e. 100– are the terms in the second group of column 2.

Similarly, comparing the terms of index 2 with the terms of index 3 in column 1,

$m_6(0110)$ and $m_7(0111)$ yield 6, 7 (1), i.e. 011–. Record it in column 2 and check off 6(0110) and 7(0111).

$m_6(0110)$ and $m_{14}(1110)$ yield 6, 14 (8), i.e. –110. Record it in column 2 and check off 6(0110) and 14(1110).

$m_9(1001)$ and $m_{13}(1101)$ yield 9, 13 (4), i.e. 1–01. Record it in column 2 and check off 9(1001) and 13(1101).

So, 6, 7 (1), i.e. 011–, and 6, 14 (8), i.e. –110 and 9, 13 (4), i.e. 1–01 are the terms in group 3 of column 2. Draw a line at the end of 9, 13 (4), i.e. 1–01.

Also, comparing the terms of index 3 with the terms of index 4 in column 1,

$m_7(0111)$ and $m_{15}(1111)$ yield 7, 15 (8), i.e. –111. Record it in column 2 and check off 7(0111) and 15(1111).

$m_{13}(1101)$ and $m_{15}(1111)$ yield 13, 15 (2), i.e. 11–1. Record it in column 2 and check off 13 and 15.

$m_{14}(1110)$ and $m_{15}(1111)$ yield 14, 15 (1), i.e. 111–. Record it in column 2 and check off 14 and 15.

So, 7, 15 (8), i.e. –111, and 13, 15 (2), i.e. 11–1 and 14, 15 (1), i.e. 111– are the terms in group 4 of column 2. Column 2 is completed now.

Comparing the terms of group 1 with the terms of group 2 in column 2, the terms 0, 1 (1), i.e. 000– and 8, 9 (1), i.e. 100– are combined to form 0, 1, 8, 9 (1, 8), i.e. –00–. Record it in group 1 of column 3 and check off 0, 1 (1), i.e. 000–, and 8, 9 (1), i.e. 100– of column 2. The terms 0, 8 (8), i.e. –000 and 1, 9 (8), i.e. –001 are combined to form 0, 1, 8, 9 (1, 8), i.e. –00–. This has already been recorded in column 3. So, no need to record again. Check off 0, 8 (8), i.e. –000 and 1, 9 (8), i.e. –001 of column 2. Draw a line below 0, 1, 8, 9 (1, 8), i.e. –00–. This is the only term in group 1 of column 3. No term of group 2 of column 2 can be combined with any term of group 3 of column 2. So, no entries are made in group 2 of column 2.

Comparing the terms of group 3 of column 2 with the terms of group 4 of column 2, the terms 6, 7 (1), i.e. 011–, and 14, 15 (1), i.e. 111– are combined to form 6, 7, 14, 15 (1, 8), i.e. –11–. Record it in group 3 of column 3 and check off 6, 7 (1), i.e. 011– and 14, 15 (1), i.e. 111– of column 2. The terms 6, 14 (8), i.e. –110 and 7, 15 (8), i.e. –111 are combined to form 6, 7, 14, 15 (1, 8), i.e. –11–. This has already been recorded in column 3; so, check off 6, 14 (8), i.e. –110 and 7, 15 (8), i.e. –111 of column 2.

Observe that the terms 9, 13 (4), i.e. 1–01 and 13, 15 (2), i.e. 11–1 cannot be combined with any other terms. Similarly in column 3, the terms 0, 1, 8, 9 (1, 8), i.e. –00– and 6, 7, 14, 15 (1, 8), i.e. –11– cannot also be combined with any other terms. So, these 4 terms are the prime implicants.

The terms, which cannot be combined further, are labelled as P, Q, R, and S. These form the set of prime implicants.

EX:

Obtain the minimal expression for $f = \Sigma\, m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$ using the tabular method.

Solution

The procedure to obtain the set of prime implicants is illustrated in Table 3.4.

