

$$T^2 = \frac{4\pi^2 a^3}{\mu}$$

$$\boxed{T^2 = \frac{4\pi^2 a^3}{\mu}}$$

$$\Rightarrow T^2 \propto a^3 \text{ ----- (*)}$$

(*) Kepler's third Law of planet motion.

The orbital period of a GEO satellite is exactly equal to the period of the earth that is 24 hour = 23 hour, 56 min, & 4.1 Sec. But, to an observer on the ground, the satellite appears to have an infinite orbital period. It always stays in the same place in the sky.

Locating the satellite in the orbit :- The equation of the orbit of satellite can be

written as-

$$r_0 = \frac{p}{1 + e \cos \phi_0} = \frac{a(1 - e^2)}{1 + e \cos \phi_0} \text{ ----- (i)}$$

The angle ϕ_0 is measured from x. axis and called — anomaly. x. such that passes through perigee. The angle ϕ_0 measured from perigee to the instantaneous position P of the satellite. The rectangular co-ordinates of the satellite are given by-

$$x_0 = r_0 \cos \phi_0$$

$$y_0 = r_0 \sin \phi_0$$

The angular velocity η of satellite can be given as-

$$\boxed{\eta = \frac{2\pi}{T} = a_1^{1/2} a^{-3/2}}$$

average angular velocity.

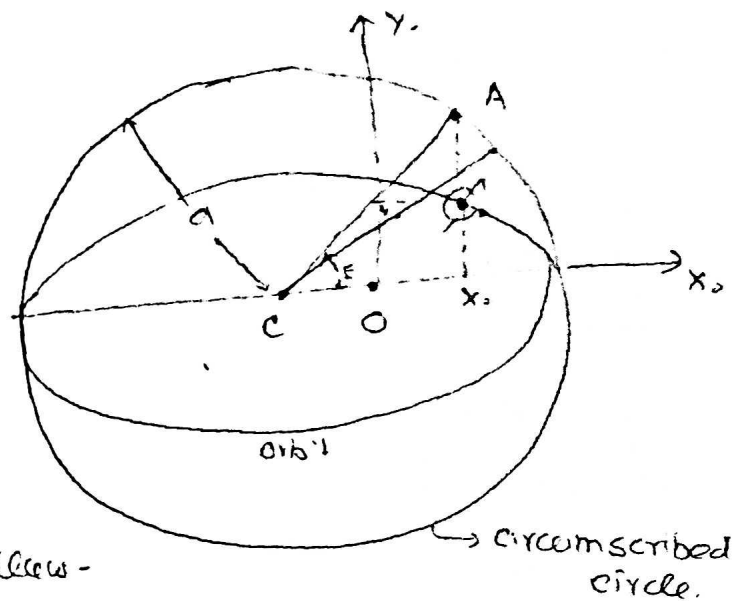
$$\boxed{\eta = \frac{1}{a} \sqrt{\frac{\mu}{a}}}$$

Where T = Orbit period of Satellite.

= Time taken by Satellite to complete a revolution.

In the Figure C is centre of both ellipse and — circumscribed circle. Point O is the centre of earth.

E = is the angle from the x_0 axis to the line joining C and A .



Now the r_0 given as below—

$$r_0 = a(1 - e \cos E)$$

$$a - r_0 = ae \cos E$$

Now we can develop a relation between eccentric anomaly E and angular velocity η —

$$\eta dt = (1 - e \cos E) dE, \quad \text{--- (ii)}$$

Let t_p is the time at perigee. This is simultaneously the time of closest approach to the earth —> The time when satellite crossing the x_0 axis —> The time when E is zero.

t_p = time at perigee
 = The time when satellite crossing the x_0 axis.
 = The time when E is zero.

Integrating equation (1) both side we get-

$$\underbrace{\eta(t - t_p)} = E - e \sin E \quad \text{--- (2)}$$

The term $\eta(t - t_p)$ called mean anomaly M .
Hence equation (2) becomes-

$$M = \eta(t - t_p) = (E - e \sin E)$$

The mean anomaly M is the arc length that the satellite would have travel since the perigee passage. If satellite moving on the circumscribed circle at the mean velocity angular velocity η .

⇒ If we know that the time of perigee t_p , the eccentricity e , and the length of semimajor axis a ,

→ Now we have the necessary equations to determine the co-ordinates (x_0, ϕ_0) and (x_0, y_0) of the satellite in the orbital plane. The process is follows-

* ⇒ Calculate avg. angular velocity by using-

$$\eta = \frac{1}{a} \sqrt{\frac{\mu}{a}}$$

* ⇒ Calculate mean anomaly M by using -

$$M = \eta(t - t_p)$$

* ⇒ Calculate eccentric anomaly E by using-

$$M = E - e \sin E$$

* $\Rightarrow$ Calculate r_0 by using following equation-

$$a - r_0 = ae \cos E$$

$$r_0 = a - ae \cos E$$

$$r_0 = a(1 - e \cos E)$$

* $\Rightarrow$ Calculating value of ϕ_0 from following equation-

$$r_0 = \frac{a(1 - e^2)}{1 + e \cos \phi_0}$$

* $\Rightarrow$ Calculate values of x_0 & y_0 by using following equations

$$x_0 = r_0 \cos \phi_0$$

$$y_0 = r_0 \sin \phi_0$$

Velocity of Satellite:- * Let us consider Satellite moving with velocity v in orbit. The velocity v can be given as below-

$$v^2 = \left(\frac{dx_0}{dt}\right)^2 + \left(\frac{dy_0}{dt}\right)^2 \quad \dots \dots \dots (i)$$

$$\text{where } x_0 = r_0 \cos \phi_0 \quad \&$$

$$y_0 = r_0 \sin \phi_0$$

$$\Rightarrow \frac{dx_0}{dt} = \cos \phi_0 \frac{dr_0}{dt} - r_0 \sin \phi_0 \frac{d\phi_0}{dt} \quad \dots \dots \dots (ii)$$

$$\Rightarrow \frac{dy_0}{dt} = \sin \phi_0 \frac{dr_0}{dt} + r_0 \cos \phi_0 \frac{d\phi_0}{dt} \quad \dots \dots \dots (iii)$$

Squaring and adding equation (ii) & (iii), we get

$$v^2 = \left(\frac{dr_0}{dt}\right)^2 + r_0^2 \left(\frac{d\phi_0}{dt}\right)^2 \quad \dots \dots \dots (iv)$$