Boolean Algebra: In 1854, George Boole developed an algebraic system now called Boolean algebra. In 1938, Claude E. Shannon introduced a two-valued Boolean algebra called switching algebra that represented the properties of bistable electrical switching circuits. For the formal definition of Boolean algebra, we shall employ the postulates formulated by E. V. Huntington in 1904.

Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators called OR, AND, and one unary operator NOT. It is the basic mathematical tool in the analysis and synthesis of switching circuits. It is a way to express logic functions algebraically.

Boolean algebra, like any other deductive mathematical system, may be defined with aset of elements, a set of operators, and a number of unproved axioms or postulates. A *set* of elements is anycollection of objects having a common property. If **S** is a set and *x* and *y* are certain objects, then **x Î S**denotes that *x* is a member of the set **S**, and *y* ÏS denotes that *y* is not an element of **S**. A set with adenumerable number of elements is specified by braces: **A** = {1,2,3,4}, *i.e.* the elements of set **A** are thenumbers 1, 2, 3, and 4. A *binary operator* defined on a set S of elements is a rule that assigns to each pair ofelements from S a unique element from S._ Example: In *a**b=*c*, we say that * is a binary operator if it specifies a rule for finding *c* from the pair (*a*,*b*)and also if *a*, *b*, *c* Î S.

Axioms and laws of Boolean algebra

Axioms or Postulates of Boolean algebra are a set of logical expressions that we accept without proof and upon which we can build a set of useful theorems.

	AND Operation	OR Operation	NOT Operation
Axiom1 :	0.0=0	0+0=0	$\overline{0}$=1
Axiom2:	0.1=0	0+1=1	$\overline{1}$=0
Axiom3:	1.0=0	1+0=1	
Axiom4:	1.1=1	1+1=1	

AND Law	OR Law
Law1: A.0=0 (Null law)	Law1: A+0=A
Law2: A.1=A (Identity law)	Law2: A+1=1
Law3: A.A=A (Impotence law)	Law3: A+A=A (Impotence law)

CLOSURE: The Boolean system is *closed* with respect to a binary operator if for every pair of Boolean values,it produces a Boolean result. For example, logical AND is closed in the Boolean system because it accepts only Boolean operands and produces only Boolean results.

_ A set *S* is closed with respect to a binary operator if, for every pair of elements of *S*, the binary operator specifies a rule for obtaining a unique element of *S*.

_ For example, the set of natural numbers N = {1, 2, 3, 4, … 9} is closed with respect to the binary operator plus (+) by the rule of arithmetic addition, since for any *a*, *b* Î N we obtain a unique *c* Î N by the operation *a* + *b* = c.

ASSOCIATIVE LAW:

A binary operator * on a set S is said to be associative whenever $(x * y) * z = x * (y * z)$ for all x, y, z Î S,

forall Boolean values x, y and z.

COMMUTATIVE LAW:

A binary operator * on a set S is said to be commutative whenever $x * y = y * x$ for all x, y, z ϵ S

IDENTITY ELEMENT:

A set S is said to have an identity element with respect to a binary operation * on S if there exists an element

e ϵ S with the property $e * x = x * e = x$ for every x ϵ S

BASIC IDENTITIES OF BOOLEAN ALGEBRA

- *Postulate 1(Definition)*: A Boolean algebra is a closed algebraic system containing a set K of two or more

  elements and the two operators · and + which refer to logical AND and logical OR •$x + 0 = x$

- $x \cdot 0 = 0$

- $x + 1 = 1$

- $x \cdot 1 = 1$

- $x + x = x$

- $x \cdot x = x$

- $x + x' = x$

- $x \cdot x' = 0$

- $x + y = y + x$

- $xy = yx$

- $x + (y + z) = (x + y) + z$

- $x (yz) = (xy) z$

- $x (y + z) = xy + xz$

- $x + yz = (x + y)(x + z)$

- $(x + y)' = x'y'$

- $(xy)' = x' + y'$

- *(x')' = x*

DeMorgan's Theorem

(a) $(a + b)' = a'b'$

(b) $(ab)' = a' + b'$

Generalized DeMorgan's Theorem

(a) $(a + b + \dots z)' = a'b' \dots z'$
(b) $(a.b \dots z)' = a' + b' + \dots z'$

Basic Theorems and Properties of Boolean algebra Commutative law

Law1: $A+B=B+A$ Law2: $A.B=B.A$

Associative law

Law1: $A + (B +C) = (A +B) +C$ Law2: $A(B.C) = (A.B)C$

Distributive law

Law1: $A.(B + C) = AB+ AC$ Law2: $A + BC = (A + B).(A +C)$

Absorption law

Law1: $A +AB =A$ Law2: $A(A +B) = A$

Solution: $\underline{A}(1+B)$ Solution: $A.A+A.B$
        A $A+A.B$
            _ $A(1+B)$
                                  A

Consensus Theorem

Theorem1. $AB+ A'C + BC = AB + A'C$ Theorem2. $(A+B). (A'+C).(B+C) =(A+B).(A'+C)$

The BC term is called the consensus term and is redundant. The consensus term is formed from a PAIR OF TERMS in which a variable (A) and its complement (A') are present; the consensus term is formed by multiplying the two terms and leaving out the selected variable and its complement

Consensus Theorem1 Proof:

  $AB+A'C+BC=AB+A'C+(A+A')BC$
          $=AB+A'C+ABC+A'BC$

$$=AB(1+C)+A'C(1+B)$$
$$= AB+ A'C$$

Principle of Duality

Each postulate consists of two expressions statement one expression is transformed into the other by interchanging the operations (+) and (·) as well as the identity elements 0 and 1. Such expressions are known as duals of each other.
If some equivalence is proved, then its dual is also immediately true.

If we prove: $(x.x)+(x'+x')=1$, then we have by duality: $(x+x)\cdot(x'.x')=0$

The Huntington postulates were listed in pairs and designated by part (a) and part (b) in below table.

Table for Postulates and Theorems of Boolean algebra

Part-A	Part-B
$A+0=A$	$A.0=0$
$A+1=1$	$A.1=A$
$A+A=A$ (Impotence law)	$A.A=A$ (Impotence law)
$A+ \bar{A}=1$	$A. \bar{A}=0$
$\bar{\bar{A}}=A$ (double inversion law)	--
Commutative law: $A+B=B+A$	$A.B=B.A$
Associative law: $A + (B +C) = (A +B) +C$	$A(B.C) = (A.B)C$
Distributive law: $A.(B + C) = AB+ AC$	$A + BC = (A + B).(A +C)$
Absorption law: $A +AB =A$	$A(A +B) = A$
DeMorgan Theorem: $\overline{(A+B)} = \bar{A}.\bar{B}$	$\overline{(A.B)} = = \bar{A}+\bar{B}$
Redundant Literal Rule: $A+ \bar{A}B=A+B$	$A.(\bar{A}+B)=AB$
Consensus Theorem: $AB+ A'C + BC = AB + A'C$	$(A+B). (A'+C).(B+C) =(A+B).(A'+C)$