Table 3.4 Example 3.30

Step 1		Step 2	Step 3	
Index 1	1 ✓	1, 3 (2) ✓	1, 3, 5, 7 (2, 4)	T
	2 ✓	1, 5 (4) ✓	1, 5, 9, 13 (4, 8)	S
	8 ✓	1, 9 (8) ✓	2, 3, 6, 7 (1, 4)	R
Index 2	3 ✓	2, 3 (1) ✓	8, 9, 12, 13 (1, 4)	Q
	5 ✓	2, 6 (4) ✓	5, 7, 13, 15 (2, 8)	P
	6 ✓	8, 9 (1) ✓		
	9 ✓	8, 12 (4) ✓		
	12 ✓	3, 7 (4) ✓		
Index 3	7 ✓	5, 7 (2) ✓		
	13 ✓	5, 13 (8) ✓		
Index 4	15 ✓	6, 7 (1) ✓		
		9, 13 (4) ✓		
		12, 13 (1) ✓		
		7, 15 (8) ✓		
		13, 15 (2) ✓		

The non-combinable terms P, Q, R, S and T are recorded as prime implicants.

$$P \rightarrow 5, 7, 13, 15\ (2, 8) = X\,1\,X\,1 = BD$$

(Literals with weights 2 and 8, i.e. C and A are deleted. The lowest minterm is $m_5 (5 = 4 + 1)$. So, literals with weights 4 and 1, i.e. B and D are present in non-complemented form. So, read it as BD.)

$$Q \rightarrow 8, 9, 12, 13\ (1, 4) = 1\,X\,0\,X = A\overline{C}$$

(Literals with weights 1 and 4, i.e. D and B are deleted. The lowest minterm is m_8. So, literal with weight 8 is present in non-complemented form and literal with weight 2 is present in complemented form. So, read it as $A\overline{C}$.)

$$R \rightarrow 2, 3, 6, 7\ (1, 4) = 0\,X\,1\,X = \overline{A}C$$

(Literals with weights 1 and 4, i.e. D and B are deleted. The lowest minterm is m_2. So, literal with weight 2 is present in non-complemented form and literal with weight 8 is present in complemented form. So, read it as $\overline{A}C$.)

$$S \rightarrow 1, 5, 9, 13\ (4, 8) = X\,X\,0\,1 = \overline{C}D$$

(Literals with weights 4 and 8, i.e. B and A are deleted. The lowest minterm is m_1. So, literal with weight 1 is present in non-complemented form and literal with weight 2 is present in complemented form. So, read it as $\overline{C}D$.)

$$T \rightarrow 1, 3, 5, 7\ (2, 4) = 0\,X\,X\,1 = \overline{A}D$$

(Literals with weights 2 and 4, i.e. C and B are deleted. The lowest minterm is 1. So, literal with weight 1 is present in non-complemented form and literal with weight 8 is present in complemented form. So, read it as $\overline{A}D$.)

The prime implicant chart of the expression

$$f = \Sigma\, m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$$

is as shown in Table 3.5. It consists of 11 columns corresponding to the number of minterms and 5 rows corresponding to the prime implicants P, Q, R, S, and T generated. Row R contains four ×s at the intersections with columns 2, 3, 6, and 7, because these minterms are covered by the prime implicant R. A row is said to cover the columns in which it has ×s. The problem now is to select a minimal subset of prime implicants, such that each column contains at least one × in the rows corresponding to the selected subset and the total number of literals in the prime implicants selected is as small as possible. These requirements guarantee that the number of unions of the selected prime implicants is equal to the original number of minterms and that, no other expression containing fewer literals can be found.

Table 3.5 Example 3.30: Prime implicant chart

		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	1	2	3	5	6	7	8	9	12	13	15
*P → 5, 7, 13, 15 (2, 8)				×		×				×	×
*Q → 8, 9, 12, 13 (1, 4)							×	×	×	×	
*R → 2, 3, 6, 7 (1, 4)		×	×		×	×					
S → 1, 5, 9, 13 (4, 8)	×			×				×		×	
T → 1, 3, 5, 7 (2, 4)	×		×	×		×					

In the prime implicant chart of Table 3.5, m_2 and m_6 are covered by R only. So, R is an essential prime implicant. So, check off all the minterms covered by it, i.e. m_2, m_3, m_6, and m_7. Q is also an essential prime implicant because only Q covers m_8 and m_{12}. Check off all the minterms covered by it, i.e. m_8, m_9, m_{12}, and m_{13}. P is also an essential prime implicant, because m_{15} is covered only by P. So check off m_{15}, m_5, m_7, and m_{13} covered by it. Thus, only minterm 1 is not covered. Either row S or row T can cover it and both have the same number of literals. Thus, two minimal expressions are possible.

$$P + Q + R + S = BD + A\overline{C} + \overline{A}C + \overline{C}D$$

or

$$P + Q + R + T = BD + A\overline{C} + \overline{A}C + \overline{A}D$$